EEE201
Circuit Analysis II

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Balanced Three-Phase Circuits

EEE201 Circuit Analysis II

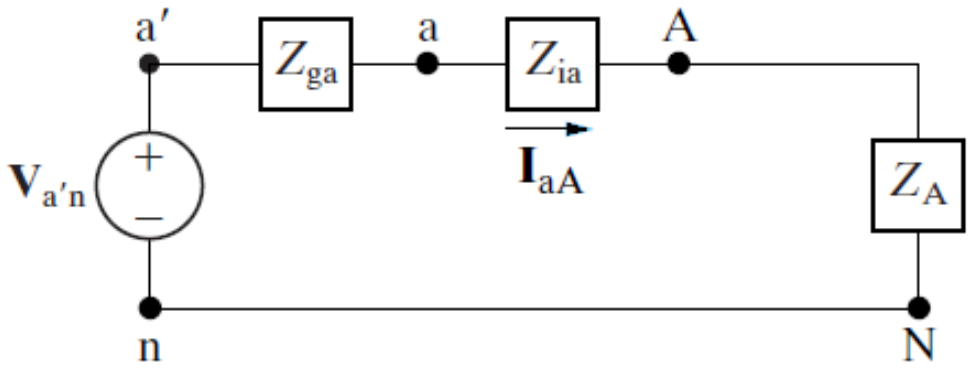
Lecture 8

Agenda

- Y-Δ Circuit
- Power Calculations in a Balanced Y Load
- Power Calculations in a Balanced Δ Load

Y-Δ Circuit

$$Z_Y = \frac{Z_\Delta}{3} \quad \rightarrow$$



Single-phase equivalent circuit

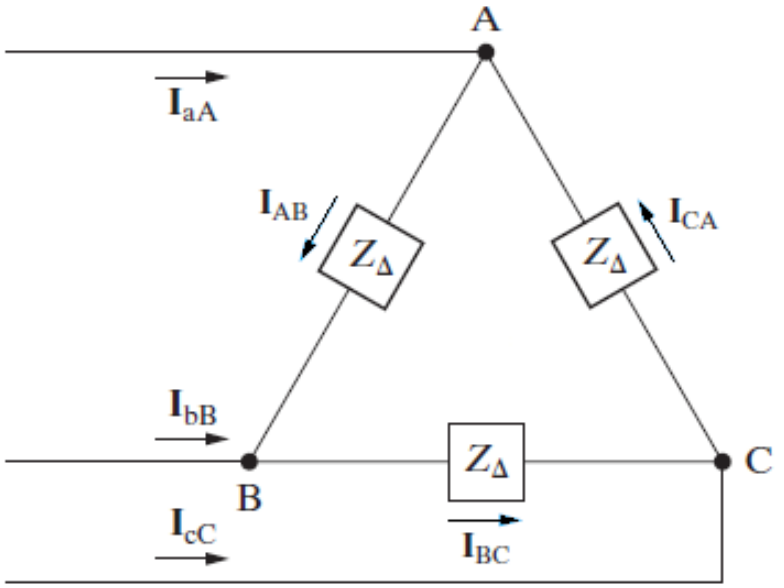
Y-Δ Circuit

Phase currents:

$$\boldsymbol{I}_{AB} = I_{\emptyset}\angle 0°$$

$$\boldsymbol{I}_{BC} = I_{\emptyset}\angle -120°$$

$$\boldsymbol{I}_{CA} = I_{\emptyset}\angle +120°$$



Line currents:

$$\boldsymbol{I}_{aA} = \sqrt{3}I_{\emptyset}\angle -30°$$

$$\boldsymbol{I}_{bB} = \sqrt{3}I_{\emptyset}\angle -150°$$

$$\boldsymbol{I}_{cC} = \sqrt{3}I_{\emptyset}\angle +90°$$

Power Calculations in a Balanced Y Load

Average power in terms of the phase voltage and current:

$$V_{\emptyset} = |\boldsymbol{V}_{AN}| = |\boldsymbol{V}_{BN}| = |\boldsymbol{V}_{CN}|$$

$$I_{\emptyset} = |\boldsymbol{I}_{aA}| = |\boldsymbol{I}_{bB}| = |\boldsymbol{I}_{cC}|$$

$$\theta_{\emptyset} = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}$$

$$P_A = P_B = P_C = P_{\emptyset} = V_{\emptyset} I_{\emptyset} \cos\theta_{\emptyset}$$

Power Calculations in a Balanced Y Load

Total average power:

$$P_T = 3P_\emptyset = 3V_\emptyset I_\emptyset \cos\theta_\emptyset$$

Total average power in terms of the line voltage and current:

$$P_T = \sqrt{3} V_L I_L \cos\theta_\emptyset$$

Power Calculations in a Balanced Y Load

Reactive power:

$$Q_{\emptyset} = V_{\emptyset} I_{\emptyset} \sin\theta_{\emptyset}$$

$$Q_T = 3Q_{\emptyset} = \sqrt{3} V_L I_L \sin\theta_{\emptyset}$$

Complex power:

$$S_{\emptyset} = P_{\emptyset} + jQ_{\emptyset} = \boldsymbol{V}_{\emptyset} \boldsymbol{I}_{\emptyset}^{*}$$

$$S_T = 3S_{\emptyset} = \sqrt{3} V_L I_L \angle\theta_{\emptyset}^{\circ}$$

Power Calculations in a Balanced Δ Load

Average power in terms of the phase voltage and current:

$$V_\emptyset = |\boldsymbol{V}_{AB}| = |\boldsymbol{V}_{BC}| = |\boldsymbol{V}_{CA}|$$

$$I_\emptyset = |\boldsymbol{I}_{AB}| = |\boldsymbol{I}_{BC}| = |\boldsymbol{I}_{CA}|$$

$$\theta_\emptyset = \theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA}$$

$$P_A = P_B = P_C = P_\emptyset = V_\emptyset I_\emptyset \cos\theta_\emptyset$$

Power Calculations in a Balanced Δ Load

Total average power:

$$P_T = 3P_\emptyset = 3V_\emptyset I_\emptyset \cos\theta_\emptyset$$

Total average power in terms of the line voltage and current:

$$P_T = \sqrt{3} V_L I_L \cos\theta_\emptyset$$

Power Calculations in a Balanced Δ Load

Reactive power:

$$Q_{\emptyset} = V_{\emptyset} I_{\emptyset} \sin\theta_{\emptyset}$$

$$Q_T = 3Q_{\emptyset} = 3V_{\emptyset} I_{\emptyset} \sin\theta_{\emptyset}$$

Complex power:

$$S_{\emptyset} = P_{\emptyset} + jQ_{\emptyset} = \boldsymbol{V}_{\emptyset} \boldsymbol{I}_{\emptyset}^{*}$$

$$S_T = 3S_{\emptyset} = \sqrt{3} V_L I_L \angle\theta_{\emptyset}^{\circ}$$

Reference

- Electric Circuits, Tenth Edition, James W. Nilsson, Susan A. Riedel Pearson, 2015