EEE328
Digital Signal Processing

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Discrete-Time Processing of Continuous-Time Signals
Continuous-Time Processing of Discrete-Time Signals

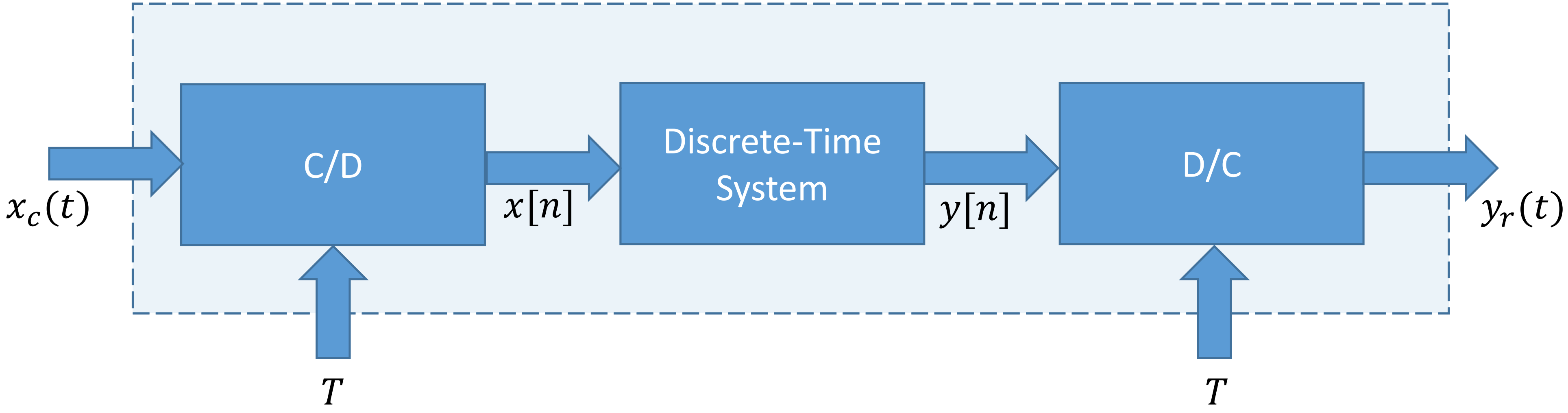
EEE328 Digital Signal Processing

Lecture 11

Agenda

- Discrete-Time Processing of Continuous-Time Signals
- Linear Time-Invariant (LTI) Discrete-Time Systems
- Impulse Invariance
- Continuous-Time Processing of Discrete-Time Signals

Discrete-Time Processing of Continuous-Time Signals



Linear Time-Invariant (LTI) Discrete-Time Systems

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$Y_r(j\Omega) = H_r(j\Omega)H(e^{j\Omega T})X(e^{j\Omega T})$$

$$\omega = \Omega T$$

$$Y_r(j\Omega) = H_r(j\Omega)H(e^{j\Omega T})\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\Omega - \frac{2\pi k}{T}))$$

If $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$, then the ideal low-pass reconstruction filter $H_r(j\Omega)$ cancels the factor $1/T$ and selects only the term in above equation for k=0, i.e.;

$$Y_r(j\Omega) = \begin{cases} H(e^{j\Omega T})X_c(j\Omega), & |\Omega| < \pi/T \\ 0, & |\Omega| \geq \pi/T \end{cases}$$

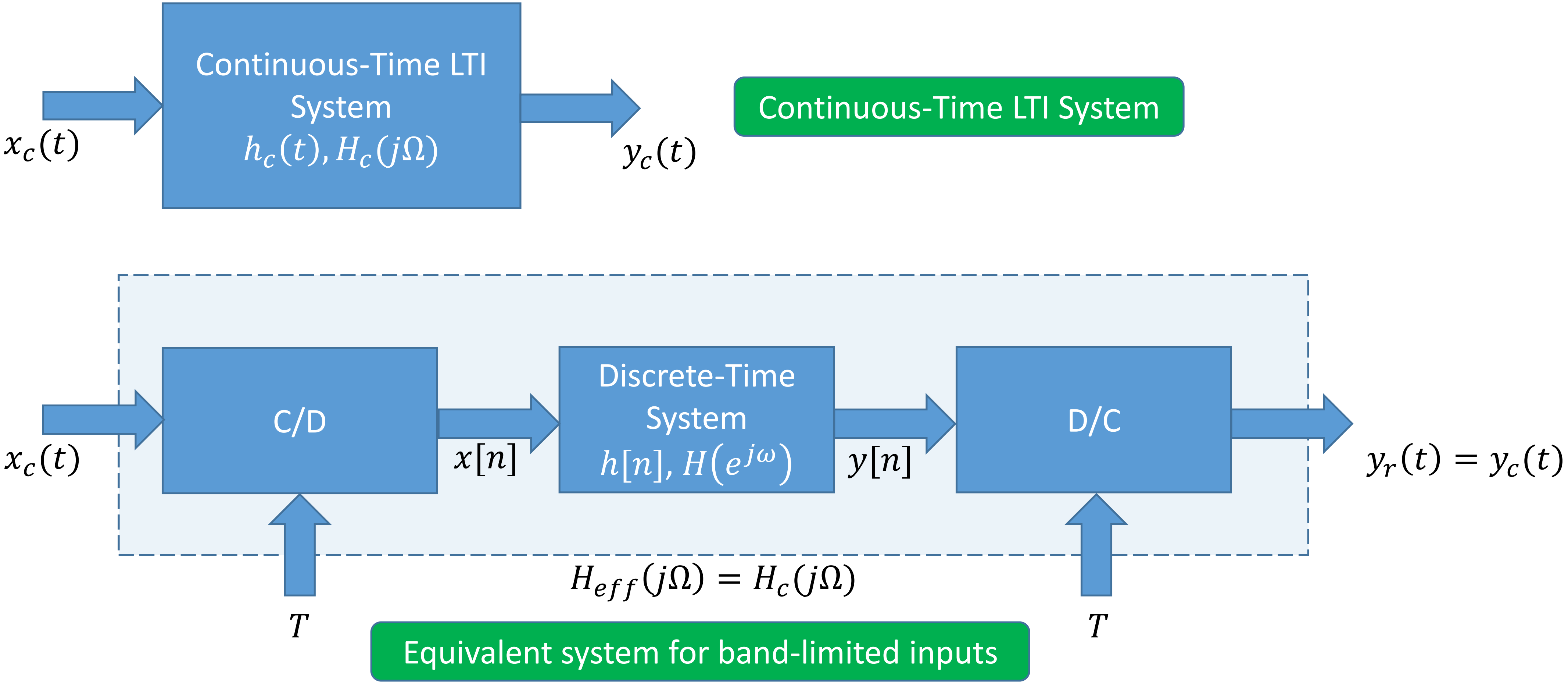
Linear Time-Invariant (LTI) Discrete-Time Systems

Thus if $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$ is bandlimited and the sampling rate is above the Nyquist rate, the output is related to the input through an equation of the form

$$Y_r(j\Omega) = H_{eff}(j\Omega)X_c(j\Omega)$$

$$H_{eff}(j\Omega) = \begin{cases} H\left(e^{j\Omega T}\right), & |\Omega| < \pi/T \\ 0, & |\Omega| \geq \pi/T \end{cases}$$

Impulse Invariance



Impulse Invariance

$$H(e^{j\omega}) = H_c(j\omega/T), \qquad |\omega| < \pi$$

$$H_c(j\Omega) = 0, \qquad |\Omega| \geq \pi/T$$

$$h[n] = Th_c(nT)$$

Continuous-Time Processing of Discrete-Time Signals

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\frac{\pi(t-nT)}{T})}{\pi(t-nT)/T}$$

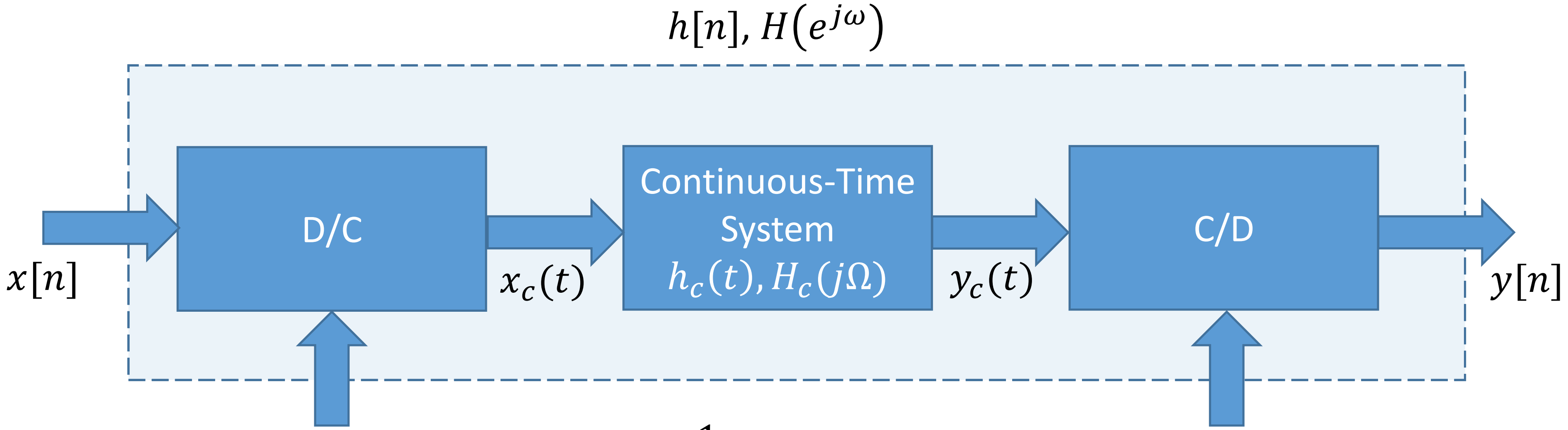
$$y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin(\frac{\pi(t-nT)}{T})}{\pi(t-nT)/T}$$

$$X_c(j\Omega) = TX\left(e^{j\Omega T}\right), \qquad |\Omega| < \pi/T$$

$$Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega), \qquad |\Omega| < \pi/T$$

$$Y\left(e^{j\omega}\right) = \frac{1}{T} Y_c(j\omega/T), \qquad |\omega| < \pi$$

Continuous-Time Processing of Discrete-Time Signals



$$H(e^{j\omega}) = \frac{1}{T} H_c(j\omega/T), \qquad |\omega| < \pi$$

$$H_c(j\Omega) = H(e^{j\Omega T}), \qquad |\Omega| < \pi/T$$

References

- Signals & Systems, Second Edition, A. V. Oppenheim, A. S. Willsky with S. H. Nawab, Prentice Hall, 1997
- Discrete-Time Signal Processing, Second Edition, A. V. Oppenheim, R. W. Schafer with J. R. Buck, Prentice Hall, 1999