PHY401
Electromagnetic Theory I

Reflection and Transmission in Oblique Incidence,

Fresnel's Equations

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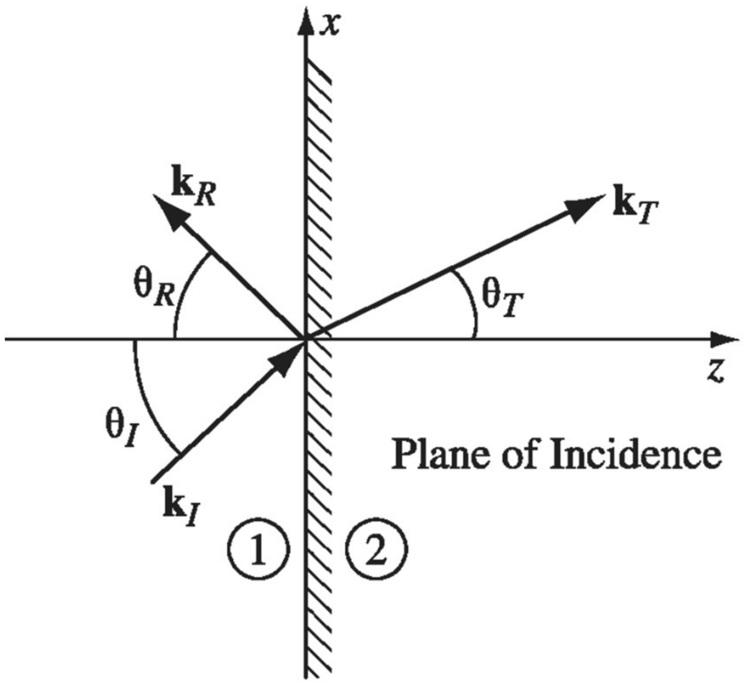
9.3.3 Reflection and Transmission at Oblique Incidence

Suppose that a monochromatic plane wave of frequency *w*, traveling in the k_I direction,



Transmitted wave:

$$\tilde{\mathbf{E}}_T(\mathbf{r},t) = \tilde{\mathbf{E}}_{0T} e^{i(\mathbf{k}_T\cdot\mathbf{r}-\omega t)}$$

$$\tilde{\mathbf{B}}_T(\mathbf{r},t) = \frac{1}{v_2}(\hat{\mathbf{k}}_T \times \tilde{\mathbf{E}}_T)$$

Reflected wave:

$$\tilde{\mathbf{E}}_R(\mathbf{r},t) = \tilde{\mathbf{E}}_{0R} e^{i(\mathbf{k}_R\cdot\mathbf{r}-\omega t)}$$

$$\tilde{\mathbf{B}}_R(\mathbf{r},t) = \frac{1}{v_1}(\hat{\mathbf{k}}_R \times \tilde{\mathbf{E}}_R)$$

Incident wave:

$$\tilde{\mathbf{E}}_I(\mathbf{r},t) = \tilde{\mathbf{E}}_{0I} e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}$$

$$\tilde{\mathbf{B}}_I(\mathbf{r},t) = \frac{1}{v_1}(\hat{\mathbf{k}}_I \times \tilde{\mathbf{E}}_I)$$

All three waves have the same frequency, ω.

$$\omega = k_I \vartheta_I = k_R \vartheta_I = k_T \vartheta_2 \quad \rightarrow \quad k_I = k_R = \frac{\vartheta_2}{\vartheta_1} k_T = \frac{n_1}{n_2} k_T$$

The combined fields in medium (1) must now be joined to the fields in medium (2), using the boundary conditions. A generic structure for the four boundary conditions,

$$()e^{i(k_I r-\omega t)} + ()e^{i(k_R r-\omega t)} = ()e^{i(k_T r-\omega t)}$$

When z=0, $\overrightarrow{k_I}.\vec{r} = \overrightarrow{k_R}.\vec{r} = \overrightarrow{k_T}.\vec{r}$

$$x(k_I)_x + y(k_I)_y = x(k_R)_x + y(k_R)_y = x(k_T)_x + y(k_T)_y$$

This Eq. can only hold if the components are separately equal.
For if x=0, we get $(k_I)_y = (k_R)_y = (k_T)_y$

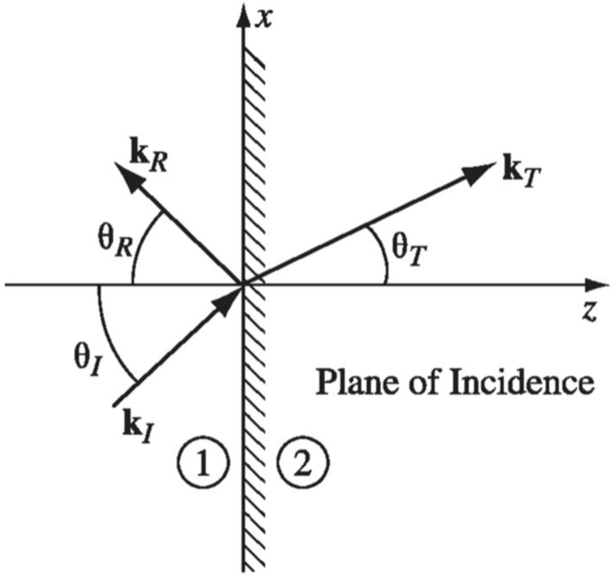
For if y=0, we get $(k_I)_x = (k_R)_x = (k_T)_x$

First Law: The incident, reflected, and transmitted wave vectors form a plane (called the **plane of incidence),** which also includes the normal to the surface.

$$(k_I)_x = (k_R)_x = (k_T)_x$$

$$k_I . \sin\theta_I = k_R . \sin\theta_R = k_T . \sin\theta_T$$

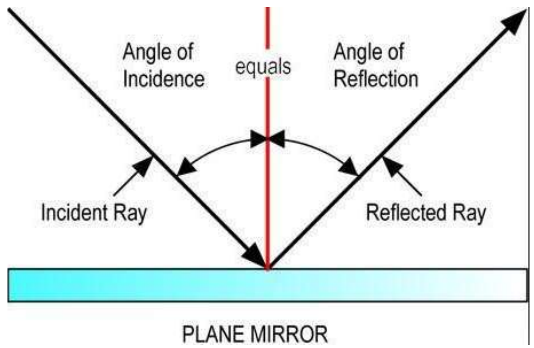
θ_I: angle of incidence
θ_R: angle of reflection
θ_T: angle of transmission



Second Law: The angle of incidence is equal to the angle of reflection.

$$k_I = k_R \Rightarrow \theta_I = \theta_R$$

This is the **law of reflection.**

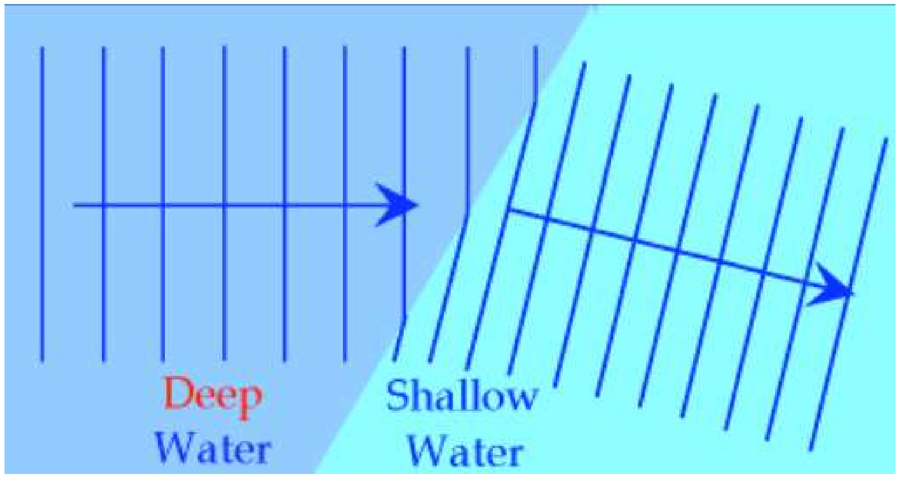


Third Law: The angle of incidence is equal to the angle of reflection.

$$k_I = \frac{n_1}{n_2} k_T \;\; so \;\; \frac{n_1}{n_2} \mathrm{k}_\mathrm{T} . \sin\theta_I = k_T . \sin\theta_T \rightarrow \qquad \frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$$

This is the **law of refraction-Snell's law.**

These are the three fundamental laws of geometrical optics. All we used was their generic form (Eq. 9.93). Therefore, any *other* waves (water waves, for instance, or sound waves) can be expected to obey the same "optical" laws when they pass from one medium into another.



$$()e^{i(k_I r-\omega t)}+()e^{i(k_R r-\omega t)}=()e^{i(k_T r-\omega t)}$$

Exponential factors are cancelled and the boundary conditions become,

(i) $\varepsilon_1(\tilde{\mathbf{E}}_{0I}+\tilde{\mathbf{E}}_{0R})_z=\varepsilon_2(\tilde{E}_{0T})_z$ Normal D

(ii) $(\tilde{\mathbf{B}}_{0I}+\tilde{\mathbf{B}}_{0R})_z=(\tilde{\mathbf{B}}_{0T})_z$ Normal B

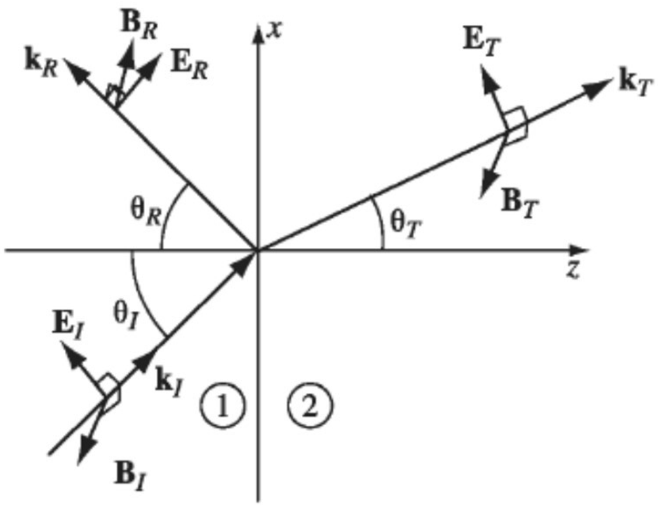
(iii) $(\tilde{\mathbf{E}}_{0I}+\tilde{\mathbf{E}}_{0R})_{x,y}=(\tilde{E}_{0T})_{x,y}$ Tangential E

(iv) $\frac{1}{\mu_1}(\tilde{\mathbf{B}}_{0I}+\tilde{\mathbf{B}}_{0R})_{x,y}=\frac{1}{\mu_2}(\tilde{\mathbf{B}}_{0T})_{x,y}$ Tangential H

where, $\widetilde{B}_0(r,t)=\frac{1}{\vartheta}\tilde{k}\times\widetilde{E}_0$

Fresnel's Equations for Parallel to the Plane of Incidence

If the polarization of the incident wave is parallel to the plane of incidence, the reflected and transmitted waves are also polarized in this plane.



Try Prob. 9.17 for perpendicular incidence.

Normal D (i) $\varepsilon_1(-\tilde{E}_{0I}\sin\theta_I + \tilde{E}_{0R}\sin\theta_R) = \varepsilon_2(-\tilde{E}_{0T}\sin\theta_T)$

Tangential E (iii) $(\tilde{E}_{0I}\cos\theta_I + \tilde{E}_{0R}\cos\theta_R) = (\tilde{E}_{0T}\cos\theta_T)$

Normal B (ii) 0=0

Tangential H (iv) $\frac{1}{\mu_1 v_1}(\tilde{E}_{0I} - \tilde{E}_{0R}) = \frac{1}{\mu_2 v_2}(\tilde{E}_{0T})$

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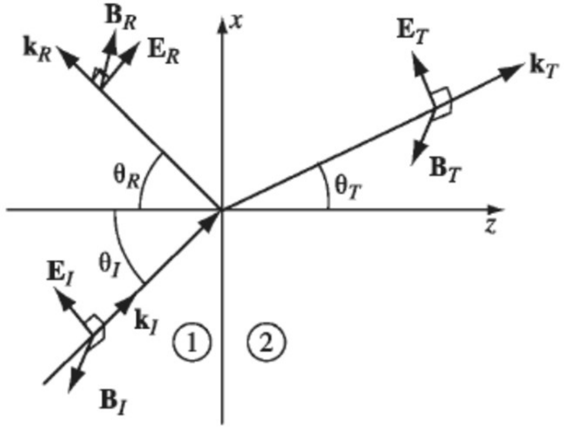
(iii) $(\tilde{E}_{0I} + \tilde{E}_{0R}) = \alpha(\tilde{E}_{0T}) \qquad \alpha \equiv \dfrac{\cos\theta_T}{\cos\theta_I}$

(iv) $(\tilde{E}_{0I} - \tilde{E}_{0R}) = \beta(\tilde{E}_{0T}) \qquad \beta \equiv \dfrac{\mu_1 v_1}{\mu_2 v_2}$

$\Rightarrow \tilde{E}_{0R} = \left(\dfrac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} \qquad \tilde{E}_{0T} = \left(\dfrac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$ Fresnel's equations



Notice that the transmitted wave is always *in phase* with the incident one; the reflected wave is either in phase ("right side up"), if $a > \beta$, or 180° out of phase ("upside down"), if $a < \beta$.

$$\Rightarrow \tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} \qquad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$$ Fresnel's equations

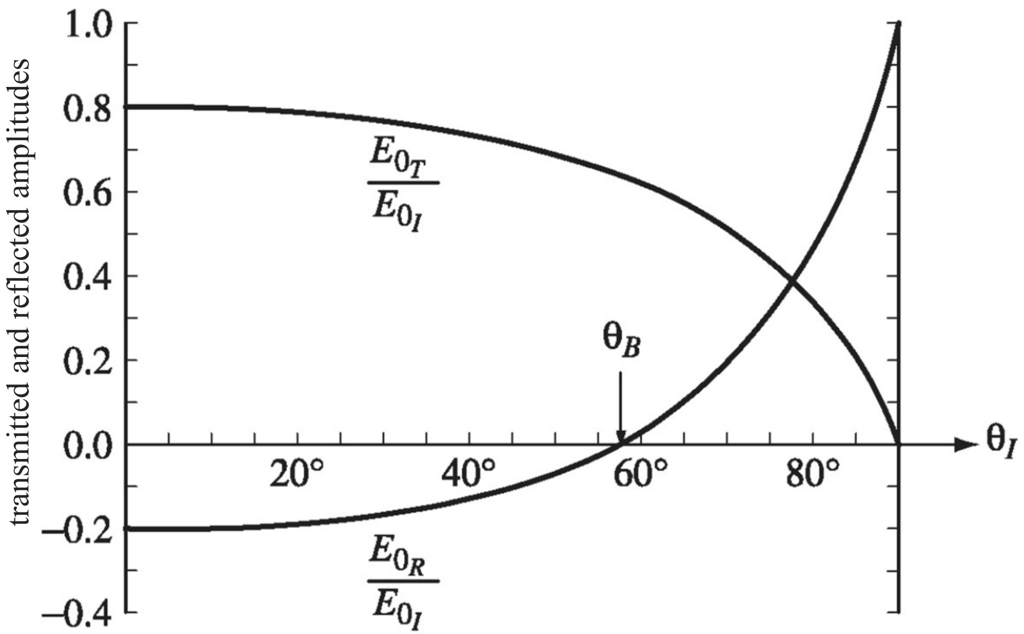
$$\alpha = \frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1-\left[\left(n_1/n_2\right)\sin\theta_I\right]^2}}{\cos\theta_I}$$

- In the case of normal incidence ($\theta = 0$), $\alpha = 1$, and we recover Eq. for normal incidence. At grazing incidence ($\theta = 90°$), α diverges, and the wave is totally reflected.
- Interestingly, there is an intermediate angle, $\boldsymbol{\theta_B}$ (called **Brewster's angle**), at which the reflected wave is completely extinguished. When does $E_{0R}=0$? Answer: When $\alpha = \beta$

At Brewster's angle

$$\theta_{B,I} + \theta_R = 90^\circ$$
$$n_1 \sin\theta_B = n_2 \sin(90^\circ - \theta_B)$$
$$n_1 \sin\theta_B = n_2 \cos(\theta_B)$$
$$\theta_B = \arctan\frac{n_2}{n_1}$$

For light incident on glass (n_2= 1.5) from air (n_1= 1)



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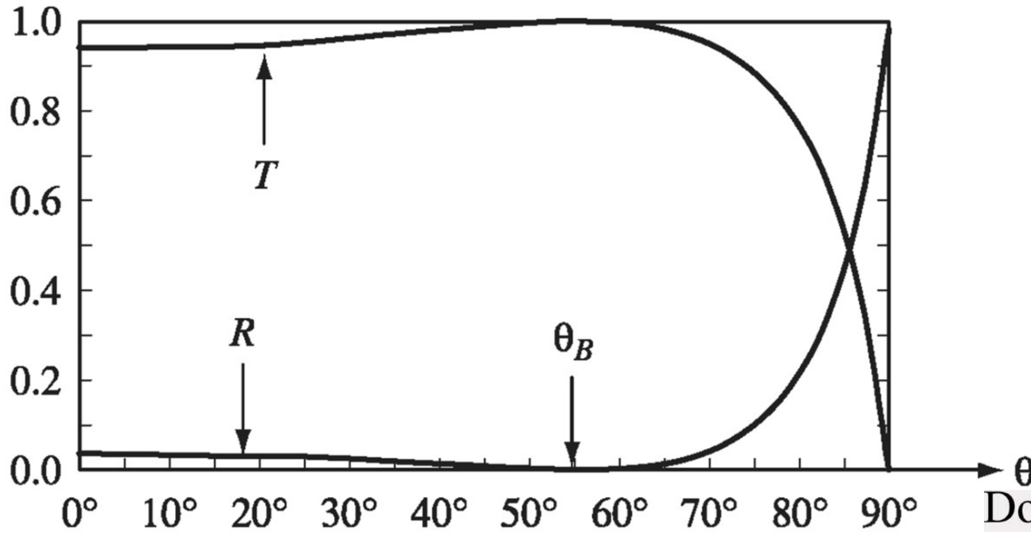
$$\Rightarrow \tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} \qquad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$$ Fresnel's equations

Reflection coefficient: $$R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$$

Transmission coefficient:

$$T = \frac{I_T}{I_I} = \frac{\varepsilon_2 \vartheta_2}{\varepsilon_1 \vartheta_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 \frac{\cos\theta_T}{\cos\theta_I} = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$

Reflection/transmission coefficient for the air/glass interface



Heavily polarized reflected sunlight is eliminated at the right figure.

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