EEE328
Digital Signal Processing

Ankara University

Faculty of Engineering

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z-Transform and The Region of Convergence (ROC) Properties & Inverse z-Transform

EEE328 Digital Signal Processing

Lecture 8

Agenda

- Properties of The Region of Convergence (ROC) for The z-Transform
- Inverse z-Transform
- z-Transform Properties

Properties of The Region of Convergence (ROC) for The z-Transform

- 1. The ROC is a ring or disk in the z-plane with a center at the origin; $0 \le r_R \le r_L \le \infty$.
- 2. Fourier transform of $x[n]$ converges absolutely if and only if the ROC of the z-transform of $x[n]$ included the unit circle.
- 3. The ROC can not includes any poles.
- 4. If $x[n]$ is a finite sequence, then the ROC is the entire z-plane, except possibly $z=0$ or $z=\infty$.
- 5. If $x[n]$ is a right-sided sequence, the ROC extends outward from the outermost finite pole in $X(z)$ to (and possibly including) $z=\infty$.

Properties of The Region of Convergence (ROC) for The z-Transform

- 6. If $x[n]$ is a left-sided sequence, the ROC extends inward from the innermost nonzero pole in $X(z)$ to $z= 0$.
- 7. A two-sided sequence is an infinite-duration which is neither right-sided nor left-sided. If $x[n]$ is a a two-sided sequence, the ROC will consist of a ring in the z-plane, bounded on the interior and exterior by a pole and consistent with property 3, not including any poles.
- 8. The ROC must be a connected region.

Properties of The Region of Convergence (ROC) for The z-Transform

- Stability

  The ROC includes the unit circle.

  ($h[n]$ is absolutely summable, BIBO)

- Causality

  ROC extends outward from the outermost finite pole

  ($h[n]=0$, $n<0$ [right-sided], Output depends only on input for $n\geq0$)

The Inverse z-Transform

- Inspection Method
- Partial Fraction Expansion
- Power Series Expansion

z-Transform Properties

$$x[n] \overset{\mathcal{Z}}{\leftrightarrow} X(z), \qquad ROC = R_x$$

$$x_1[n] \overset{\mathcal{Z}}{\leftrightarrow} X_1(z), \qquad ROC = R_{x_1}$$

$$x_2[n] \overset{\mathcal{Z}}{\leftrightarrow} X_2(z), \qquad ROC = R_{x_2}$$

z-Transform Properties

- Linearity

$$x[n] \overset{z}{\leftrightarrow} X(z), \qquad ROC = R_x$$

$$ax_1[n] + bx_2[n] \overset{z}{\leftrightarrow} aX_1(z) + bX_2(z), \qquad ROC = R_{x_1} \cap R_{x_2}$$

z-Transform Properties

- Time-Shifting

$$x[n-n_0] \overset{z}{\leftrightarrow} z^{-n_0}X(z), \qquad ROC = R_x$$ except for the possible addition or deletion of z=0 and z=∞

- Multiplication by an Exponential Sequence

$$z_0^n x[n] \overset{z}{\leftrightarrow} X\left(\frac{z}{z_0}\right), \qquad ROC = |z_0|R_x$$

z-Transform Properties

- Differentiation

$$nx[n] \overset{z}{\leftrightarrow} -z\frac{dX(z)}{dz}, \qquad ROC = R_x$$

- Conjugation of a Complex Sequence

$$x^*[n] \overset{z}{\leftrightarrow} X^*(z^*), \qquad ROC = R_x$$

z-Transform Properties

- Time Reversal

$$x^*[-n] \overset{\mathcal{Z}}{\leftrightarrow} X^*(1/z^*), \qquad ROC = 1/R_x$$

$$x[-n] \overset{\mathcal{Z}}{\leftrightarrow} X(1/z), \qquad ROC = 1/R_x$$

z-Transform Properties

- Convolution of Sequences

$$x_1[n] * x_2[n] \overset{z}{\leftrightarrow} X_1(z)X_2(z), \qquad ROC = R_{x_1} \cap R_{x_2}$$

- Initial Value Theorem

$$x[n] = 0, \qquad n < 0$$

$$\Rightarrow \quad x[0] = \lim_{z \to \infty} X(z)$$

References

- Signals & Systems, Second Edition, A. V. Oppenheim, A. S. Willsky with S. H. Nawab, Prentice Hall, 1997
- Discrete-Time Signal Processing, Second Edition, A. V. Oppenheim, R. W. Schafer with J. R. Buck, Prentice Hall, 1999