Physics 122: Electricity & Magnetism – Lecture 10 Capacitance

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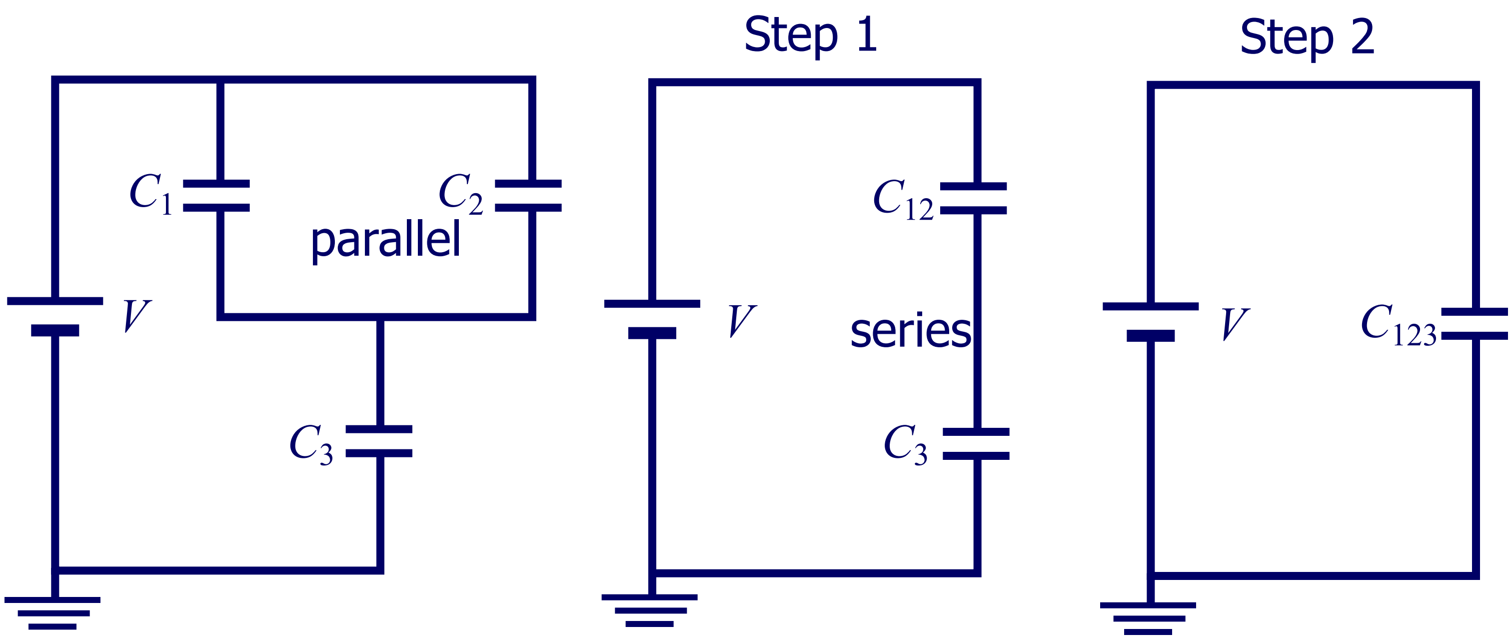
Example Capacitor Circuit



$$C_{12} = C_1 + C_2$$

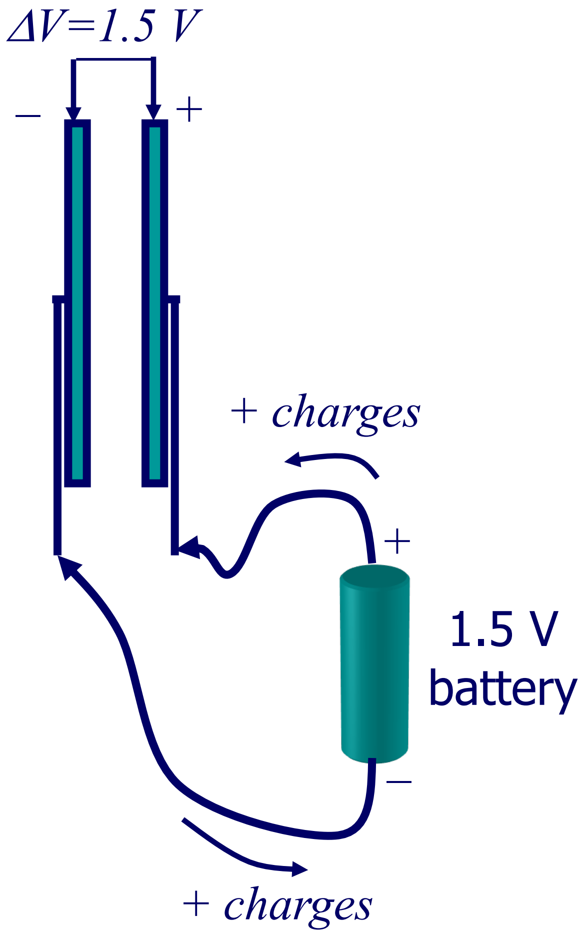
$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$C_{123} = \frac{C_{12} C_3}{C_{12} + C_3}$$

$C_1 = 12.0\ \mu\text{F}$, $C_2 = 5.3\ \mu\text{F}$, $C_3 = 4.5\ \mu\text{F}$

$C_{123} = (12 + 5.3)4.5/(12+5.3+4.5)\ \mu\text{F} = 3.57\ \mu\text{F}$

Capacitors Store Energy



$$U = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$

Capacitors Store Energy

$$U = \tfrac{1}{2} C V^2 = \frac{\varepsilon_0 A}{2d} V^2$$

$$u = \frac{U}{vol} = \frac{\varepsilon_0 A}{2dAd} V^2 = \tfrac{1}{2} \varepsilon_0 \left(\frac{V}{d} \right)^2$$

$$V = -\int \vec{E} \cdot d\vec{s} = Ed$$

$$u = \tfrac{1}{2} \varepsilon_0 E^2$$

Energy stored in electric field

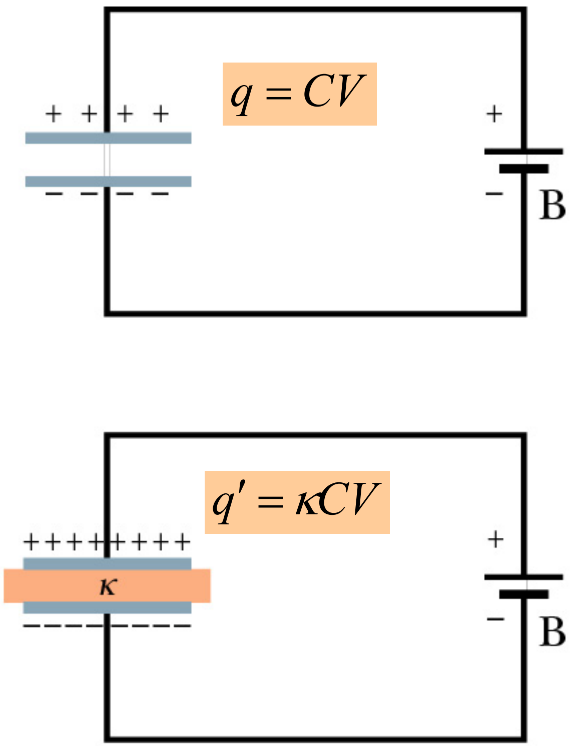
Dielectrics

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer Oil	4.5	
Pyrex	4.7	14
Ruby Mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20º C)	80.4	
Water (50º C)	78.5	
Titania Ceramic	130	
Strontium Titanate	310	8

$$C' = \frac{\kappa \varepsilon_0 A}{d} = \kappa C$$

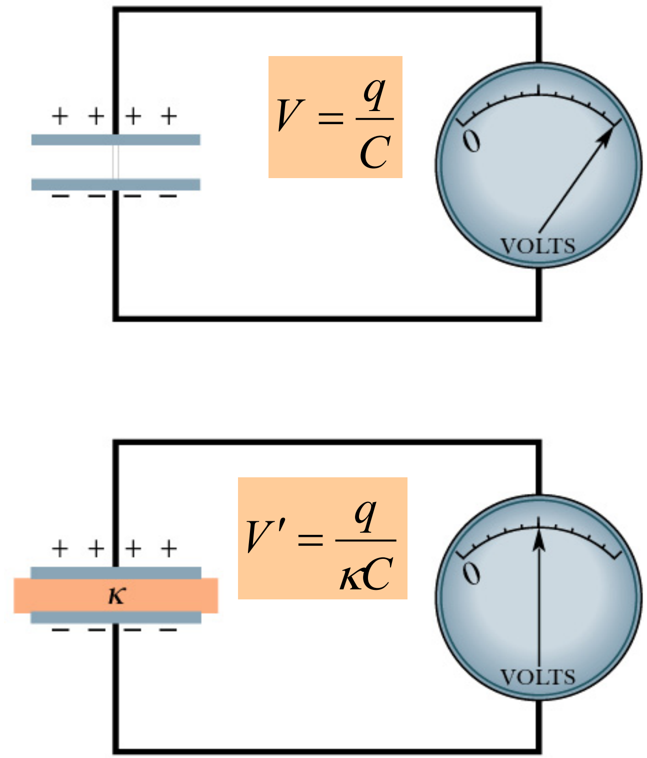
What Happens When You Insert a Dielectric?

- With battery attached, V=const, so more charge flows to the capacitor



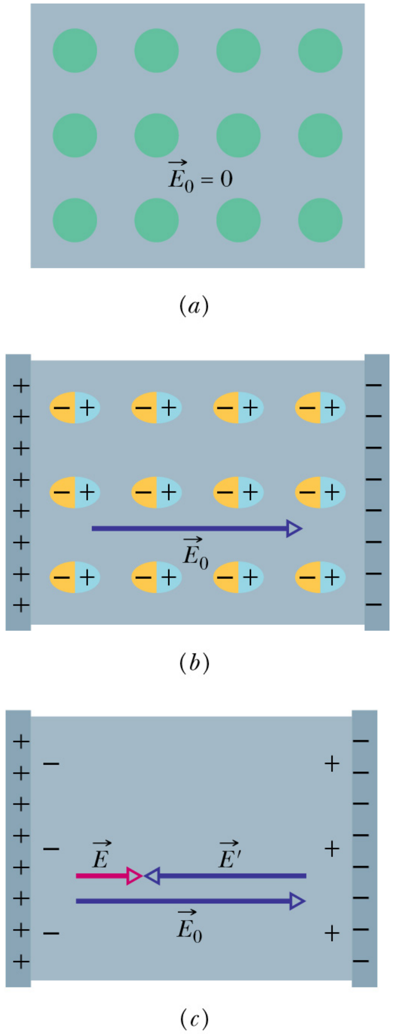
V = a constant

- With battery disconnected, q=const, so voltage (for given q) drops.



q = a constant

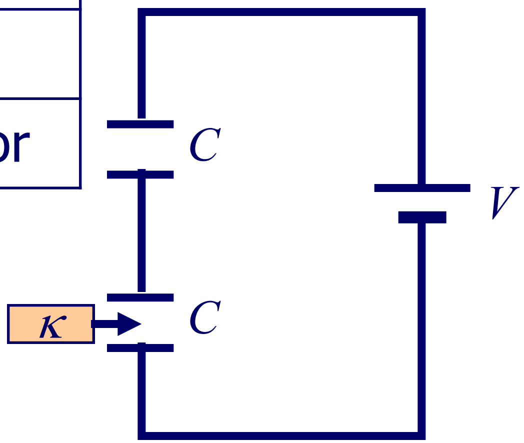
What Does the Dielectric Do?



Two identical parallel plate capacitors are connected in series to a battery as shown below. If a dielectric is inserted in the lower capacitor, which of the following increase for that capacitor?

A. I and III.
B. I, II and IV.
C. I, II and III.
D. All except II.
E. All increase.

I.	Capacitance of capacitor
II.	Voltage across capacitor
III.	Charge on capacitor
IV.	Energy stored on capacitor



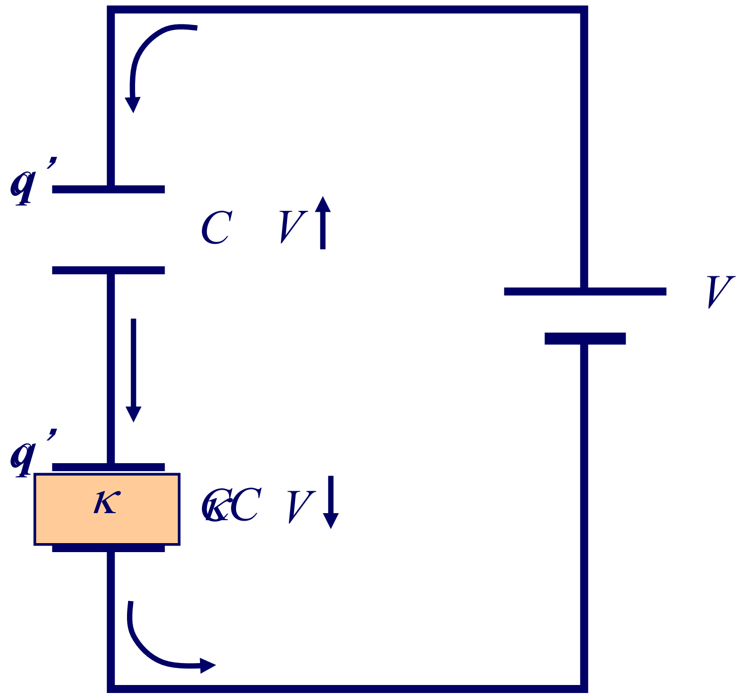
$q = CV$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$U = \frac{q^2}{2C} = \frac{1}{2} C V^2$

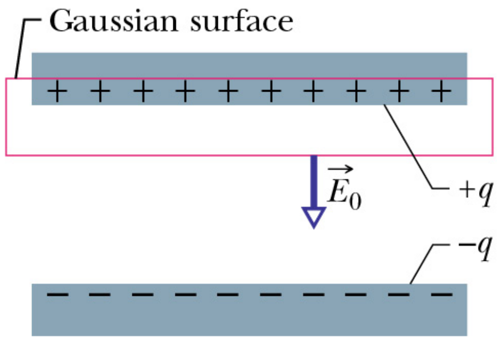
A Closer Look

- Insert dielectric
- Capacitance goes up by κ
- Charge increases
- Charge on upper plate comes from upper capacitor, so its charge also increases.
- Since $q' = CV_1$ increases on upper capacitor, V_1 must increase on upper capacitor.
- Since total $V = V_1 + V_2$ = constant, V_2 must decrease.

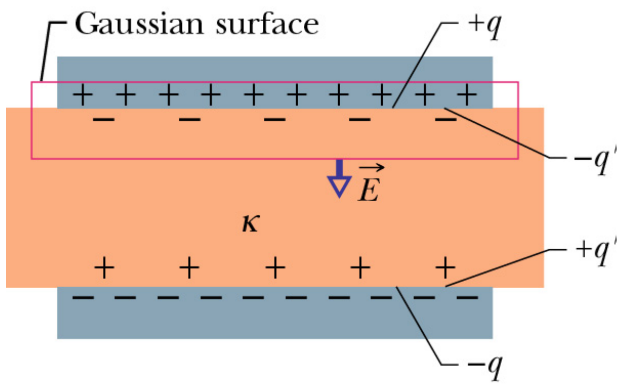


Dielectrics and Gauss' Law

- Gauss' Law holds without modification, but notice that the charge enclosed by our gaussian surface is less, because it includes the induced charge q' on the dielectric.
- For a given charge q on the plate, the charge enclosed is $q - q'$, which means that the electric field must be smaller. The effect is to weaken the field.
- When attached to a battery, of course, more charge will flow onto the plates until the electric field is again E_0.



(a)



(b)

Summary

- Capacitance says how much charge is on an arrangement of conductors for a given potential. $q = CV$
- Capacitance depends only on geometry
  - Parallel Plate Capacitor
  - Cylindrical Capacitor
  - Spherical Capacitor
  - Isolated Sphere

$$C = \frac{\varepsilon_0 A}{d} \qquad C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \qquad C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad C = 4\pi\varepsilon_0 R$$

- Units, F (farad) = C^2/Nm or C/V (note ε_0 = 8.85 pF/m)
- Capacitors in parallel in series

$$C_{eq} = \sum_{j=1}^{n} C_j \qquad \frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

- Energy and energy density stored by capacitor

$$U = \tfrac{1}{2} CV^2 \qquad u = \tfrac{1}{2}\varepsilon_0 E^2$$

- Dielectric constant increases capacitance due to induced, opposing field. $C' = \kappa C$ κ is a unitless number.