Physics 122: Electricity & Magnetism – Lecture 4
Electric Field

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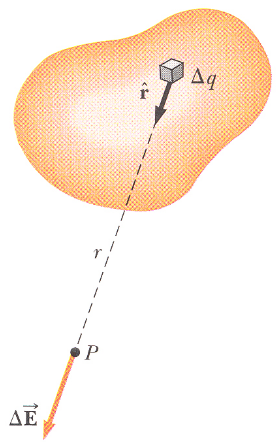
Electric Field of a Continuous Charge Distribution

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r^2}\hat{r}$$

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\lim_{\Delta q\to 0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

- Find an expression for dq:
  - $dq = \lambda dl$ for a line distribution
  - $dq = \sigma dA$ for a surface distribution
  - $dq = \rho dV$ for a volume distribution
- Represent field contributions at P due to point charges dq located in the distribution. Use symmetry,

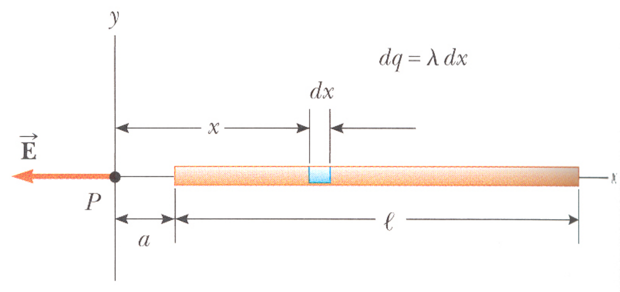
$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

- Add up (integrate the contributions) over the whole distribution, varying the displacement as needed,

$$\vec{E} = \int dE$$

Example: Electric Field Due to a Charged Rod

- A rod of length l has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



- Start with

$$dq = \lambda dx$$

- then,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$$

- So

$$E = \int_a^{l+a} \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{l+a}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_a^{l+a}$$

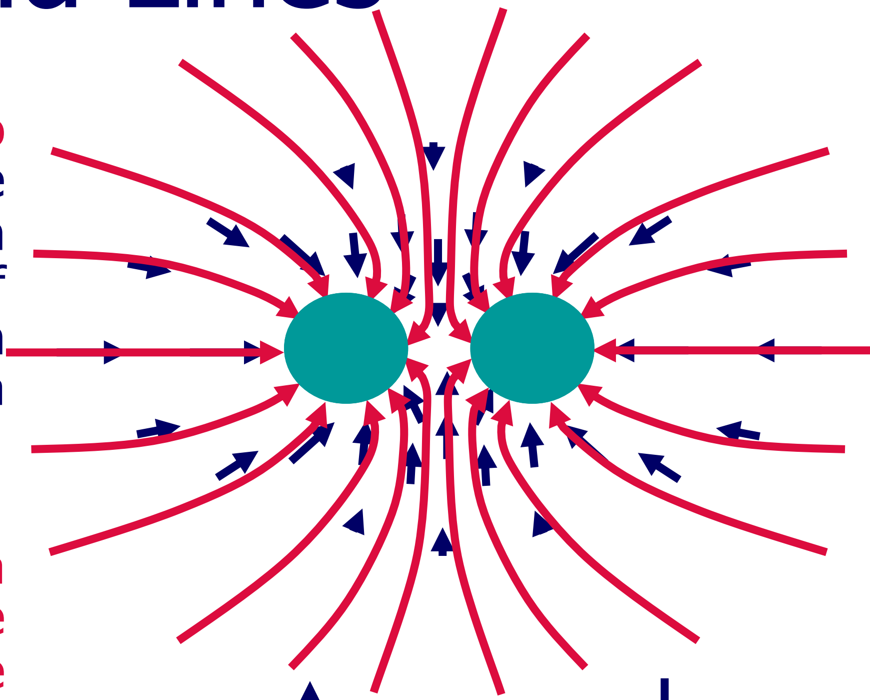
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{l}\left(\frac{1}{a} - \frac{1}{l+a}\right) = \frac{Q}{4\pi\varepsilon_0 a(l+a)}$$

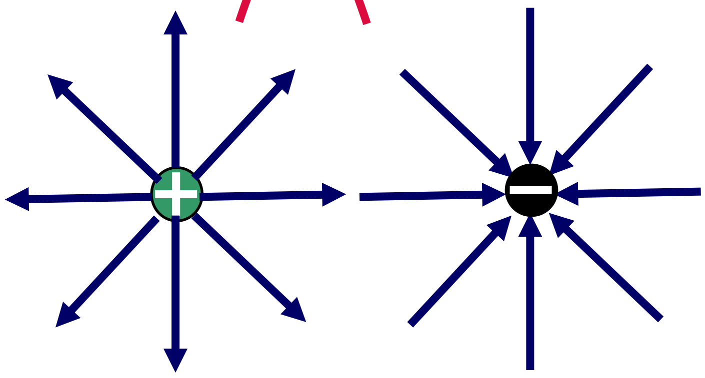
- Finalize
  - *l* => 0 ?
  - *a* >> *l* ?

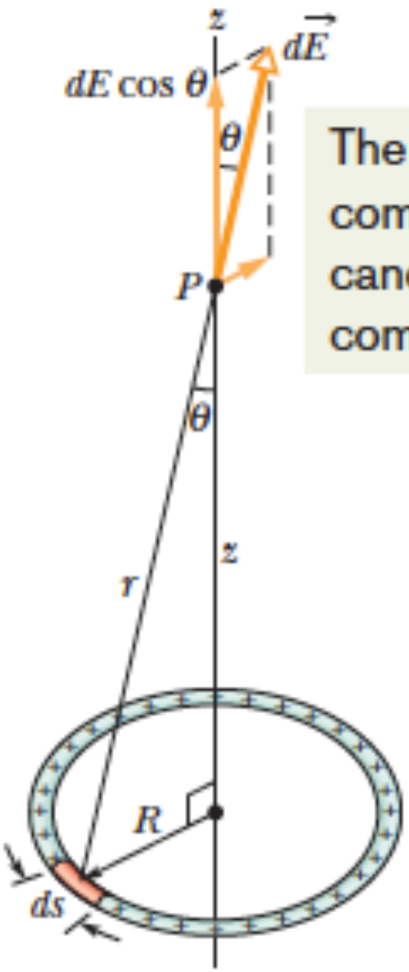
Electric Field Lines

- The electric field vector is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector. The direction of the line is that of the force on a positive test charge placed in the field.

- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.







The perpendicular components just cancel but the parallel components add.

$$dq = \lambda\, ds.$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}.$$

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}.$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds.$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

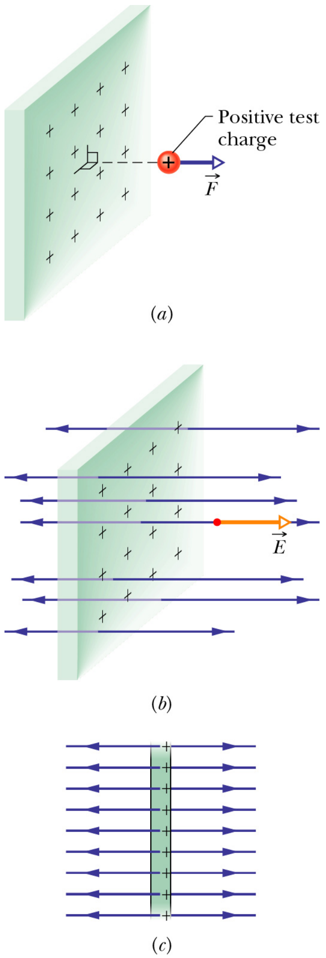
$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring).}$$

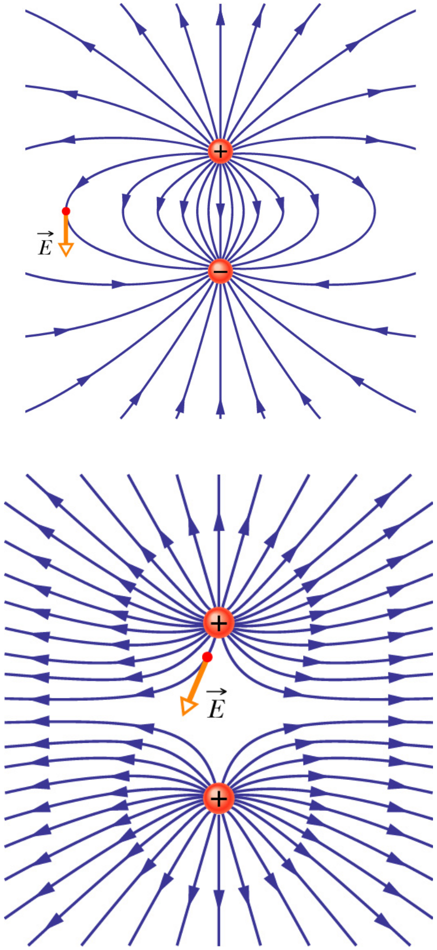
Electric Field Lines

- The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.

- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

- No two field lines can cross.



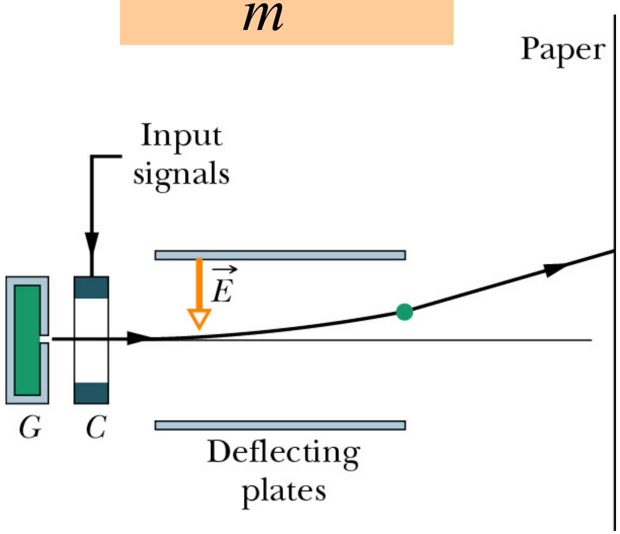


Motion of a Charged Particle in a Uniform Electric Field

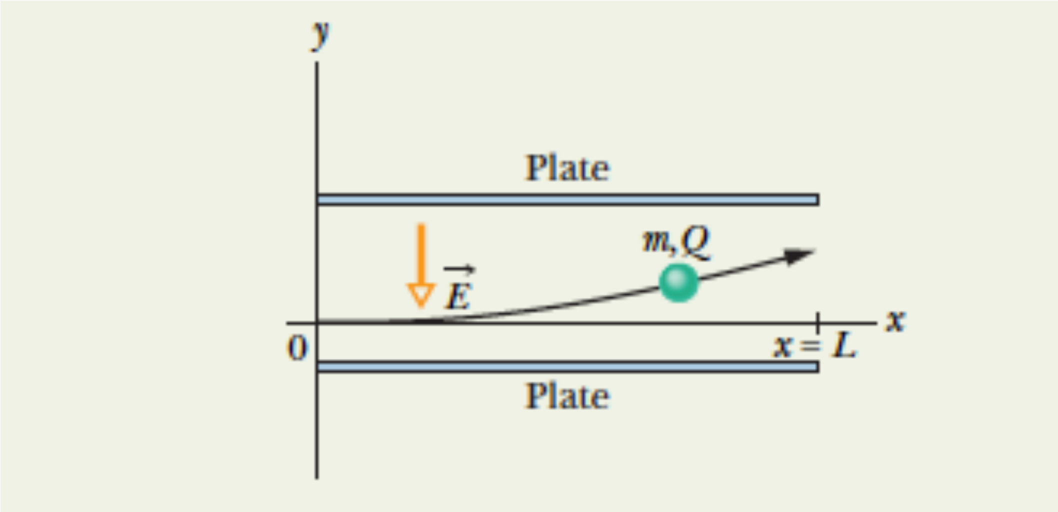
$$\vec{F} = q\vec{E}$$

$$\vec{F} = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q\vec{E}}{m}$$



- If the electric field E is uniform (magnitude and direction), the electric force F on the particle is constant.
- If the particle has a positive charge, its acceleration a and electric force F are in the direction of the electric field E.
- If the particle has a negative charge, its acceleration a and electric force F are in the direction opposite the electric field E.



printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

$$a_y = \frac{F}{m} = \frac{QE}{m}.$$

$$y = \tfrac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t,$$

$$\begin{aligned} y &= \frac{QEL^2}{2mv_x^2} \\ &= \frac{(1.5 \times 10^{-13}\ \mathrm{C})(1.4 \times 10^{6}\ \mathrm{N/C})(1.6 \times 10^{-2}\ \mathrm{m})^2}{(2)(1.3 \times 10^{-10}\ \mathrm{kg})(18\ \mathrm{m/s})^2} \\ &= 6.4 \times 10^{-4}\ \mathrm{m} \end{aligned}$$

A Dipole in an Electric Field

- Start with

$$\tau = Fx\sin\theta + F(d-x)\sin\theta = Fd\sin\theta$$

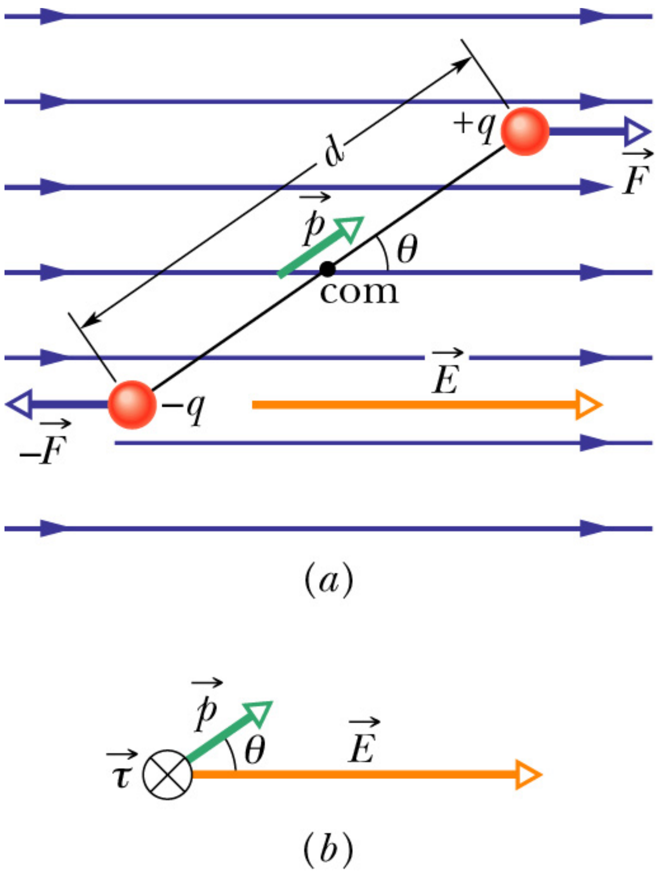
- Then

$$F = qE \quad \text{and} \quad p = qd$$

- So

$$|\tau| = pE\sin\theta$$

$$\vec{\tau} = \vec{p}\times\vec{E}$$



A Dipole in an Electric Field

- Start with

$$dW = \tau d\theta$$

- Since

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} pE \sin\theta d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta d\theta$$

$$= pE[-\cos\theta]_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

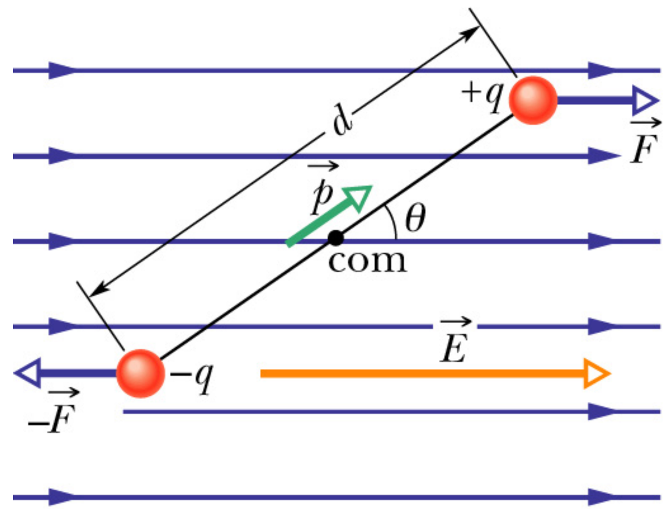
- Choose

$U_i = 0$ at $\theta_i = 90°$

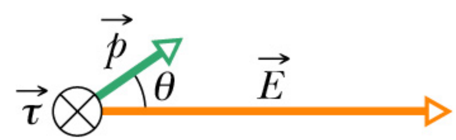
- So

$$U = -pE\cos\theta$$

$$U = -\vec{p} \cdot \vec{E}$$

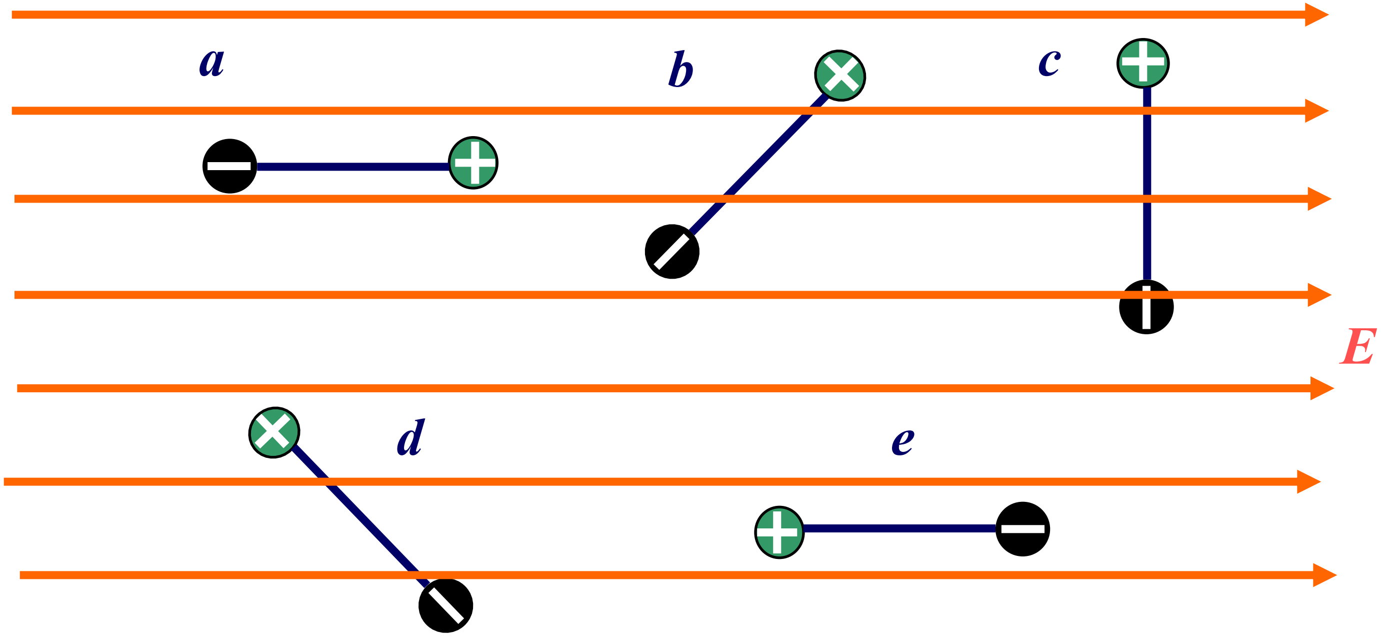


(a)



(b)

4. In which configuration, the potential energy of the dipole is the greatest?



Summary

- Electric field E at any point is defined in terms of the electric force F that acts on a small positive test charge placed at that point divided by the magnitude q_0 of the test charge:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

- Electric field lines provide a means for visualizing the direction and magnitude of electric fields. The electric field vector at any point is tangent to a field line through that point. The density of field lines in any region is proportional to the magnitude of the electric field in that region.
- Field lines originate on positive charge and terminate on negative charge.
- Field due to a point charge:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

The direction is away from the point charge if the charge is positive and toward it if the charge is negative.

- Field due to an electric dipole:

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

- Field due to a continuous charge distribution: treat charge elements as point charges and then summing via inegration, the electric field vectors produced by all the charge elements.
- Force on a point charge in an electric field:

$$\vec{F} = q\vec{E}$$

- Dipole in an electric field:
  - The field exerts a torque on the dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

  - The dipole has a potential energy U associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}$$