EEE328
Digital Signal Processing

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

The z-Transform

EEE328 Digital Signal Processing

Lecture 7

Agenda

- The z-Transform
- Bilateral z-Transform
- Unilateral z-Transform

- The z-Transform

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$z = e^{j\omega}$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

(Bilateral) z-Transform

- The z-Transform

$$x[n] \overset{z}{\leftrightarrow} X(z)$$

$$Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = X(z)$$

- The z-Transform

$$\mathcal{X}(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

Unilateral z-Transform

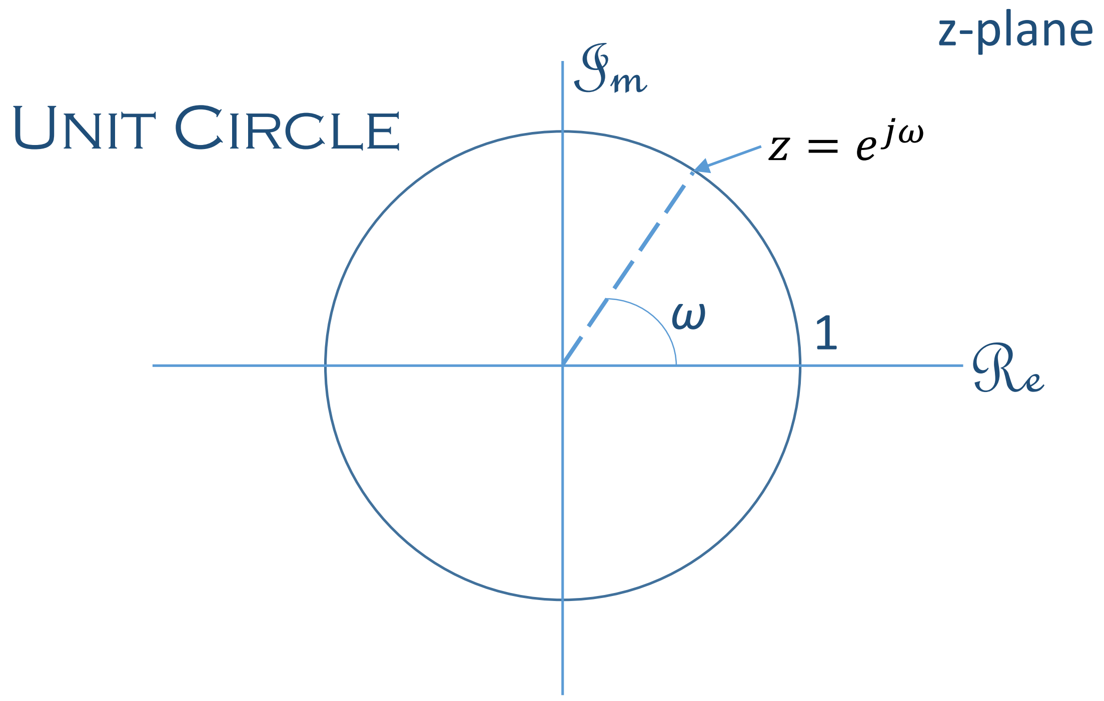
- The z-Transform

$$z = re^{j\omega}$$

$$X\left(re^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n] \left(re^{j\omega}\right)^{-n}$$

$$X\left(re^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n] r^{-n} \, e^{-j\omega n}$$

- The z-Transform



The unit circle in the complex z-plane

- The z-Transform

$$\sum_{n=-\infty}^{\infty} |x[n]\,r^{-n}| < \infty$$

Convergenge

$$\sum_{n=-\infty}^{\infty} |x[n]|\,|r|^{-n} < \infty$$

- The z-Transform

Example:

$$x[n] = a^n u[n]$$ Right-sided sequence

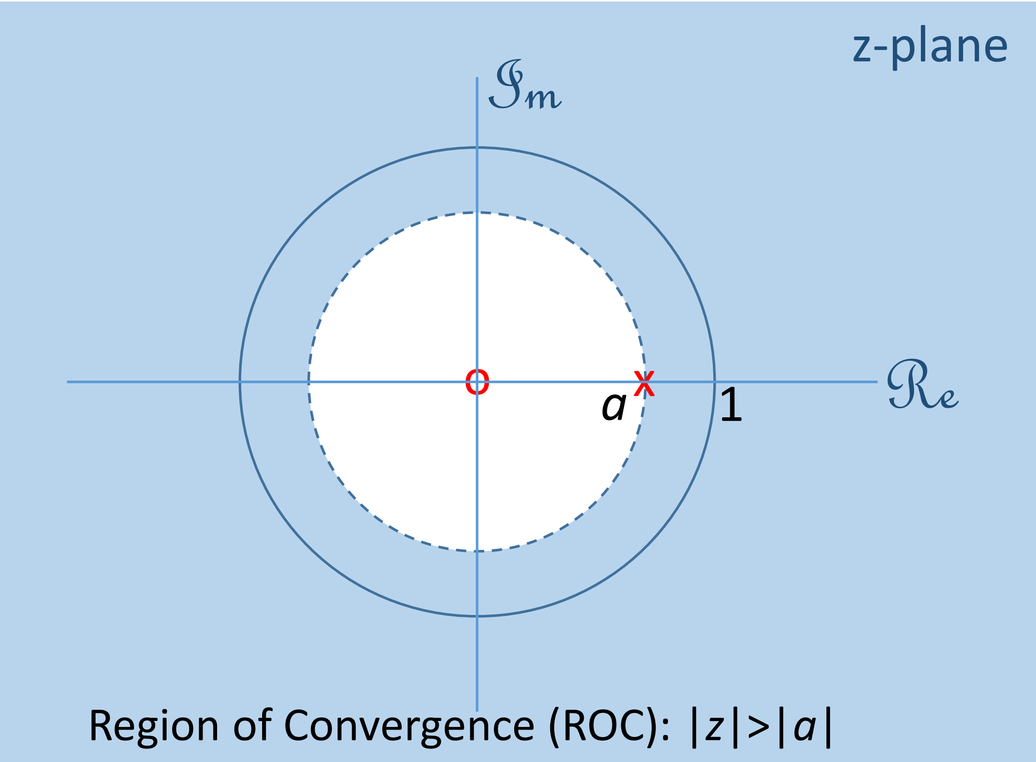
$$X(z) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

For convergence of $X(z)$

$$\sum_{n=0}^{\infty} |az^{-1}|^n < \infty \Rightarrow |az^{-1}| < 1 \Rightarrow |z| < |a|$$

$$X(z) = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}, \qquad |z| < |a|$$

- Example (Cont.) for $|a|<1$



o : Zero

x : Pole

References

- Signals & Systems, Second Edition, A. V. Oppenheim, A. S. Willsky with S. H. Nawab, Prentice Hall, 1997
- Discrete-Time Signal Processing, Second Edition, A. V. Oppenheim, R. W. Schafer with J. R. Buck, Prentice Hall, 1999