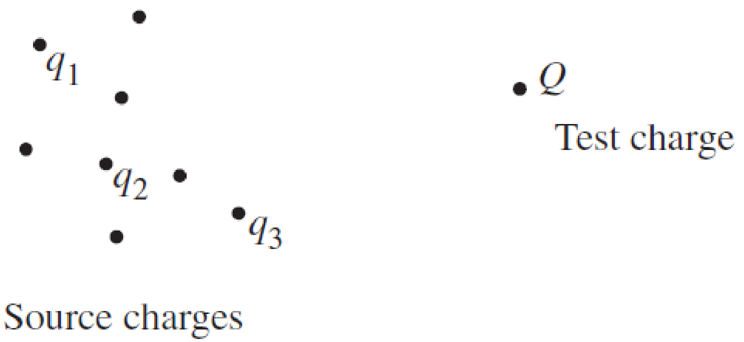
Magnetostatics

Electrostatics (the source charge is at rest)



the principle of superposition:

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots$$

The forces between charges *in motion*:

Currents in same directions; attract.

Figure

Currents in opposite directions; repel.

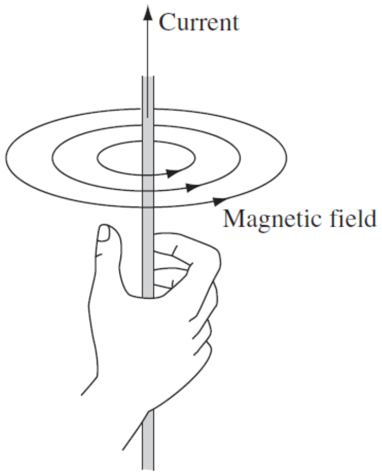
Figure

a stationary charge produces only an electric field **E** in the space around it,

a *moving* charge generates, in addition, a magnetic field **B**.

Direction of magnetic field; right hand rule;

if you grab the wire with your right hand—thumb in the direction of the current—your fingers curl around in the direction of the magnetic field.



Magnetic Forces

Electric Force: $\mathbf{F}_{elec}=Q\mathbf{E}$

The magnetic force on a charge *Q*, moving with velocity **v** in a magnetic field **B**,

Lorentz force law $\mathbf{F}_{\text{mag}} = Q(\mathbf{v} \times \mathbf{B})$

In the presence of both electric *and* magnetic fields;

$$\mathbf{F} = \mathbf{F}_{\text{elec}} + \mathbf{F}_{\text{mag}} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{F}_{\mathrm{mag}} = Q(\mathbf{v} \times \mathbf{B})$$

Figure

Work done by **magnetic forces:**

if Q moves an amount $d\mathbf{l} = \mathbf{v}\, dt$, the work done is

$$\mathbf{W}_{\text{mag}} = \int \mathbf{F}_{\text{mag}} \cdot d\mathbf{l} = \int Q(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$= \int Q(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v}\, dt$$

$$= 0$$

Magnetic forces do no work.

Magnetic forces may alter the *direction* in which a particle moves, but they cannot speed it up or slow it down.

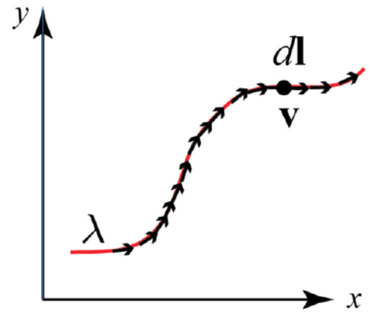
Currents

The **current** in a wire is the *charge per unit time* passing a given point. $I = \frac{dq}{dt}$

Current is measured in coulombs-per-second, or **amperes** (A): 1 A = 1 C/s.

A line charge λ flowing in a wire at speed **v** is described by **current;**

$$\mathbf{I} = \frac{dq}{dt} = \frac{\lambda\, d\mathbf{l}}{dt} = \lambda \mathbf{v}$$



The direction of current is in the direction of charge-flow.
(Current is actually a vector.)

The magnetic force on a segment of current-carrying wire is:

$$\mathbf{F}_{\text{mag}} = (\mathbf{v} \times \mathbf{B})Q = \int (\mathbf{v} \times \mathbf{B})\, dq = \int (\mathbf{v} \times \mathbf{B})\, \lambda dl = \int (\mathbf{I} \times \mathbf{B})\, dl$$

I and d**l** both point in the same direction;

$$\mathbf{F}_{\text{mag}} = \int (\mathbf{I} \times \mathbf{B})\, dl = I \int (d\mathbf{l} \times \mathbf{B})$$

If charge flowing on a surface is described by **surface current density**

$$I = \frac{dq}{dt}$$

$$\mathbf{K} = \frac{d\mathbf{I}}{dl_{\perp}} = \sigma \mathbf{v}$$

Current density is a vector quantity.

Figure 13

(the current in this ribbon is $d\mathbf{I}$;
K is the current per unit width;
The mobile surface charge density is σ and its velocity is **v**,)

The magnetic force on the surface current is

$$\mathbf{F}_{\text{mag}} = (\mathbf{v} \times \mathbf{B})Q = \int (\mathbf{v} \times \mathbf{B})\, \sigma da = \int (\mathbf{K} \times \mathbf{B})\, da$$

Charge flowing in a volume is described by **volume current density**

$$\mathbf{J} = \frac{d\mathbf{I}}{da_\perp} = \rho\mathbf{v}$$

Figure 14

Current density is a vector quantity.

(***J*** is the *current per unit area*; a “tube” of infinitesimal cross section $da_\perp$,
If the (mobile) volume charge density is ρ and the velocity is **v**)

Magnetic force on the volume current:

$$\mathbf{F}_{\mathrm{mag}} = (\mathbf{v} \times \mathbf{B})Q = \int (\mathbf{v} \times \mathbf{B})\,\rho d\tau = \int (\mathbf{J} \times \mathbf{B})\,d\tau$$

Example 4. **(a)** A current *I* is uniformly distributed over a wire of circular cross section, with radius *a*. Find the volume current density *J* .

The area (perpendicular to the flow) is πa^2,

Figure 15

$$J = \frac{I}{\pi a^2}$$

(b) Suppose the current density in the wire is proportional to the distance from the axis, for some constant k:

$$J = ks$$

Find the total current in the wire.

Figure

The current through the shaded patch is $J da_\perp$, and $da_\perp = s\, ds\, d\Phi$. Hence,

$$\mathbf{J} = \frac{d\mathbf{I}}{da_\perp} \Longrightarrow I = \int (ks)(s\, ds\, d\phi) = 2\pi k \int_0^a s^2\, ds = \frac{2\pi k a^3}{3}$$

The Continuity Equation (Conservation of Charge)

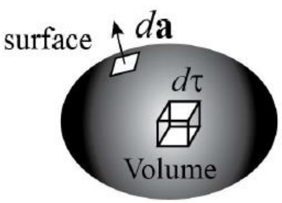
$\mathbf{J} = \dfrac{d\mathbf{I}}{da_\perp}$ ⟹ the total current crossing a surface S can be written as;

$$I = \int_S J \, da_\perp = \int_S \mathbf{J} \cdot d\mathbf{a}$$

Current crossing a surface
||
Total charge per unit time crossing a surface

For a closed surface :

$$\oint_S \mathbf{J} \cdot d\mathbf{a} = \int_\mathcal{V} (\nabla \cdot \mathbf{J}) \, d\tau$$



Total charge per unit time leaving the volume V.

Because charge is conserved, whatever flows out through the surface must come at the expense of what remains inside:

$$\int_\mathcal{V} (\nabla \cdot \mathbf{J}) \, d\tau = -\frac{d}{dt} \int_\mathcal{V} \rho \, d\tau = -\int_\mathcal{V} \left(\frac{\partial \rho}{\partial t} \right) d\tau$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

The Continuity Equation