PHY401
Electromagnetic Theory I

The Wave Equation for
Electromagnetic Waves inVacuum

Assoc. Prof. Dr. Fulya Bagci
Department of Physics
Engineering/Ankara University
fbagci@eng.ankara.edu.tr

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9.2 Electromagnetic Waves in Vacuum

9.2.1 The wave equation for **E** and **B**

In regions of space where there is no charge or current, Maxwell's equations read

$$(i)\vec{\nabla}.\vec{E}=0 \qquad (iii)\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$(ii)\vec{\nabla}.\vec{B}=0 \qquad (iv)\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

They constitute a set of coupled, first-order, partial differential equations for E and B.

Apply curl to (iii) $(iii)\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

$$(iii)\vec{\nabla}\times\vec{\nabla}\times\vec{E}=\vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}\vec{E}\right)-\vec{\nabla}^{2}\vec{E}=\vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$(i)\vec{\nabla}.\vec{E}=0$$

$$\vec{\nabla}^{2}\vec{E}=\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)$$

Use (iv) $(iv)\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$$\vec{\nabla}^{2}\vec{E}=\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\Rightarrow\boxed{\vec{\nabla}^{2}\vec{E}=\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}}$$

Similarly, you can show that

$$\boxed{\vec{\nabla}^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$$

We have now separate equations for E and B but they are of second order.

In vacuum, each Cartesian component of E and B satisfies the three-dimensional wave equation.

$$\boxed{\vec{\nabla}^2 \vec{f} = \frac{1}{\vartheta^2} \frac{\partial^2 f}{\partial t^2}}$$

Maxwell's equations imply that empty space supports the propagation of electromagnetic waves travelling at a speed of light:

$$\boxed{\vartheta = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 m / s}$$

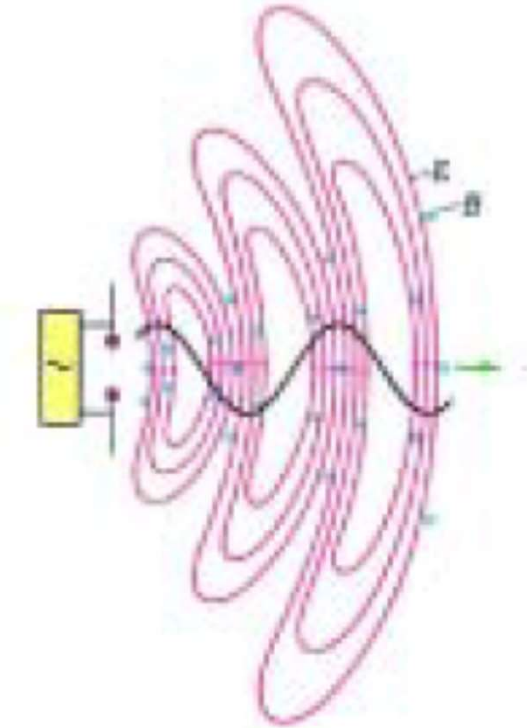
Light is an electromagnetic wave!

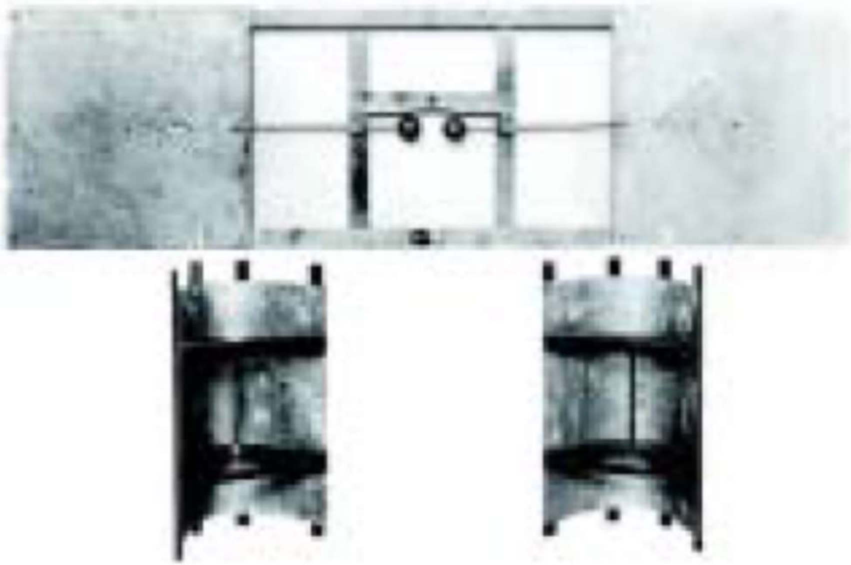
Notice the crucial role played by Maxwell's contribution to Ampere's law,

$$\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

Without it the wave equation would not emerge, and there will be no electromagnetic theory of light.

When Maxwell’s work was published in 1867 it did not receive immediate acceptance. It is Hertz who conclusively demonstrated the existence of electromagnetic waves.





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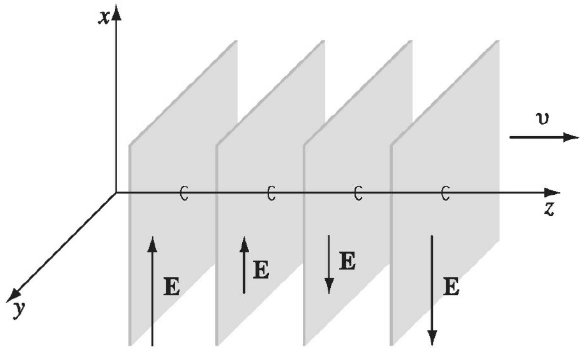
9.2.2 Monochromatic Plane Waves

A wave of single frequency is called a **monochromatic wave**.
If the waves are traveling in the z direction and have no x or y dependence, because the fields are uniform for every plane, these waves are called **plane waves**.

$$\tilde{E}(z,t)=\tilde{E}_0 e^{i(kz-\omega t)}$$

$$\tilde{B}(z,t)=\tilde{B}_0 e^{i(kz-\omega t)}$$



Since $(i)\vec{\nabla}.\vec{E}=0$ and $(ii)\vec{\nabla}.\vec{B}=0$

$$\tilde{E}_{0z}=\tilde{B}_{0z}=0 \qquad \tilde{E}(z,t)=\tilde{E}_{0x} e^{i(kz-\omega t)}$$

That is, the electromagnetic waves are transverse.

Moreover, $(iii)\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$ implies a relation between the electric and magnetic amplitudes

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \widetilde{E}_0 e^{i(kz-\omega t)} & 0 & 0 \end{vmatrix} = -\omega B_0 e^{i(kz-\omega t)}$$

$$k(\tilde{E}_0)_x = \omega(\tilde{B}_0)_y$$

More compactly, $\widetilde{B}_0 = \dfrac{k}{\omega}\left(\hat{z}\times\widetilde{E}_0\right)$

E and B are in phase and mutually perpendicular.
Their amplitudes are related by,

$$B_0 = \frac{k}{\omega}E_0 = \frac{1}{c}E_0$$

By convention, we use the direction of ***E*** to specify the polarization of an electromagnetic wave.
Propagation (wave) vector points in the direction of propagation, whose magnitude is the wave number.

$$\tilde{\mathbf{E}}(\mathbf{r}, t) = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{n}},$$

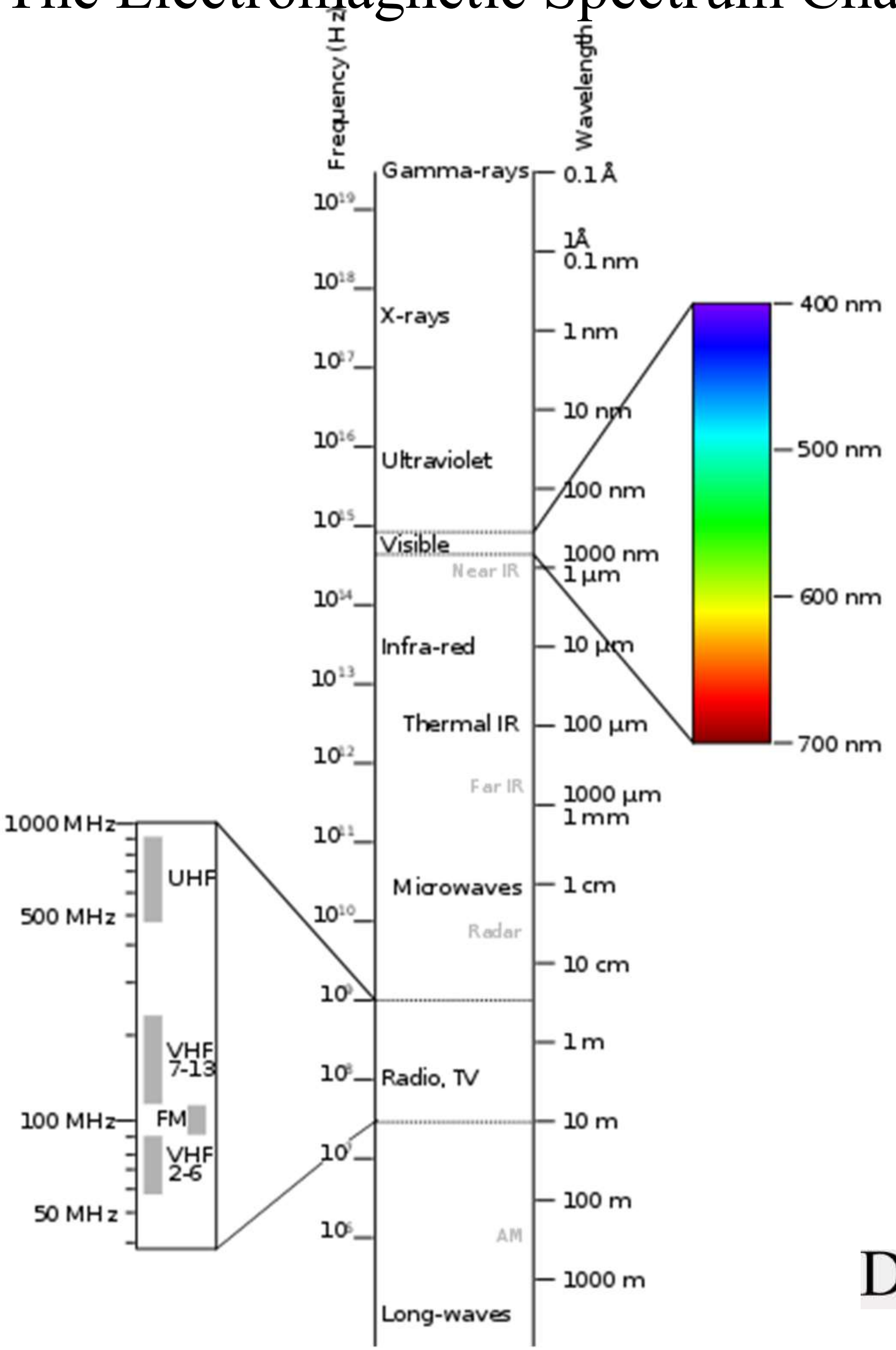
$$\tilde{\mathbf{B}}(\mathbf{r}, t) = \frac{1}{c}\tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}(\hat{\mathbf{k}} \times \hat{\mathbf{n}}) = \frac{1}{c}\hat{\mathbf{k}} \times \tilde{\mathbf{E}},$$

Because ***E*** is transverse, $\hat{n}.\hat{k} = 0$

$$\mathbf{E}(\mathbf{r}, t) = E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,\hat{\mathbf{n}},$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c}E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)(\hat{\mathbf{k}} \times \hat{\mathbf{n}}).$$

The Electromagnetic Spectrum Chart



Doç.Dr. Fulya Bağcı