HİPERBOLİK FONKSİYONLAR

$$\sinh z = \frac{e^{z} - e^{-z}}{2}$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\frac{d}{dz}\sinh z = \cosh z$$

$$\frac{d}{dz}\cosh z = \sinh z$$

FİZ202 - AYŞE KAŞKAŞ

$$-\boldsymbol{i}\ \sinh(iz)=\sin z \qquad \cosh(\boldsymbol{i}z)=\cos z$$

$$\sinh(-z)=-\sinh z \qquad \cosh(-z)=\cosh z$$

$$-\boldsymbol{i}\sin(\boldsymbol{i}z)=\sinh z \qquad \cos(\boldsymbol{i}z)=\cosh z$$

$$\cosh^2 z-\sinh^2 z=1$$

$$\sinh(z_1+z_2)=\sinh z_1\ \cosh z_2+\cosh z_1\ \sinh z_2$$

$$\cosh(z_1+z_2)=\cosh z_1\cosh z_2+\sinh z_1\sinh z_2$$

$$\sinh z=\sinh x\ \cos y+\boldsymbol{i}\cosh x\ \sin y$$

$$\cosh z=\cosh x\ \cos y+\boldsymbol{i}\sinh x\ \sin y$$

FİZ202 - AYŞE KAŞKAŞ

$$|\sinh z|^2 = \sinh^2 x + \sin^2 y$$

$$|\cosh z|^2 = \sinh^2 x + \cos^2 y$$

$$\frac{d}{dz}\tanh z = \operatorname{sech}^2 z$$

$$\frac{d}{dz}\coth z = -\operatorname{cosec}^2 z$$

$$\frac{d}{dz}\operatorname{sech} z = -\operatorname{sech} z \ \tanh z$$

$$\frac{d}{dz}\operatorname{cosech} z = -\operatorname{cosec} z \ \coth z$$

FİZ202 - AYŞE KAŞKAŞ

LOGARİTMİK FONKSİYONLAR

$z = e^{w}$ Þ $\log z = w$

$e^{\log z} = z$

$e^{i2n\pi} = 1 \qquad n = 0, \pm 1, \pm 2....$

$z = r\, e^{i\Theta} e^{i2n\pi} \qquad \theta = \Theta + 2n\pi \qquad (n = 0, \pm 1, \pm 2.....)$

$z = r\, e^{i(\Theta + 2n\pi)}$ Þ Þ $\log z = \ln r + i(\Theta + 2n\pi)$

$\arg(z) = \theta$

Prensip argümanı $\mathrm{Arg}(z) = \Theta$

FİZ202 - AYŞE KAŞKAŞ

$$\log(z_1 z_2) = \log z_1 + \log z_2$$

$$\log\left(\frac{z_1}{z_2}\right) = \log z_1 - \log z_2$$

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2$$

$$\log\left(\frac{1}{z}\right) = -\log z$$

$$|z_1 z_2| = |z_1||z_2| \quad \Rightarrow \quad \ln|z_1 z_2| = \ln|z_1| + \ln|z_2|$$

$$\ln|z_1 z_2| + \boldsymbol{i}\arg(z_1\ z_2)$$
$$= (\ln|z_1| + i\arg z_1) + (\ln|z_2| + \boldsymbol{i}\arg z_2)$$

FİZ202 - AYŞE KAŞKAŞ

TERS TRİGONOMETRİK VE HİPERBOLİK FONKSİYONLAR

$$w = \sin^{-1} z \quad \Rightarrow \quad z = \sin w$$

$$z = \frac{e^{iw} - e^{-iw}}{2i} \quad \Rightarrow \quad \left(e^{iw}\right)^2 - 2iz\left(e^{iw}\right) - 1 = 0$$

$$e^{iw} = z + (1 - z^2)^{1/2}$$

$$\sin^{-1} z = -i \log\left[iz + \left(1 - z^2\right)^{1/2}\right]$$

$$\cos^{-1} z = -i \log\left[z + i\left(1 - z^2\right)^{1/2}\right]$$

$$\tan^{-1} z = \frac{i}{2} \log\left[\frac{i + z}{i - z}\right]$$

FİZ202 - AYŞE KASKAŞ

$$\sinh^{-1} z = \log\left[z + (z^2 + 1)^{1/2}\right]$$

$$\cosh^{-1} z = \log\left[z + (z^2 - 1)^{1/2}\right]$$

$$\tanh^{-1} z = \frac{1}{2}\log\left[\frac{1+z}{1-z}\right]$$

$$\frac{d}{dz}\sin^{-1} z = \frac{1}{(1-z^2)^{1/2}}$$

$$\frac{d}{dz}\cos^{-1} z = \frac{-1}{(1-z^2)^{1/2}}$$

$$\frac{d}{dz}\tan^{-1} z = \frac{1}{1+z^2}$$

FİZ202 - AYŞE KAŞKAŞ

KAYNAKLAR

- Complex Variables and Applications, J.W. Brown and R.V. Churchill, 1990.
- Kısmi Diferansiyel Denklemler, Schaum's Outlines, P. Duchateu ve D.W. Zachmann, 2000.
- Complex Analysis, Theodore W. Gamelin, 2001.

FİZ202 AYŞE KASKAŞ