Electrodynamics

For most substances, the current density **J** is proportional to the *force per unit charge*, **f**:

$$\mathbf{J} = \sigma \mathbf{f}$$

σ : **conductivity** of the medium. The **resistivity**: $\rho = 1/\sigma$

The force that drives the charges to produce the current, here is an electromagnetic;

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The velocity of the charges is sufficiently small that the magnetic term can be ignored:

$$\mathbf{J} = \sigma \mathbf{E}$$

Data from Handbook of Chemistry and Physics

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Sea water	0.2
Copper	1.68×10^{-8}	Germanium	0.46
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2500
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.61×10^{-7}	Water (pure)	8.3×10^{3}
Nichrome	1.08×10^{-6}	Glass	$10^{9} - 10^{14}$
Manganese	1.44×10^{-6}	Rubber	$10^{13} - 10^{15}$
Graphite	1.6×10^{-5}	Teflon	$10^{22} - 10^{24}$

Electromotive Force (*emf*)

Source of electromotive force: any device that supply electrical energy

$$\mathcal{E} \equiv \oint \mathbf{f} \cdot d\mathbf{l}$$

Figure 7

$\mathcal{E}$ is called the **electromotive force**, or **emf**, of the circuit.

It's not a *force* at all—it's the *integral* of a *force per unit charge*.

Motional emf

Suppose that in the shaded region there is a uniform magnetic field **B**, pointing into the page.

Figure 10

$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh$$

h is the width of the loop.

The horizontal segments *bc* and *ad* contribute nothing, since the force there is perpendicular to the wire.

(Let ***v*** in y- and ***B*** in x-direction; then ***F*** will be in ***z***-direction.)

Let Φ be the flux of **B** through the loop:

$$\Phi \equiv \int \mathbf{B} \cdot d\mathbf{a}$$

Figure 10

$$\Phi = Bhx$$

As the loop moves, the flux decreases:

$$\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv$$

(The minus sign accounts for the fact that *dx/dt* is negative.)

$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh \quad \longleftrightarrow \quad \mathcal{E} = -\frac{d\Phi}{dt}$$

The emf generated in the loop is minus the rate of change of flux through the loop.

Generators exploit **motional emfs**, which arise when you *move a wire through a magnetic field*.

ELECTROMAGNETIC INDUCTION: Faraday's Law

In 1831 Michael Faraday reported on a series of experiments:

Figure

1. Pulled a loop of wire to the right through a magnetic field; a current flowed in the loop.

Figure

2. Moved the *magnet* to the *left*, holding the loop still; a current flowed in the loop.

Figure

3. With both the loop and the magnet at rest, changed the *strength* of the field; a current flowed in the loop.

RESULT:

A changing magnetic field induces an electric field.

Since the emf of (2) equal to the emf of (1) – at static B moving loop:
induced electric field that accounts for the emf is equal to the rate of change of the flux;

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

$$\Phi \equiv \int \mathbf{B} \cdot d\mathbf{a}$$

E is related to the change in **B** by the equation;

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

Faraday's law

applying Stokes' theorem:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

E.g.: 5. A long cylindrical magnet of length L and radius a carries a uniform magnetization $\mathbf{M}$ parallel to its axis. It passes at constant velocity v through a circular wire ring of slightly larger diameter. Graph the emf induced in the ring, as a function of time.

Figure 22

As in the case of long solenoid with surface bound current, the field inside the solenoid;

$$\mathbf{B} = \mu_0 \mathbf{M}$$

(except near the ends, where it starts to spread out.)

$$\Phi \equiv \int \mathbf{B} \cdot d\mathbf{a}. \longrightarrow \Phi = \mu_0 M \pi a^2$$

Figure 23

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

Lenz's law: The induced current will flow in such a direction that the flux *it* produces tends to cancel the change.

E.g.: 6. The "jumping ring" demonstration. If you wind a solenoidal coil around an iron core (the iron is there to beef up the magnetic field), place a metal ring on top, and plug it in, the ring will jump several feet in the air. Why?

Figure 24

E.g.: 7. A uniform magnetic field **B***(t)*, pointing straight up, fills the shaded circular region as given below. If **B** is changing with time, what is the induced electric field?

Draw an Amperian loop of radius *s*, and apply Faraday's law:

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

Figure 25

$$\oint \mathbf{E} \cdot d\mathbf{l} = E(2\pi s) = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\pi s^2 B(t)\right) = -\pi s^2 \frac{dB}{dt}$$

$$\mathbf{E} = -\frac{s}{2}\frac{dB}{dt}\,\hat{\boldsymbol{\phi}}$$

If **B** is *increasing*, **E** runs *clockwise*, as viewed from above.

Electrodynamics Before Maxwell

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \text{(Gauss's law)},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \qquad \text{(no name)},$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law)},$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \text{(Ampère's law)}.$$

The divergence of curl is always zero; apply the divergence to number (iii), it works out:

$$\nabla \cdot (\nabla \times \mathbf{E}) = \nabla \cdot \left(-\frac{\partial \mathbf{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\overbrace{(\nabla \cdot \mathbf{B})} \quad \checkmark$$

However, the divergence of (iv);

$$\underbrace{\nabla \cdot (\nabla \times \mathbf{B})}_{= 0} = \mu_0 \underbrace{(\nabla \cdot \mathbf{J})}_{\neq 0} \quad \times$$

beyond magnetostatics (*steady* currents),
Ampère's law cannot be right.

How Maxwell Fixed Ampère's Law

Maxwell fixed it by purely theoretical arguments: the continuity equation and Gauss's law

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\epsilon_0 \nabla \cdot \mathbf{E}) = -\nabla \cdot \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Result: A changing electric field induces a magnetic field.

Maxwell's equations:

(i) $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$ → (Gauss's law),

(ii) $\nabla \cdot \mathbf{B} = 0$ → (no name),

(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ → (Faraday's law),

(iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ (Ampère's law with Maxwell's correction).

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$