



Linear Algebra with MATLAB - 1

Lecture 12

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Numbers:

- ▶ Natural numbers: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$
- ▶ Integers numbers: $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$
- ▶ Rational numbers are composed of ratios of integers:
$$\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \right\}$$
- ▶ Irrational numbers are numbers like π , $\sqrt{2}$, Euler's number, golden ratio, etc.
- ▶ Real numbers contains \mathbb{Q} and irrational numbers but not imaginary numbers.
- ▶ let i be $\sqrt{-1}$, complex numbers are: $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$

Vectors:

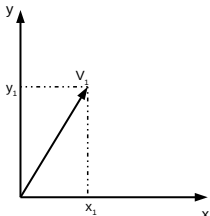


Definition: A vector is an element in a vector space that has a direction and a magnitude.

Throughout the course, vectors will be considered to be in column form:

$$\vec{v} = (x_1, x_2, \dots, x_n) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Let $\vec{v}_1 = (x_1, y_1)$ be a vector in \mathbb{R}^2 :





Vector Operations:

Summation:

```
>> x1 = [1 2 3];  
>> disp(x1)  
    1    2    3
```

```
>> x2 = [4; 5; 6];  
>> disp(x2)  
    4  
    5  
    6
```

```
>> x = x1+x2
```

```
x =
```

```
    5    6    7  
    6    7    8  
    7    8    9
```



Dot-Product:

Dot Product: Let $\vec{x} = (x_1, x_2, \dots, x_n)$ and $\vec{y} = (y_1, y_2, \dots, y_n)$ be two vectors in \mathbb{R}^n :

$$\begin{aligned}\vec{x} \cdot \vec{y} &= \sum_{k=1}^n x_k y_k \\ \vec{x} \cdot \vec{y} &= \|\vec{x}\| \|\vec{y}\| \cos(\theta)\end{aligned}\tag{1}$$

where, θ is the angle between two vectors and $\|\vec{x}\|$ is the norm (length) of the vector \vec{x} .



Vector Product in MATLAB:

```
>> disp([1 2 3]*[4 5 6]');  
32
```

```
>> disp([1 2 3]'*[4 5 6]);  
4    5    6  
8    10   12  
12   15   18
```

```
>> disp([1 2 3].*[4 5 6]);  
4    10   18
```

```
>> disp([1 2 3]'.*[4 5 6]);  
4    5    6  
8    10   12  
12   15   18
```

```
>> disp([1 2 3].*[4 5 6]');  
4    8    12  
5    10   15  
6    12   18
```

Matrices:



Definition: Identity matrix is an $n \times n$ matrix where all its diagonal elements are ones and the remaining elements are zeros.

$$\mathcal{I}_{n \times n} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, A_{m \times n} \mathcal{I}_{n \times n} = \mathcal{I}_{m \times m} A_{m \times n} = A_{m \times n}$$

Matrix Multiplication:

$$A_{m \times n} \vec{c} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} c_1 a_{11} + \cdots + c_n a_{1n} \\ c_1 a_{21} + \cdots + c_n a_{2n} \\ \vdots \\ c_1 a_{m1} + \cdots + c_n a_{mn} \end{bmatrix}$$

Creating Matrices in MATLAB:



```
>> A = zeros(5)
```

```
A =
```

```
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
0 0 0 0 0
```

```
>> A = ones(5)
```

```
A =
```

```
1 1 1 1 1
1 1 1 1 1
1 1 1 1 1
1 1 1 1 1
1 1 1 1 1
```

```
>> A = eye(5)
```

```
A =
```

```
1 0 0 0 0
0 1 0 0 0
0 0 1 0 0
0 0 0 1 0
0 0 0 0 1
```


Matrix Transpose:



Matrix Transpose: Transpose of matrix is equivalent to the original matrix flipped around its diagonal. Hence, via taking the transpose, each column of a matrix becomes the corresponding row of that matrix. The transpose of A in (??):

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Some properties of matrix transpose:

$$(A^T)^T = A, (A + B)^T = A^T + B^T, (AB)^T = B^T A^T$$



Matrix Transpose in MATLAB:

```
>> A = [1 2 3; 4 5 6; 7 8 9]
```

```
A =
```

```
    1    2    3
    4    5    6
    7    8    9
```

```
>> A'
```

```
ans =
```

```
    1    4    7
    2    5    8
    3    6    9
```



Determinant of a Matrix:

Determinant of 2×2 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \det|A| = a_{11}a_{22} - a_{21}a_{12} \quad (2)$$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 8 \end{bmatrix}, \quad |\det(A)| = 2 \times 8 - 1 \times 6 = 10 \quad (3)$$



Matrix Determinant in MATLAB:

```
>> A = [2 1; 6 8]
```

```
A =
```

```
     2     1  
     6     8
```

```
>> det(A)
```

```
ans =
```

```
    10
```



Solving Linear System of Equations (LSE):

Let's consider the following equations:

$$2x_1 - 4x_2 + 2x_3 = 0$$

$$3x_1 - 5x_2 + 4x_3 = 5$$

$$x_1 + 2x_2 + 2x_3 = 11$$

How do we solve this equation? How can we find the values for x_1 , x_2 and x_3 such that the equations are satisfied?



Representation as $Ax = b$:

Such a system can be represented as:

$$A = \begin{bmatrix} 2 & -4 & 2 \\ 3 & -5 & 4 \\ 1 & 2 & 2 \end{bmatrix} \quad (4)$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 5 \\ 11 \end{bmatrix} \quad (5)$$

$$Ax = b \quad (6)$$



Gaussian Elimination:

Gaussian elimination is one of the fundamental algorithms to solve linear system of equations. Gaussian elimination consists of two parts. The first part is called forward substitution. The aim is to obtain an upper triangular matrix by applying permutations, row scaling and elimination starting from the pivot element.

$$\left[\begin{array}{ccc|c} 2 & -4 & 2 & 0 \\ 3 & -5 & 4 & 5 \\ 1 & 2 & 2 & 11 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 3 & -5 & 4 & 5 \\ 1 & 2 & 2 & 11 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 4 & 1 & 11 \end{array} \right]$$
$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & -3 & -9 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$



Gaussian Elimination:

After upper triangular matrix is obtained, the second step of Gaussian elimination is started. This step is called Back substitution. Starting from the last element.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

The back substitution is done until identity matrix is obtained on the left side.

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$



Solving LSE in MATLAB:

```
>> A = [2 -4 2; 3 -5 4; 1 2 2];  
>> b = [0, 5, 11]';  
>> x = A\b
```

```
x =
```

```
1.0000  
2.0000  
3.0000
```

```
>> linsolve(A,b)
```

```
ans =
```

```
1.0000  
2.0000  
3.0000
```



Using Matrix Inverse:

```
>> A = [2 -4 2; 3 -5 4; 1 2 2];  
>> b = [0, 5, 11]';  
>> A*pinv(A)
```

ans =

```
1.0000    0.0000   -0.0000  
0.0000    1.0000   -0.0000  
0.0000   -0.0000    1.0000
```

```
>> x = pinv(A)*b
```

x =

```
1.0000  
2.0000  
3.0000
```



Matrix Operations in MATLAB:

Matrix to Vector:

```
>> A = [2 -4 2; 3 -5 4; 1 2 2]
```

```
A =
```

```
    2    -4     2  
    3    -5     4  
    1     2     2
```

```
>> A(:)'
```

```
ans =
```

```
    2     3     1    -4    -5     2     2     4     2
```



Matrix Sorting in MATLAB:

```
>> A = [2 -3; -6 9]
```

```
A =
```

```
     2     -3  
    -6     9
```

```
>> sort(A,1)
```

```
ans =
```

```
    -6    -3  
     2     9
```

```
>> sort(A,2)
```

```
ans =
```

```
    -3     2  
    -6     9
```



Max & Min:

```
>> A
```

```
A =
```

```
     2    -3  
    -6     9
```

```
>> max(A, [], 1)
```

```
ans =
```

```
     2     9
```

```
>> max(A, [], 2)
```

```
ans =
```

```
     2  
     9
```



Max & Min:

```
>> A
```

```
A =
```

```
     2    -3  
    -6     9
```

```
>> min(min(A))
```

```
ans =
```

```
    -6
```

```
>> min(A(:))
```

```
ans =
```

```
    -6
```