Physics 122: Electricity & Magnetism – Lecture 8 Electric Potential

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Potential due to a group of point charges

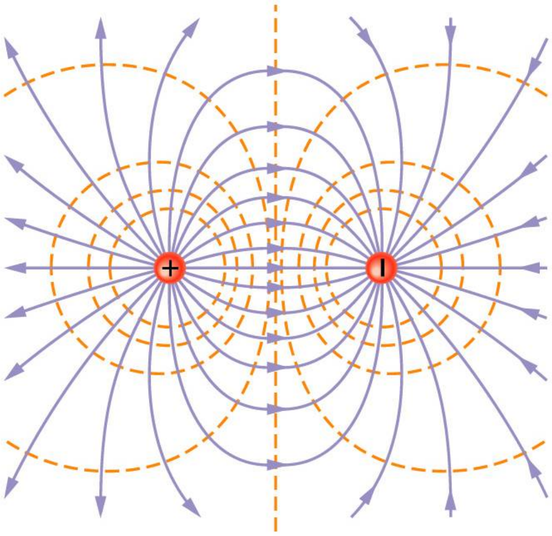
- Use superposition

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{s} = -\sum_{i=1}^{n} \int_{\infty}^{r} \vec{E}_i \cdot d\vec{s} = \sum_{i=1}^{n} V_i$$

- For point charges

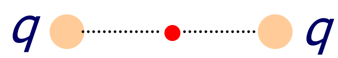
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

- The sum is an algebraic sum, not a vector sum.
- E may be zero where V does not equal to zero.
- V may be zero where E does not equal to zero.

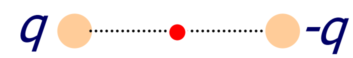


Electric Field and Electric Potential

4. Which of the following figures have V=0 and E=0 at red point?



A



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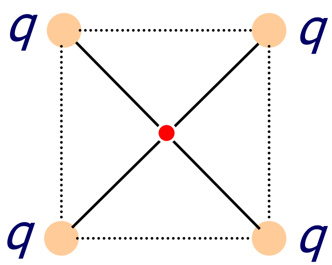

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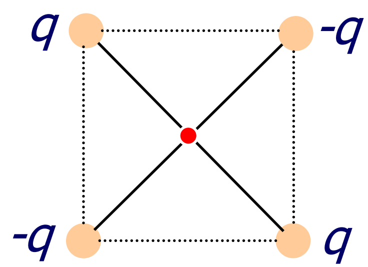

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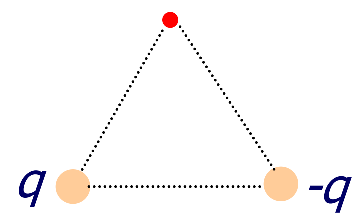
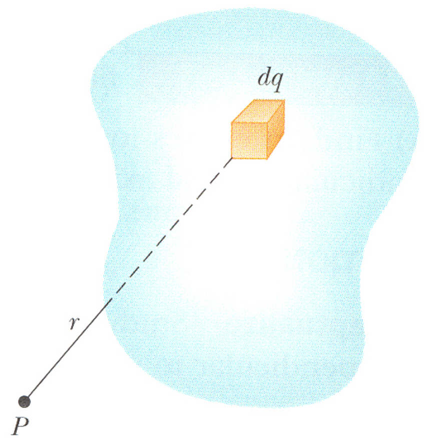


E

Potential due to a Continuous Charge Distribution



- Find an expression for dq:
  - $dq = \lambda dl$ for a line distribution
  - $dq = \sigma dA$ for a surface distribution
  - $dq = \rho dV$ for a volume distribution
- Represent field contributions at P due to point charges dq located in the distribution.

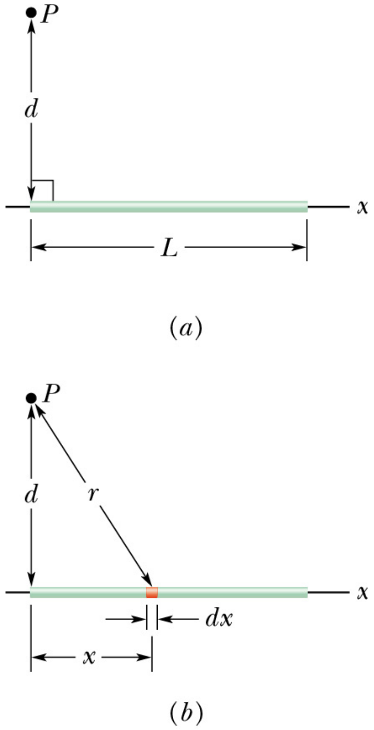
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

- Integrate the contributions over the whole distribution, varying the displacement as needed,

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

Example: Potential Due to a Charged Rod

- A rod of length L located along the x axis has a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance d from the origin.



- Start with

$$dq = \lambda dx$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+d^2)^{1/2}}$$

- then,

$$V = \int dV = \int_0^L \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{(x^2+d^2)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x+(x^2+d^2)^{1/2}\right)\right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L+(L^2+d^2)^{1/2}\right)-\ln d\right]$$

- So

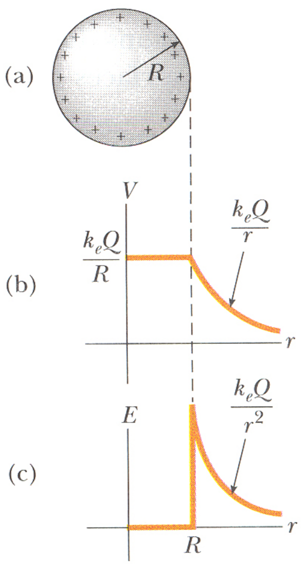
$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L+(L^2+d^2)^{1/2}}{d}\right]$$

Potential Due to a Charged Isolated Conductor

- According to Gauss' law, the charge resides on the conductor's outer surface.
- Furthermore, the electric field just outside the conductor is perpendicular to the surface and field inside is zero.
- Since

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} = 0$$

- Every point on the surface of a charged conductor in equilibrium is at the same electric potential.
- Furthermore, the electric potential is constant everywhere inside the conductor and equal to its value to its value at the surface.



Calculating the Field from the Potential

- Suppose that a positive test charge q_0 moves through a displacement ds from on equipotential surface to the adjacent surface.
- The work done by the electric field on the test charge is $W = -dU = -q_0 dV$.
- The work done by the electric field may also be written as $W = q_0 \vec{E} \cdot d\vec{s}$
- Then, we have $$-q_0 dV = q_0 E(\cos\theta)ds$$ $$E\cos\theta = -\frac{dV}{ds}$$
- So, the component of E in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

$$E_s = -\frac{\partial V}{\partial s}$$

- If we know V(x, y, z),

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

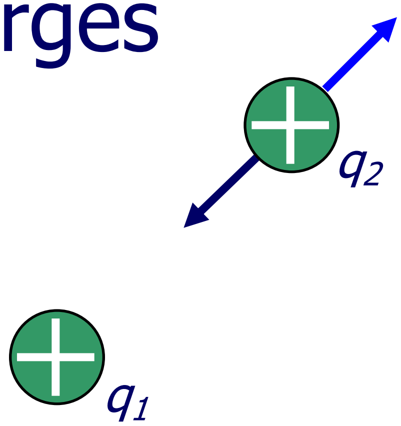
Electric Potential Energy of a System of Point Charges

$$\Delta U = U_f - U_i = -W$$

$$W = \vec{F} \cdot \Delta \vec{r} = q\vec{E} \cdot \Delta \vec{r}$$

$$W_{app} = -W$$

$$\Delta U = U_f - U_i = W_{app}$$



- Start with (set $U_i=0$ at ∞ and $U_f=U$ at r)

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r}$$

- We have

$$U = q_2 V = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

- If the system consists of more than two charged particles, calculate U for each pair of charges and sum the terms algebraically.

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

Summary

- Electric Potential Energy: a point charge moves from i to f in an electric field, the change in electric potential energy is
  $$\Delta U = U_f - U_i = -W$$
- Electric Potential Difference between two points i and f in an electric field:
  $$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$
- Equipotential surface: the points on it all have the same electric potential. No work is done while moving charge on it. The electric field is always directed perpendicularly to corresponding equipotential surfaces.
- Finding V from E:
  $$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_i^f \vec{E} \cdot d\vec{s}$$
- Potential due to point charges:
  $$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$
- Potential due to a collection of point charges:
  $$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$
- Potential due to a continuous charge distribution:
  $$V = \int dV = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$
- Potential of a charged conductor is constant everywhere inside the conductor and equal to its value to its value at the surface.
- Calculatiing E from V:
  $$E_s = -\frac{\partial V}{\partial s} \quad E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$
- Electric potential energy of system of point charges:
  $$U = q_2 V = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$