Physics 122: Electricity & Magnetism – Lecture 3
Electric Field

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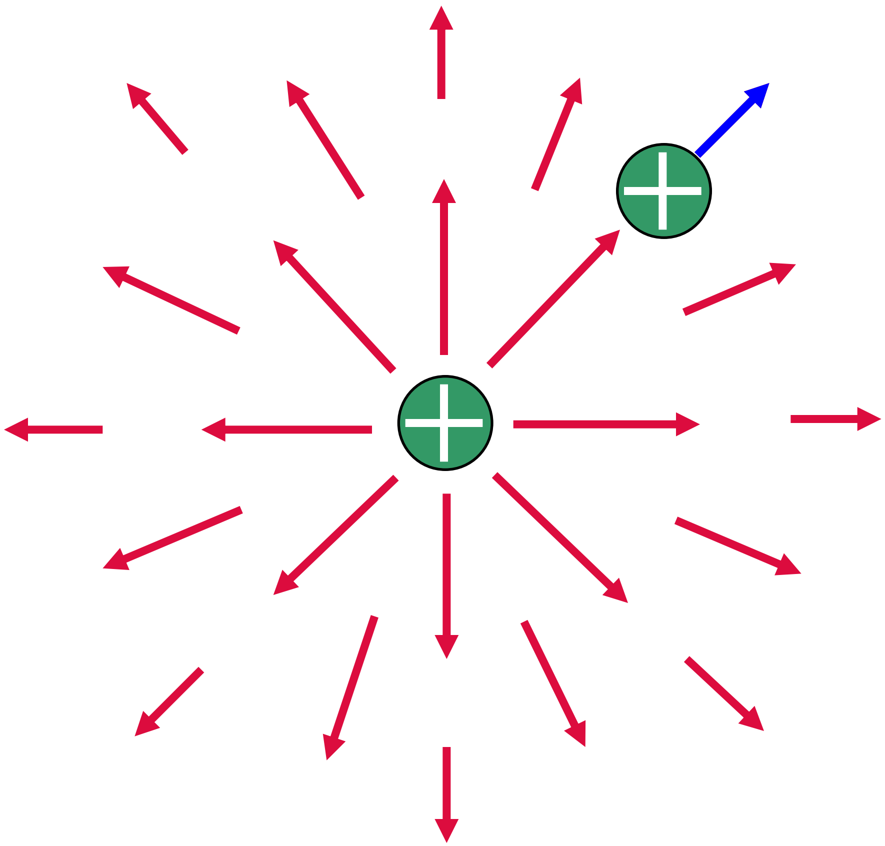
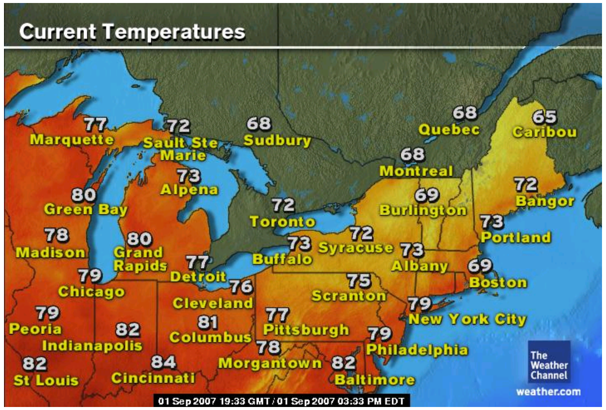
Electric Force and Field Force

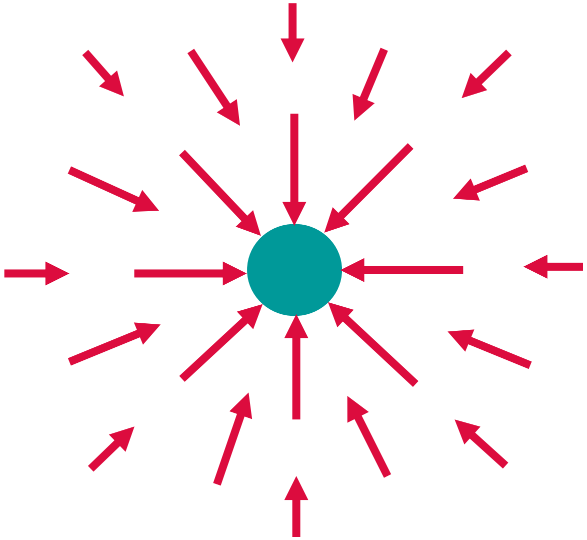
- What? -- Action on a distance
- How? – Electric Field
- Why? – Field Force
- Where? – in the space surrounding charges



Fields

- Scalar Fields:
  - Temperature – T(r)
  - Potential energy – U(r)
- Vector Fields:
  - Velocity field – $\vec{v}(\vec{r})$
  - Gravitational field – $\vec{g}(\vec{r})$
  - Electric field – $\vec{E}(\vec{r})$
  - Magnetic field – $\vec{B}(\vec{r})$



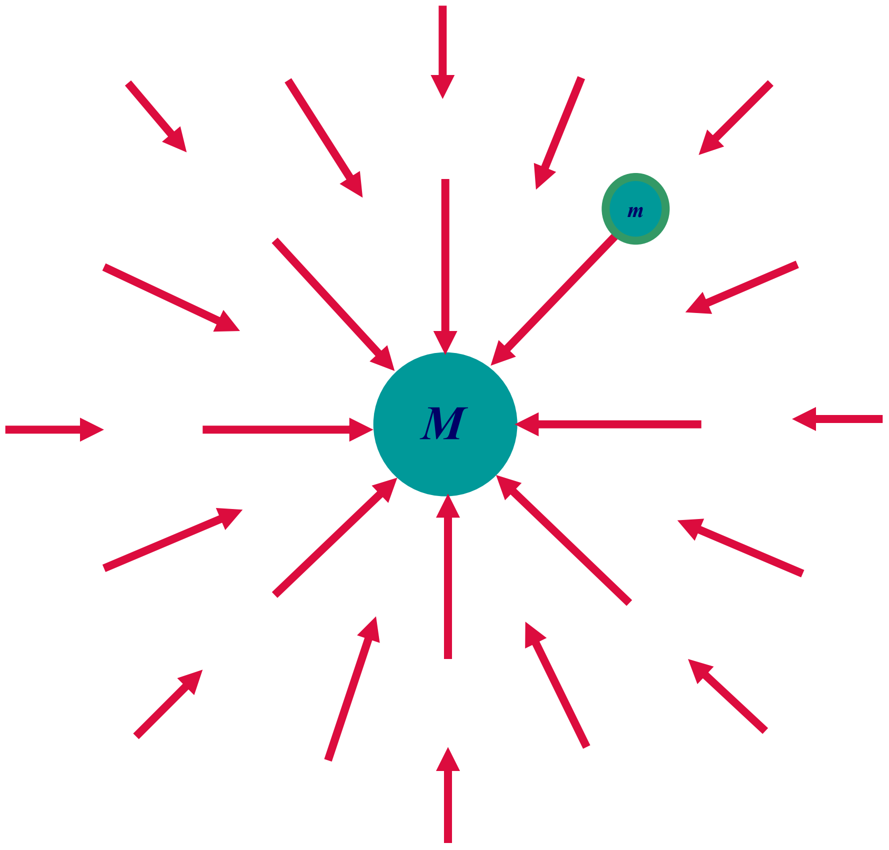


Vector Field Due to Gravity

- When you consider the force of Earth's gravity in space, it points everywhere in the direction of the center of the Earth. But remember that the strength is:

$$\vec{F} = -G\frac{Mm}{r^2}\hat{r}$$

- This is an example of an inverse-square force (proportional to the inverse square of the distance).



Idea of Test Mass

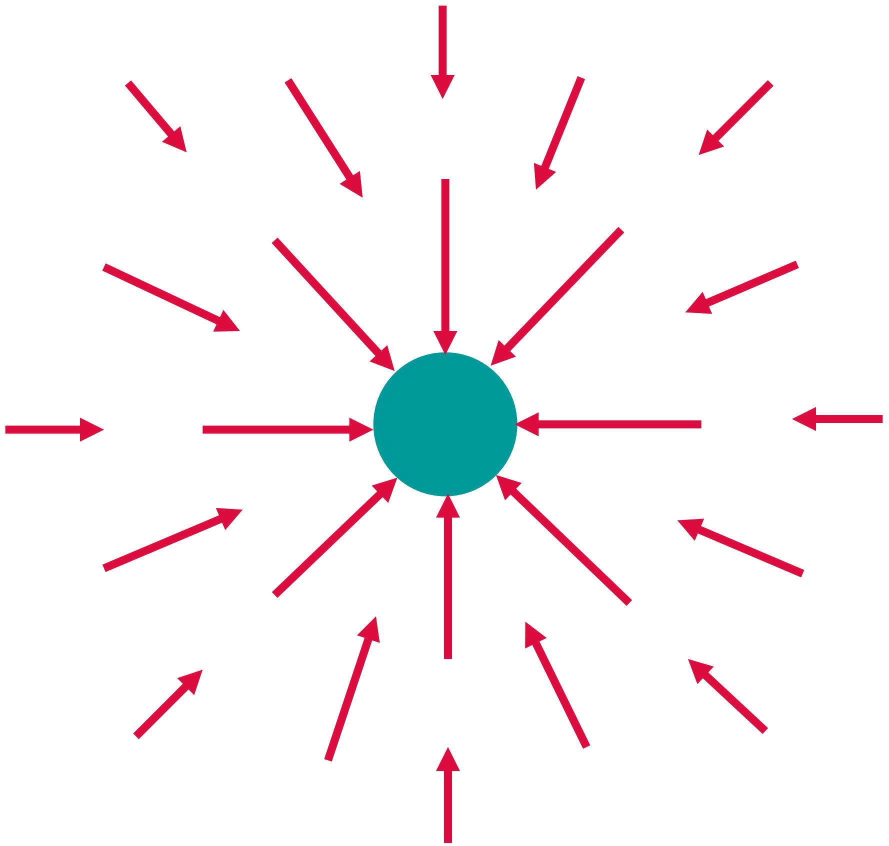
- Notice that the actual amount of force depends on the mass, m:

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

- It is convenient to ask what is the force per unit mass. The idea is to imagine putting a unit test mass near the Earth, and observe the effect on it:

$$\frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{r} = -g(r)\hat{r}$$

- $g(r)$ is the "gravitational field."

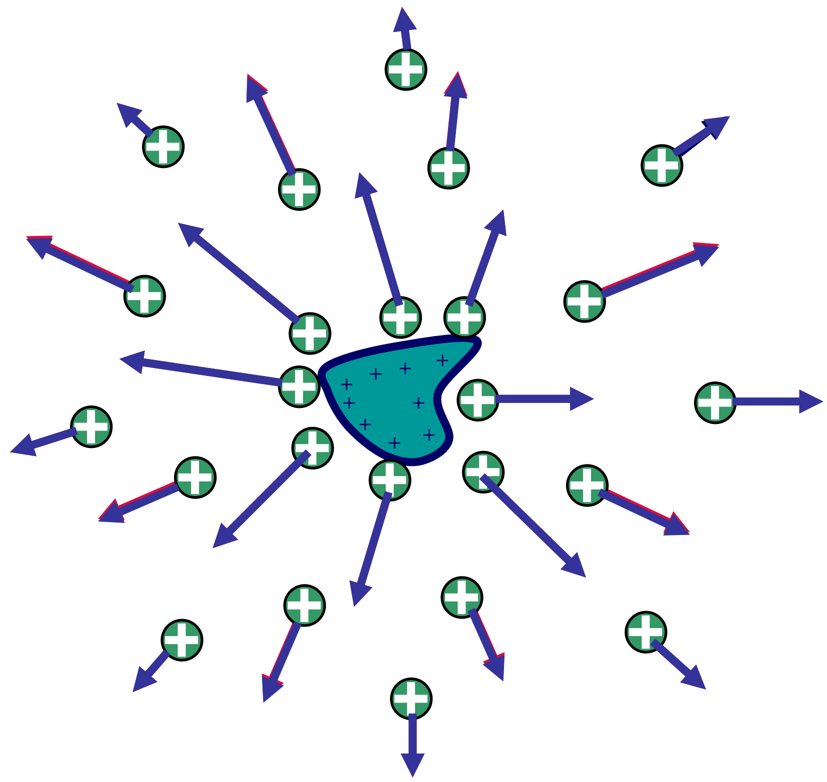


Electric Field

- Electric field is said to exist in the region of space around a charged object: the source charge.
- Concept of test charge:
  - Small and positive
  - Does not affect charge distribution
- Electric field:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

  - Existence of an electric field is a property of its source;
  - Presence of test charge is not necessary for the field to exist;

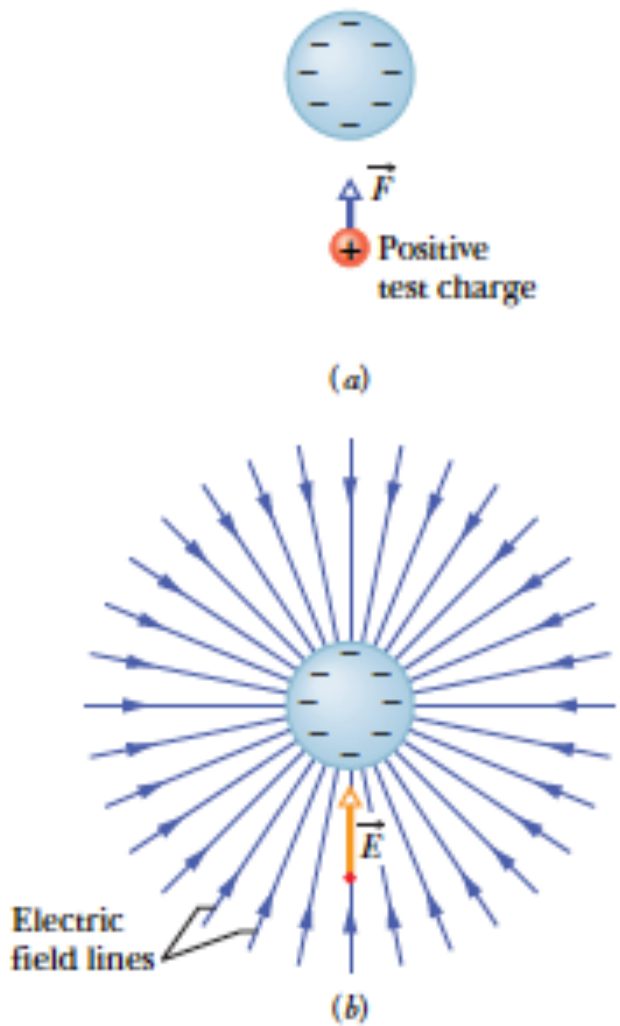


Electric Field

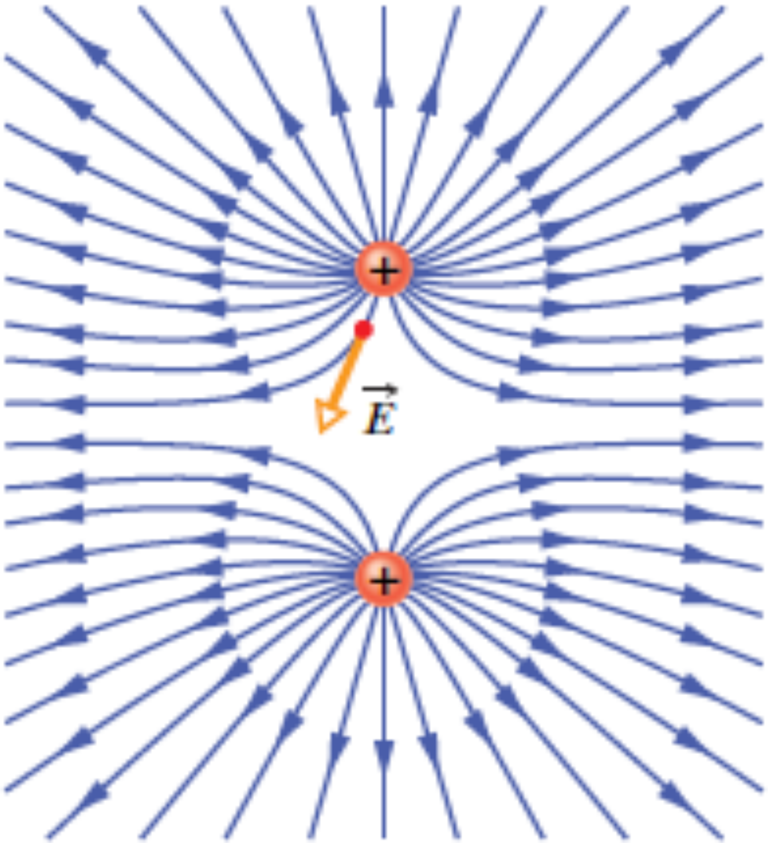
$$\vec{E} = \frac{\vec{F}}{q_0}$$

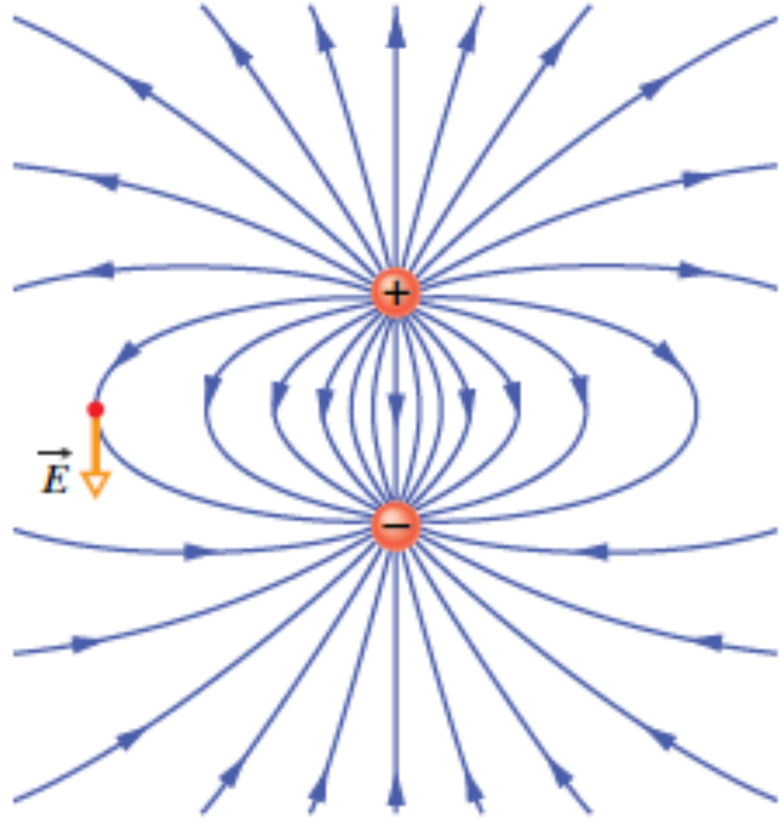
- Magnitude: $E=F/q_0$
- Direction: is that of the force that acts on the positive test charge
- SI unit: N/C

Situation	Value
Inside a copper wire of household circuits	10^{-2} N/C
Near a charged comb	10^{3} N/C
Inside a TV picture tube	10^{5} N/C
Near the charged drum of a photocopier	10^{5} N/C
Electric breakdown across an air gap	3×10^{6} N/C
At the electron’s orbit in a hydrogen atom	5×10^{11} N/C
On the suface of a Uranium nucleus	3×10^{21} N/C



$\vec{F}$
Positive
test charge
(a)
$\vec{E}$
Electric
field lines
(b)

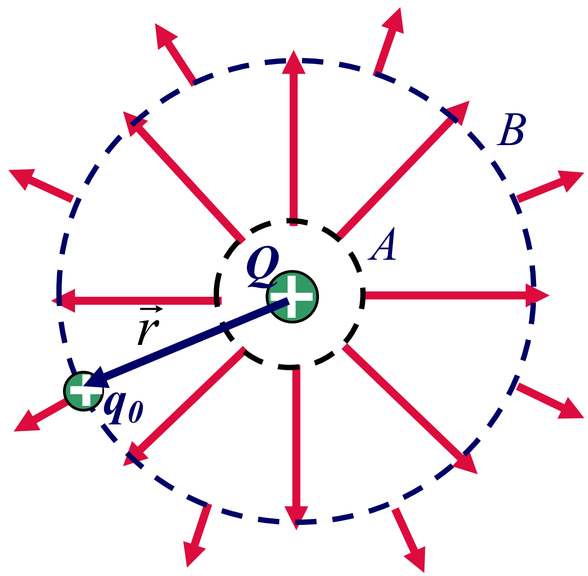

E


E

Electric Field due to a Point Charge Q

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



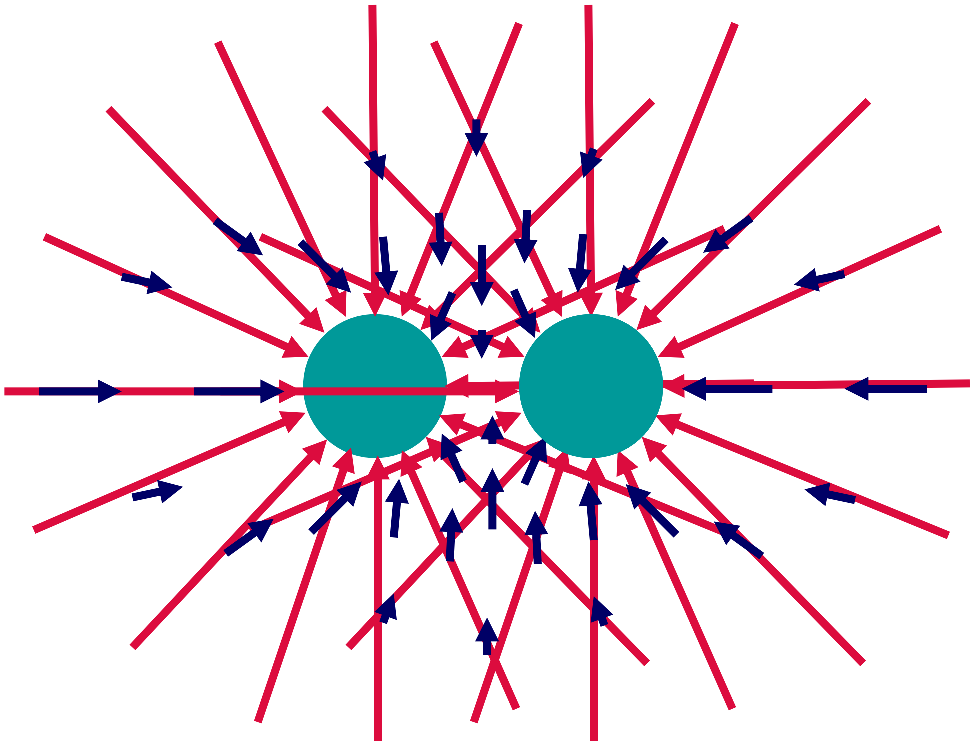
- Direction is radial: outward for +|Q|
  inward for -|Q|
- Magnitude: constant on any spherical shell
- Flux through any shell enclosing Q is the same: $E_A A_A = E_B A_B$

Electric Field due to a group of individual charge

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \ldots + \vec{F}_{0n}$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \ldots + \frac{\vec{F}_{0n}}{q_0}$$
$$= \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$



Example: Electric Field of a Dipole

- Start with

$$E = E_+ - E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-d/2)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(z+d/2)^2}$$
$$= \frac{q}{4\pi\varepsilon_0 z^2}[(1-\frac{d}{2z})^{-2} - (1+\frac{d}{2z})^{-2}]$$

- If d << z, then,

$$[(1-\frac{d}{2z})^{-2} - (1+\frac{d}{2z})^{-2}] = [(1+\frac{2d}{2z(1!)}+\ldots) - (1-\frac{2d}{2z(1!)}+\ldots)] \approx \frac{2d}{z}$$

- So

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d}{z} = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}$$
$$p = qd$$
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

- ✓ $E \sim 1/z^3$
- ✓ E =>0 as d =>0
- ✓ Valid for "far field"