CHE/CEN138

COMPUTER PROGRAMMING

MATRICES AND VECTORS, MATRICE AND ARRAY OPERATIONS, LINEAR ALGEBRA

References

1.Pratap, R. “Getting Started with MATLAB: A Quick Introduction for Scientists and Engineers”Oxford University Press, 2010.

2.Hunt, B.R., Lipsman, L.R. and Rosemberg J. M. “A guide to MATLAB for Beginners and ExperiencedUsers"Cambridge University Press, 2001.

3.Kubat, C. “MATLAB Yapay Zeka ve Mühendislik Uygulamaları” İkinci Baskı, Pusula Yayıncılık, 2014McGraw Hill, International Edition 2012.

• Matrix Algebra

- Scalar multiplication
- Matrix addition
- Matrix subtraction
- Vector addition
- Vector multiplication
- Matrix multiplication

Scalar multiplication

$$2 \cdot \begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 12 \\ 2 & -2 \end{bmatrix}$$

Matrix addition

$$\begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 5 & 9 \\ 3 & -3 \end{bmatrix}$$

!!! Addition is possible if matrices are of the same order.

Matrix subtraction

$$\begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 3 \\ -1 & 1 \end{bmatrix}$$

!!! Subtraction is possible if matrices are of the same order.

Vector addition

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

!!! Addition is possible if vectors are of the same size.

Vector multiplication

You can multiply a **row** vector with a **column** vector. To multiply two vectors, multiply the corresponding elements, then add the results.

$$\begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} = 2 + (-4) + 0 = 2$$

!!! Multiplication is possible if vectors are of the same order.

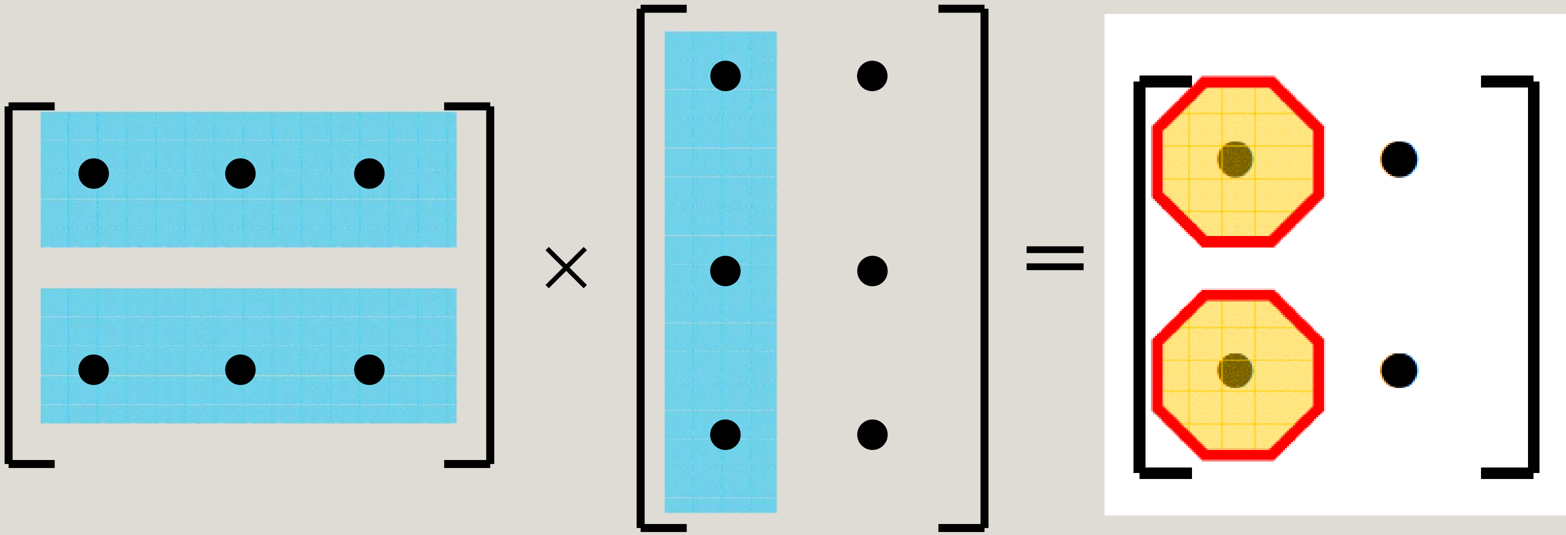
Matrix multiplication

Let **A** be a k x m matrix and

B be an m x n matrix,

$$\mathbf{A_{KM}} \times \mathbf{B_{MN}} = \mathbf{C_{KN}}$$

Matrix multiplication (2)



Determinant and inverse

- Minor
- Cofactor
- Determinant
- Inverse

Finding the inverse of a matrix

Only square matrices have inverses. To find the inverse of $\mathbf{A}$,

... you need to find $|\mathbf{A}|$.

To find the determinant $|\mathbf{A}|$,

... you need to find the cofactors of elements of $\mathbf{A}$.

... you need to find the minors of $|\mathbf{A}|$

Minor of a matrix

$$\text{Minor}(\mathbf{a_{12}}) = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}$$

Finding cofactor of element $\mathbf{a_{ij}}$

The cofactor of element $\mathbf{a_{ij}}$ is

Cofactor ($\mathbf{a_{ij}}$) = $\mathbf{C_{ij}}$ = $\mathbf{(-1)^{i+j}}$ $|$minor($\mathbf{a_{ij}}$)$|$

Finding the inverse of A

The inverse of A:

$$A^{-1} = 1/|A|\,C^T$$

Determinant of a matrix

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{11}a_{22} - a_{21}a_{12}$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} =$$

$$a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Example

$$3x_1 - x_2 + x_3 = 2$$
$$2x_1 + x_2 = 1$$
$$x_1 + 2x_2 - x_3 = 3$$

$$\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Example_continued

$$X = A^{-1}B = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ -1.0 & 2.0 & -1.0 \\ -1.5 & 3.5 & -2.5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}$$

$$x_1 = 2,$$

$$x_2 = -3,$$

$$x_3 = -7$$