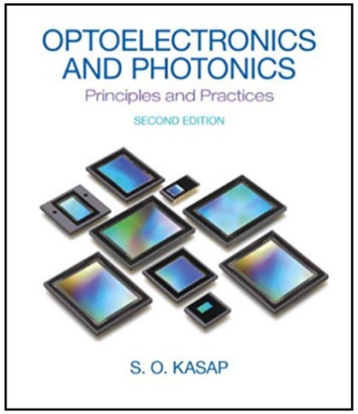
Optoelectronics-I

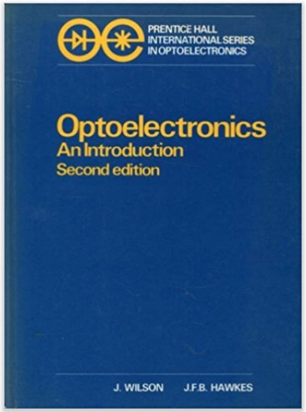
Chapter-12

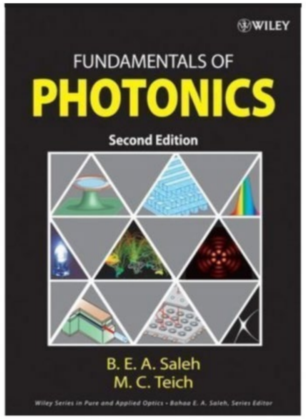
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







Department of Electrical and Electronics Enginnering, Ankara University
Golbasi, ANKARA

Optical Resonators
and
Interferometers

Objectives

When you finish this lesson you will be able to:

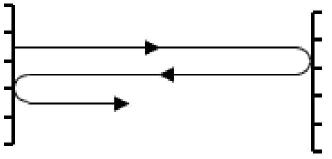
- ✓ Describe the Fundemantals of Optical Resonators
- ✓ Define the Planar Mirror Resonators
- ✓ Explain the Mode number and Free Spectral range
- ✓ Explain the Fabry-Perot (Etalon) Resonator
- ✓ Define the Quality factor (Q)
- ✓ Explain the Mach–Zehnder Interferometer
- ✓ Explain Michelson interferometer
- ✓ Explain Sagnac Interferometer and Fiber Optical Gyroscope

Optical Resonators

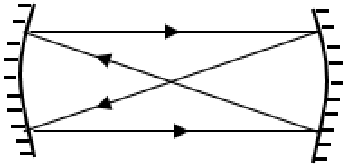
An optical resonator is an optical device that is used to confine and concentrate light. As an optical counterpart of an electronic resonant circuit, this device confines and stores light at certain resonance frequencies.

Most important application is producing a laser light. *Optical resonators* or *Optical cavities* are a major component of lasers, surrounding the gain medium and providing feedback of the laser light.

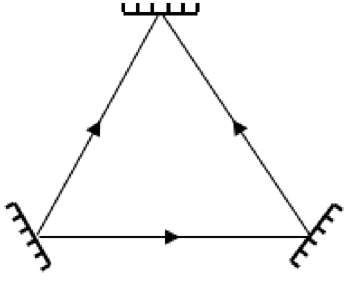
The light circulates or is repeatedly reflected within an optical resonators, without escaping.



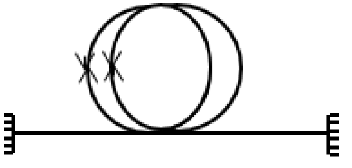
Planar Mirror



Spherical Mirror



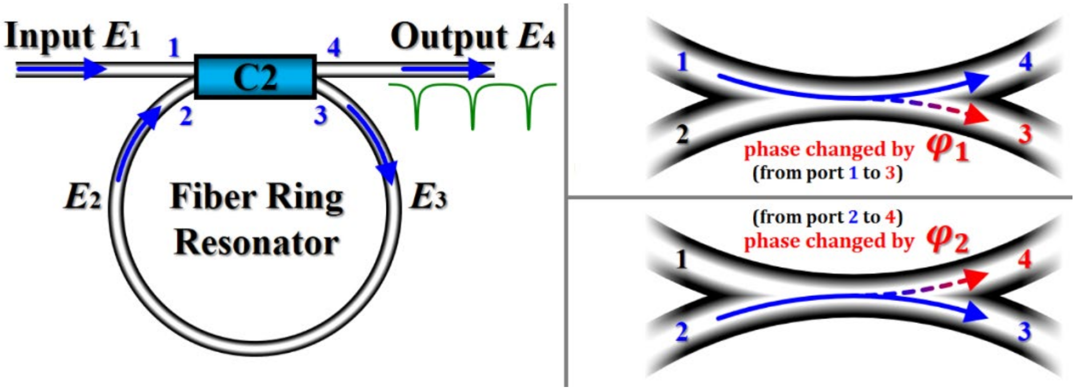
Ring Mirror



Fiber Ring

Optical Resonators

- ✓ The Mirror resonators include 2 or 3 mirrors
- ✓ TIR (Total Internal Reflection) are used in the Dielectric Resonators instead of mirrors, for example Fiber rings, integrated optic rings, Microdisks, microspheres



From Zhuoyan Li at. al, “Analysis of Resonance Asymmetry Phenomenon in Resonant Fiber Optic Gyro”, Sensor 2018.

There are two key parameters:

Modal volume (V)

- ✓ V is the volume occupied by confined optical mode

Quality factor (Q)

- ✓ Q is proportional to storage time in units of optical period
- ✓ V and Q represent the degrees of spatial and temporal confinements, respectively
- ✓ Large Q means low-loss resonator

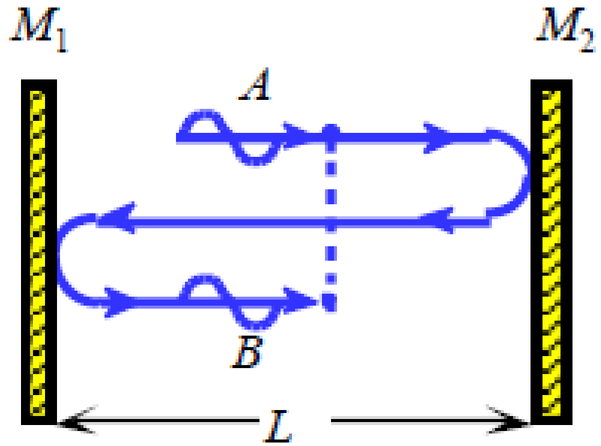
Optical Resonators

Planar Mirror Resonators

Imagine that two perfect mirrors (R_m = 1) separated by a distance L. Reflections between mirror surfaces M_1 and M_2 lead to constructive and destructive interference the cavity.

Due to boundary conditions, the tangential component of the electric field of the corresponding resonator modes must be zero at the mirror surfaces.

If the half-wavelength or its multiples equal the cavity length (L), a constructive interference occurs.



Reflected waves interfere with each other and thus the resultant wave is obtained.

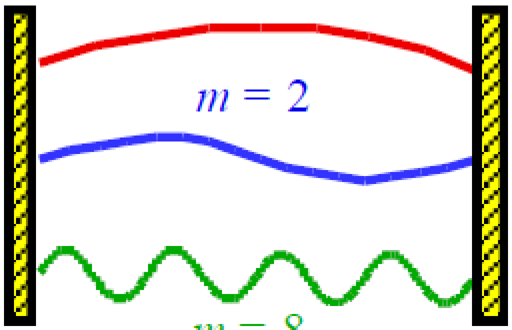
$$\begin{aligned} E &= E_o e^{-i(\omega t - kz)} + rE_o e^{-i(\omega t + kz)} \\ &= E_0 e^{-i\omega t}\left(e^{-ikz} + re^{ikz}\right) \end{aligned}$$

where r is the field reflection coefficient

Optical Resonators

Planar Mirror Resonators

The resultant wave is formed by cavity mode and each cavity mode is defined by the m value.



For a constructive interference, we can write that

$$kz = m\pi \quad \text{and} \quad z = L$$

$$\frac{2\pi}{\lambda} L = m\pi$$

$$m\frac{\lambda}{2} = L \qquad m = 1,2,3\ldots$$

m is the mode number. The mode frequencies corresponding allowed frequencies of the resonator modes are can be calculated as,

$$\nu_m = m\frac{c}{2L}$$

superposition of modes ⇒ $E = \sum_m E_m \sin(k_m z)$

The frequency difference between adjacent modes (free spectral range): $$\nu_f = \frac{c}{2L}$$

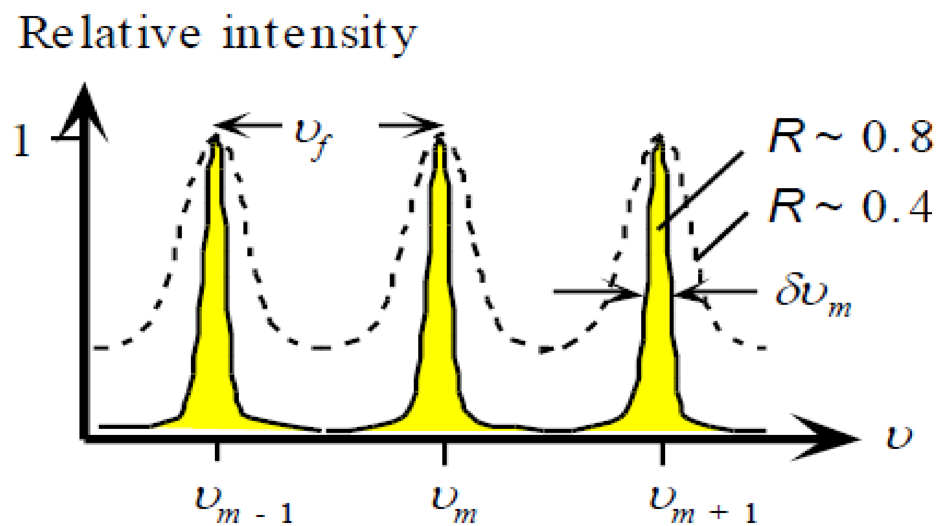
Optical Resonators

Planar Mirror Resonators

Free spectral range: $$\Delta v_m = v_{m+1} - v_m = v_f$$

The resonance wavelengths in the optical cavity is $$\lambda_m = \frac{c}{\upsilon_m}$$

Attention: $c = c_0 / n$ Where n is the refractive index in the resonator



Intensity vs. frequency for various modes

R is mirror reflectance and lower *R* means higher loss from the cavity.

Optical Resonators

Planar Mirror Resonators

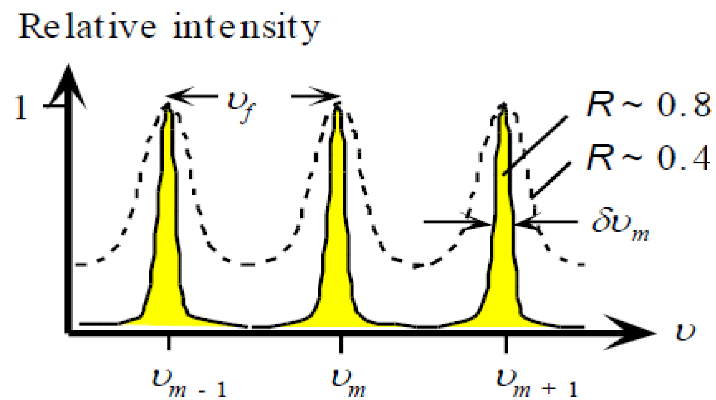
An Example:

Consider a 50 µm long resonator filled with air (n=1).
a) Calculate the mode frequencies corresponding first five modes.
b) Calculate the resonance wavelengths for these modes.
c) Calculate the free spectral range.

Optical Resonators

Fabry-Perot (Etalon) Resonator

Consider a two parallel mirror separated by a distance L.



If the cavity medium is lossless and the mirror are perfectly reflecting ($R_m \simeq 1$), then the peaks at frequency spectrum exhibit sharp lines.

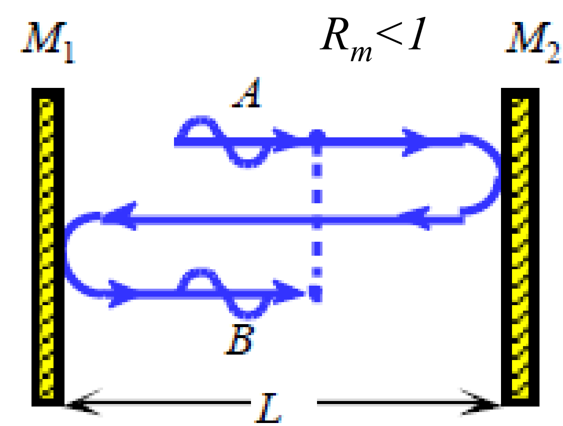
If the mirror does not reflect perfectly, the mode peaks are not sharp and have a finite width ($R_m < 1$). When A and B interfere,

$$|E_A| = A \qquad R = R_m = r^2$$

$$A + B = A + Ar^2 \exp(-j2kL)$$

$$E_{cavity} = A + B + \cdots$$

$$E_{cavity} = A + Ar^2 \exp(-j2kL) + Ar^4 \exp(-j4kL) + \cdots$$



This series can be evaluated as

$$E_{cavity} = \frac{A}{1 - r^2 \exp(-j2kL)}$$

Optical Resonators

Fabry-Perot (Etalon) Resonator

$$E_{cavity} = \frac{A}{1 - r^2 \exp(-j2kL)}$$

$$I \;\alpha\; |E_{cavity}|^2 \;\Rightarrow\; I = \frac{I_o}{(1-R)^2 + 4R\sin^2(kL)}$$

$$I_{max} = \frac{I_o}{(1-R)^2}$$

The finesse F is a measure of the sharpness of the interference fringes,

$$\text{Finesse of resonator} = \frac{\text{Intermode spacing}}{\text{Width of a mode}} = \frac{v_f}{\delta v_m}$$

$$F = \frac{v_f}{\delta v_m}$$

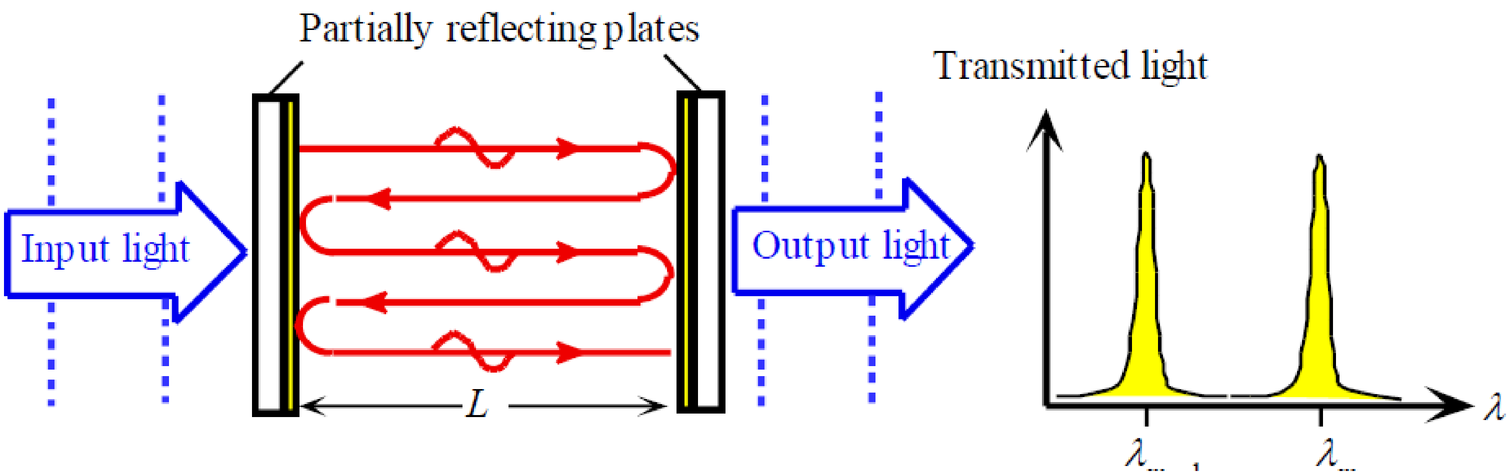
δv_m : is the spectral width of the cavity

$$F = \frac{\pi\sqrt{R}}{1-R}$$

Larger finesses lead to sharper mode peaks

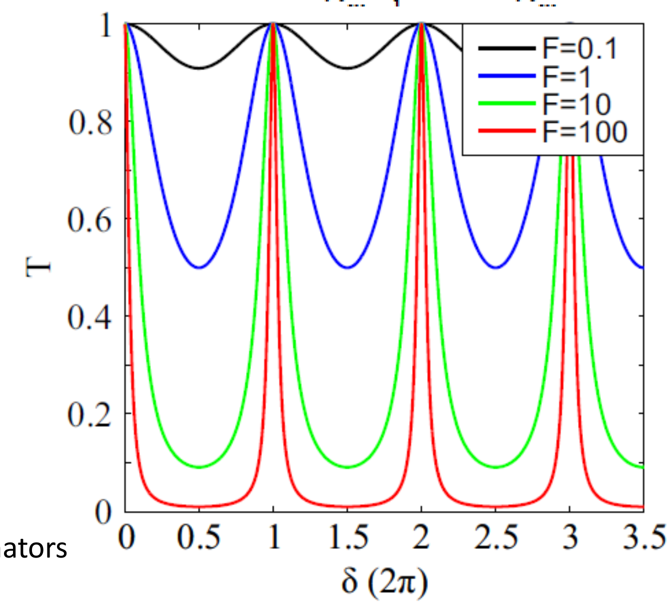
Optical Resonators

Fabry-Perot (Etalon) Resonator



Fabry-Perot etalon

$$I_{trans} = I_{incident} \frac{(1-R)^2}{(I-R)^2 + 4R\sin^2(kL)}$$



From https://www.nanophotonik.uni-bonn.de/lehre/sommersemester-2015/chapter-4-optical-resonators

Optical Resonators

Fabry-Perot (Etalon) Resonator

Example:
Consider a Fabry-Perot optical cavity of air length = 100 microns with mirrors that have a 0.9 reflectance.
a) Calculate the cavity mode nearest to 900 nm.
b) Calculate the separation of the modes and the spectral width of each mode.

Solution:

a)

$$m = \frac{2L}{\lambda} = \frac{2\left(100\times10^{-6}m\right)}{\left(900\times10^{-9}m\right)} = 222.22$$

$$\lambda_m = \frac{2L}{m} = \frac{2\left(100\times10^{-6}m\right)}{222} = 900.90nm$$

b)

$$\Delta v_m = v_f = \frac{c}{2L} = \frac{3\times10^{8}m/s}{2\left(100\times10^{-6}m\right)} = 1.5\times10^{12}Hz$$

$$F = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{0.9}}{1-0.9} = 29.8$$

$$\delta v_m = \frac{v_f}{F} = 5.03\times10^{10}Hz$$

$$\delta\lambda_m = \left|\frac{c}{{v_m}^2}\right|\delta v_m = \left|\frac{\lambda_m^2}{c}\right|\delta v_m = 0.136nm$$

Optical Resonators

Quality factor (Q)

The quality factor Q is often used to characterize electrical resonance circuits and microwave resonators. For optical resonators, large Q factors are associated with low-loss resonators and it can be defines as,

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy lost per cycle}} \qquad I(t) = I_0 e^{-\frac{\omega}{Q}t} = I_0 e^{-\left(\frac{t}{\tau_c}\right)}$$

In the case of a Fabry-Perot cavity with R1 and R2 the reflection coefficient of the mirror, after one round trip the intensity can be calculated as,

$$I = I_0 R_1 R_2 (1 - \alpha L)^2$$

α is the loss per unit length

After p round-trip,

$$I = I_0 \left[R_1 R_2 (1 - \alpha L)^2\right]^p = I_0 \exp\left(-\frac{t}{\tau_c}\right)$$

τ_c is the lifetime of the photons in the cavity

where p can be expressed by $p = \frac{(c/n)\, t}{2L}$

We can write that

$$\frac{1}{\tau_c} = -\left(\frac{c}{2nL}\right) \ln\left[R_1 R_2 (1 - \alpha L)^2\right] = \frac{\omega}{Q}$$

$$Q = \frac{-2n\ell\omega}{c} \frac{1}{\ln\left[R_1 R_2 (1 - \alpha L)^2\right]} = \frac{\nu_0}{\Delta\nu_c} = 2\pi\nu\tau_c$$

Optical Resonators

Quality factor (Q)

Consider a 90 cm-long He-Ne laser (λ = 633 nm) composed of 2 mirrors with equal coefficient of reflection (R = 0.98) and no additional loss (α = 0) and n = 1.
Calculate the cavity lifetime τ_c
Calculate the quality factor Q.

$$\frac{1}{\tau_c} = -\left(\frac{c}{2nL}\right) \ln\left[R_1 R_2 (1-\alpha L)^2\right]$$

$$\frac{1}{\tau_c} = -\frac{2\ln R}{\left(\frac{2L}{c}\right)} \approx 1/150 \text{ ns}^{-1}$$

The optical frequency of the laser is $\nu_0 = {}^{c}/_{\lambda} = 5 \times 10^{14}$ Hz

The quality factor

$$Q = 2\pi\nu_0\tau_c = 2\pi \times 5 \times 10^{14} \times 150 \text{ ns}$$
$$= 4.7 \times 10^8$$

Optical Interferometers

When two waves of the same frequency are combined, the resulting intensity pattern is formed by interfering of the two waves. **Interferometry** is a measurement method using the phenomenon of interference of waves.

Interferometers are widely used in science and industry to measure of small displacements, refractive index changes and surface irregularities. Common applications are astronomy, fiber optics, engineering metrology, optical metrology, oceanography, seismology, spectroscopy etc.

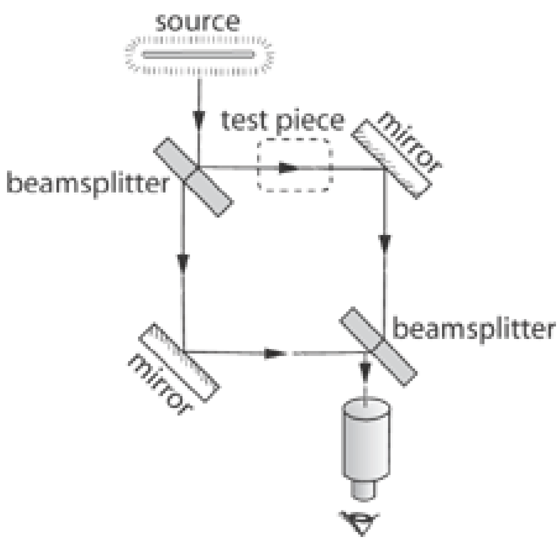
There are many interferometers and interferometric techniques. The most popular interferometers are given below.

Amplitude splitting interferometers: Mach–Zehnder, Fabry–Perot, Michelson

Common path interferometers: Sagnac, Fiber optic gyroscope, Point diffraction

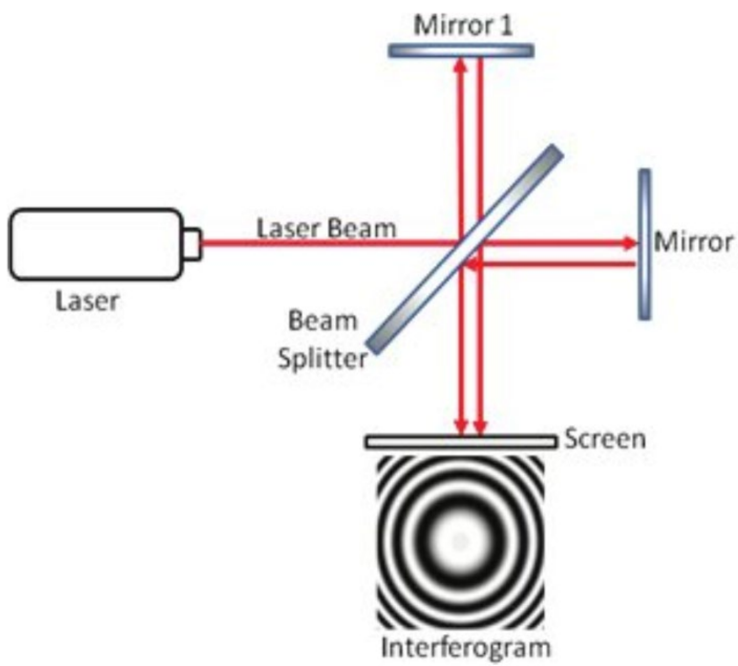
Optical Interferometers

Mach–Zehnder Interferometer



Mach–Zehnder Interferometer consists of two beam splitters and two fully reflecting mirrors.
When a light ray travels through test piece (sample) its phase will be shifted by an amount that depends on the index of refraction of the sample. The resulting fringe pattern is used to characterize the transparent objects.

Michelson interferometer

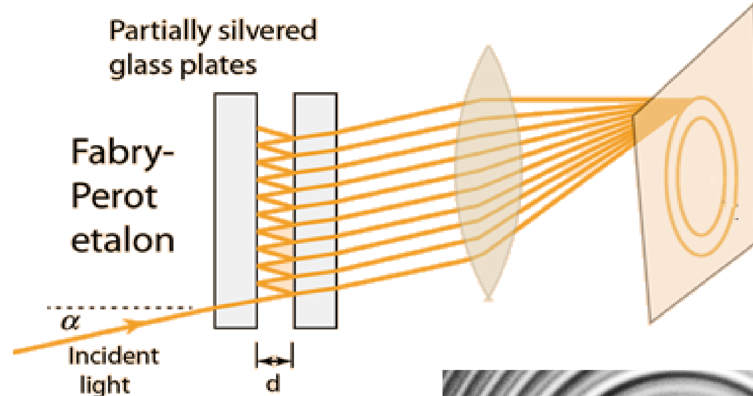


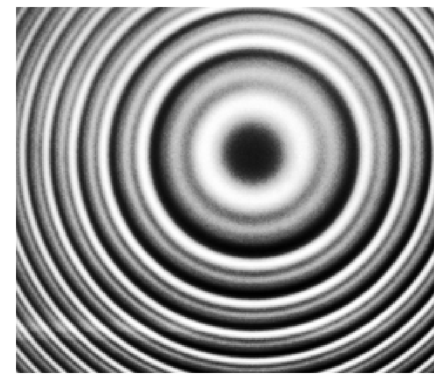
Michelson interferometer is another tool to produce interference between two beams of light. This technique allows wavelength of a light to be measured, the refractive index of transparent materials and small changes in length.

Optical Interferometers

Fabry perot interferometer

The Fabry Perot interferometer makes use of multiple reflections between two closely spaced partially silvered surfaces.





In order to observe the fringe pattern, the parallel transmitted light is focused to an image plane by a lens .

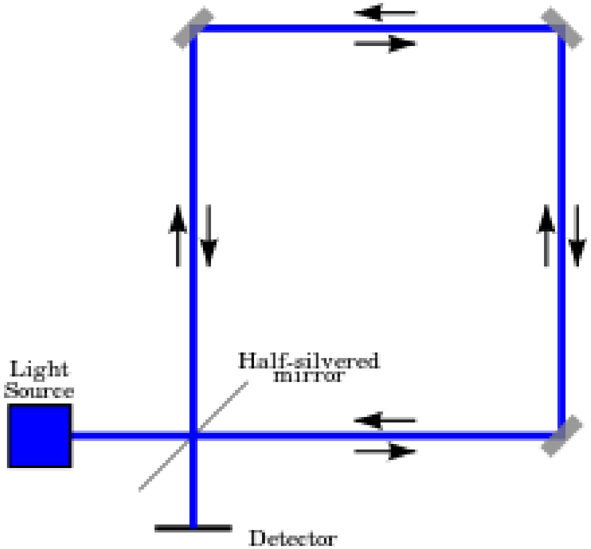
The large number of interfering rays produces an interferometer with extremely high resolution,

In the Fabry - Perot interferometer, the multiple beam interference fringes from a plane parallel plate illuminated near normal incidence are used.

Optical Interferometers

Sagnac Interferometer

A beam of light is split and the two beams are made to follow the same path but in opposite directions. On return to the point of entry the two light beams are allowed to exit the ring and undergo interference.



named after French physicist Georges Sagnac

When the interferometer is at rest, two lights travels at the same speed. In this case, the detector output is maximum because there is no phase difference between the two lights.

When the interferometer system is rotated, one beam of light will slow with respect to the other beam of light, so the detector output will change due to the phase difference between the two lights.

The phase shift of the interference fringes is proportional to the platform's angular velocity

Optical Interferometers

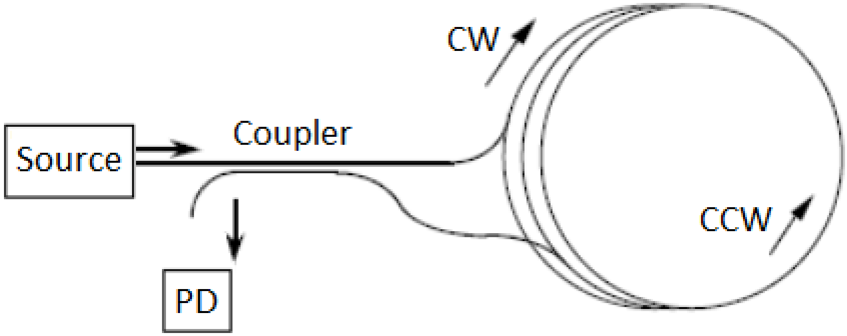
Fiber Optic Gyroscope

Fiber optic gyroscopes are used to measure the angular velocity in many different applications.

The working principle is based on Sagnac interferometer where a single beam is split into two parts and directed in opposite directions along the fiber cable.

The Sangac effect is increased by the number of turns in the optical fiber.

Since the optical fiber gyroscopes are very sensitive to rotational movement, they can measure the angular velocity with high accuracy.



The phase shift of the interference fringes is proportional to the platform's angular velocity