EEE201
Circuit Analysis II

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Introduction to the Laplace Transform

EEE201 Circuit Analysis II

Lecture 9

Agenda

- Definition of the Laplace Transform
- Laplace Transform of the Unit Step Function
- Laplace Transform of the Unit Impulse Function
- Functional Transforms
- Operational Transforms

Definition of the Laplace Transform

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

A set of integro-differential equations in the time domain

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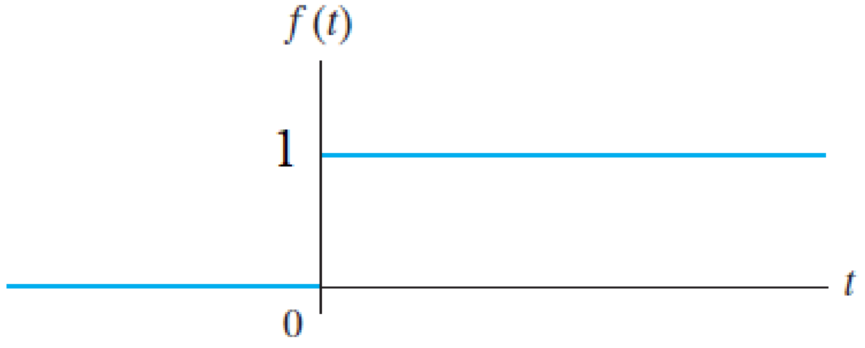
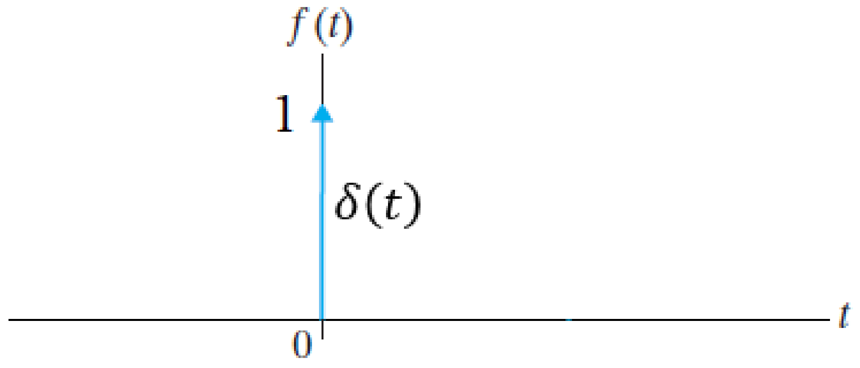
A set of algebraic equations in the frequency domain

Laplace Transform of the Unit Step Function



$$\mathcal{L}\{u(t)\} = \int_0^{\infty} 1e^{-st}\, dt = \frac{1}{s}$$

Laplace Transform of the Unit Impulse Function



$$\mathcal{L}\{\delta(t)\} = \int_{0}^{\infty} \delta(t)e^{-st}\,dt = \int_{0}^{\infty} \delta(t)\,dt = 1$$

Functional Transforms

Decaying exponential function:

$$f(t) = \begin{cases} e^{-at}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$\mathcal{L}\{e^{-at}\} = \int_0^\infty e^{-at} e^{-st}\, dt = \int_0^\infty e^{-(a+s)t}\, dt = \frac{1}{s+a}$$

Functional Transforms

Sinusoidal function:

$$f(t) = \sin(wt)$$

$$\mathcal{L}\{\sin(wt)\} = \int_{0}^{\infty} \sin(wt)\, e^{-st}\, dt = \frac{w}{s^2 + w^2}$$

Operational Transforms

Multiplication by a constant

$$\mathcal{L}\{f(t)\} = F(s) \implies \mathcal{L}\{Kf(t)\} = KF(s)$$

Addition (Subtraction)

$$\mathcal{L}\{f_1(t)\} = F_1(s), \;\; \mathcal{L}\{f_2(t)\} = F_2(s), \;\; \mathcal{L}\{f_3(t)\} = F_3(s) \implies$$

$$\mathcal{L}\{f_1(t) + f_2(t) - f_3(t)\} = F_1(s) + F_2(s) - F_3(s)$$

Operational Transforms

Differentiation

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-), \qquad f(0^-)\text{: initial value of } f(t)$$

Integration

$$\mathcal{L}\left\{\int_0^t f(x)\,dx\right\} = \frac{F(s)}{s}$$

Operational Transforms

Translation in the time domain

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s), \qquad a > 0$$

Translation in the frequency domain

$$\mathcal{L}\{e^{-at}f(t)\} = F(s+a)$$

Scale changing

$$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right), \qquad a > 0$$

Reference

- Electric Circuits, Tenth Edition, James W. Nilsson, Susan A. Riedel Pearson, 2015