Linear Algebra with MATLAB - 2

Lecture 13

Dr. Görkem Saygılı

Department of Biomedical Engineering
Ankara University

Introduction to MATLAB, 2017-2018 Spring

Matrix Determinant in MATLAB:

```
>> A = [2 1; 6 8]

A =

     2     1
     6     8

>> det(A)

ans =

    10
```

Solving Linear System of Equations (LSE):

Let's consider the following equations:

$$\begin{aligned} 2x_1 - 4x_2 + 2x_3 &= 0 \\ 3x_1 - 5x_2 + 4x_3 &= 5 \\ x_1 + 2x_2 + 2x_3 &= 11 \end{aligned}$$

How do we solve this equation? How can we find the values for x_1, x_2 and x_3 such that the equations are satisfied?

Representation as $Ax = b$:

Such a system can be represented as:

$$A = \begin{bmatrix} 2 & -4 & 2 \\ 3 & -5 & 4 \\ 1 & 2 & 2 \end{bmatrix} \tag{1}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 5 \\ 11 \end{bmatrix} \tag{2}$$

$$Ax = b \tag{3}$$

Gaussian Elimination:

Gaussian elimination is one of the fundamental algorithms to solve linear system of equations. Gaussian elimination consists of two parts. The first part is called forward substitution. The aim is to obtain an upper triangular matrix by applying permutations, row scaling and elimination starting from the pivot element.

$$\left[\begin{array}{ccc|c} 2 & -4 & 2 & 0 \\ 3 & -5 & 4 & 5 \\ 1 & 2 & 2 & 11 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 3 & -5 & 4 & 5 \\ 1 & 2 & 2 & 11 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 4 & 1 & 11 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & -3 & -9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Gaussian Elimination:

After upper triangular matrix is obtained, the second step of Gaussian elimination is started. This step is called Back substitution. Starting from the last element.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

The back substitution is done until indentity matrix is obtained on the left side.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Solving LSE in MATLAB:

```
>> A = [2 -4 2; 3 -5 4; 1 2 2];
>> b = [0, 5, 11]';
>> x = A\b

x =

    1.0000
    2.0000
    3.0000

>> linsolve(A,b)

ans =

    1.0000
    2.0000
    3.0000
```

Using Matrix Inverse:

```
>> A = [2 -4 2; 3 -5 4; 1 2 2];
>> b = [0, 5, 11]';
>> A*pinv(A)

ans =

    1.0000    0.0000   -0.0000
    0.0000    1.0000   -0.0000
    0.0000   -0.0000    1.0000

>> x = pinv(A)*b

x =

    1.0000
    2.0000
    3.0000
```

Matrix Operations in MATLAB:

Matrix to Vector:

```
>> A = [2 -4 2; 3 -5 4; 1 2 2]

A =

     2    -4     2
     3    -5     4
     1     2     2

>> A(:)'

ans =

     2     3     1    -4    -5     2     2     4     2
```

Matrix Sorting in MATLAB:

```
>> A = [2 -3; -6 9]

A =

     2    -3
    -6     9

>> sort(A,1)

ans =

    -6    -3
     2     9

>> sort(A,2)

ans =

    -3     2
    -6     9
```

Max & Min:

```
>> A

A =

     2    -3
    -6     9

>> max(A,[],1)

ans =

     2     9

>> max(A,[],2)

ans =

     2
     9
```

Max & Min:

```
>> A

A =

     2    -3
    -6     9

>> min(min(A))

ans =

    -6

>> min(A(:))

ans =

    -6
```