Signals and Systems

Lecture 4. Basic Signals-1

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2019-2020 Fall Semester

Introduction

Continuous Time

Sürekli Zaman

Discrete Time

Ayrık Zaman

- Sinusoidal Signals

  Sinüzoidal Sinyaller
- Complex Exponential Signals

  Karmaşık üstel sinyaller
- Unit Step Function

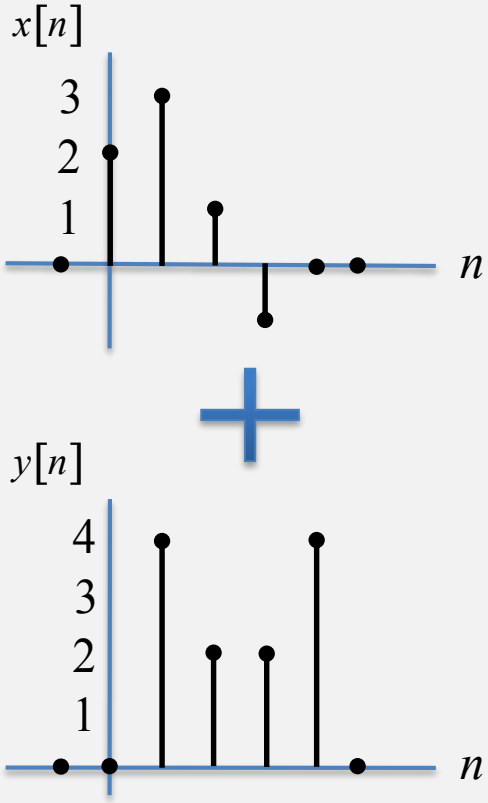
  Birim basamak fonksiyonu
- Unit Impulse Function

  Birim dürtü fonksiyonu

Basic Signal Operations

1. *Addition of two signals*: addition of values corresponding to the same time instants

example:

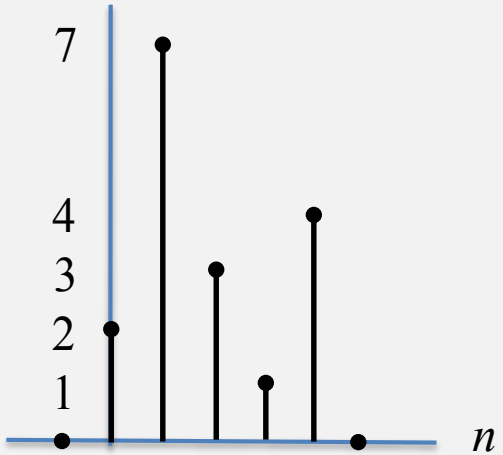


$$z[0] = x[0] + y[0]$$

$$z[1] = x[1] + y[1]$$

$$\vdots$$

$$z[n] = x[n] + y[n]$$



Basic Signal Operations

2. *Multiplication of two signals*: multiplication of values corresponding to the same time instants.

$$z[n] = x[n] \cdot y[n]$$

$$z[0] = x[0] \cdot y[0], z[1] = x[1] \cdot y[1], \cdots$$

3. *Multiplication of a signal with a constant (scale)*: multiplication of values corresponding to all time instants with the same constant.

$$y[n] = ax[n]$$

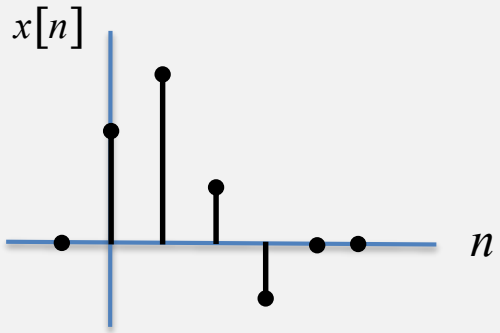
$$y[0] = ax[0],\ y[1] = ax[1], \cdots$$

Basic Signal Operations

4. *Shifting a signal in time*: when y[*n*] is version of x[*n*] shifted in time by n_o

$$y[n] = x[n - n_o]$$

example:

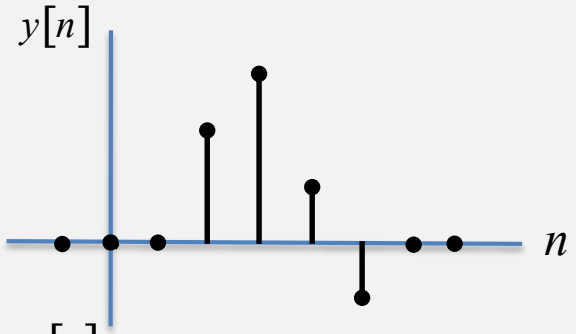


Quesiton: for $n_o = 2$

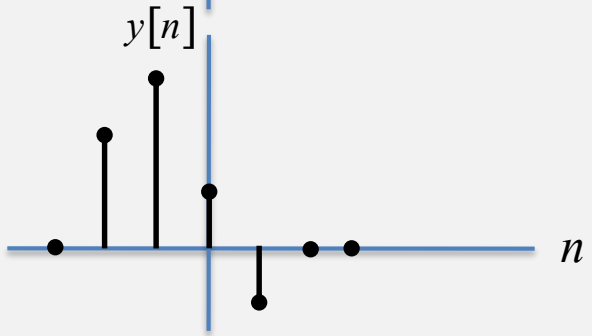
which one on the next column is

$$y[n] = x[n - n_o]$$

(a)



(b)



Answer: .....

Basic Signal Operations

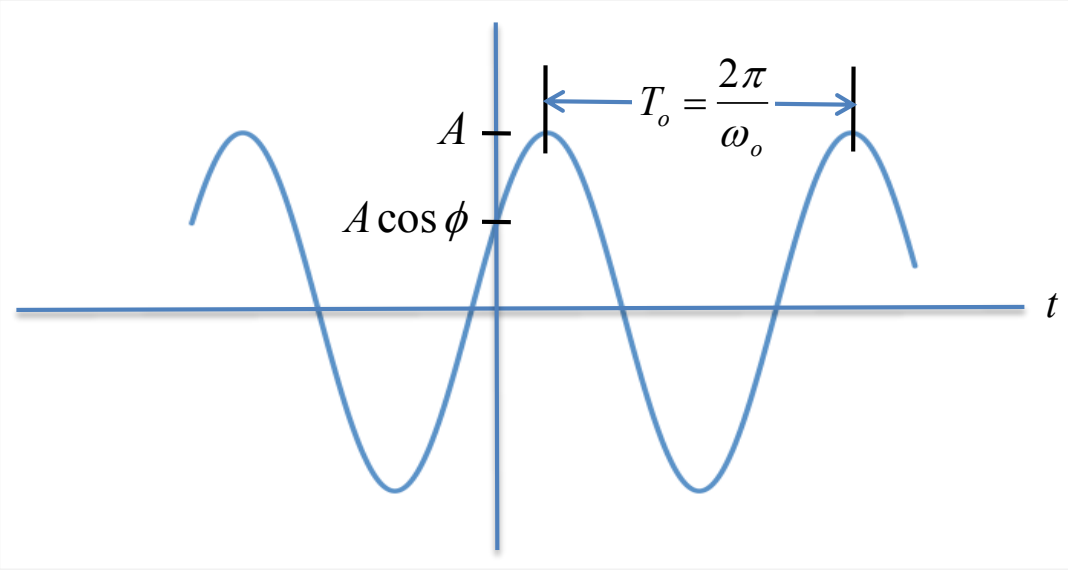
Note 1: Addition, multiplication and scaling are applied in the same way for both continuous and discrete time signals.

Note 2: For time shift, t_o can be any real number whereas in discrete time n_o must be an integer.

Continuous Time Sinusoidal Signals

$$x(t) = A\cos(\omega_o t + \phi)$$

amplitude *(genlik)* frequency *(frekans)* phase *(faz)*



Continuous Time Sinusoidal Signals

Periodicity

$$x(t) = x(t + T_o)$$

period $\triangleq$ smallest T_o

$$A\cos(\omega_o t + \phi) = A\cos(\omega_o t + \underbrace{\omega_o T_o}_{2\pi m} + \phi)$$

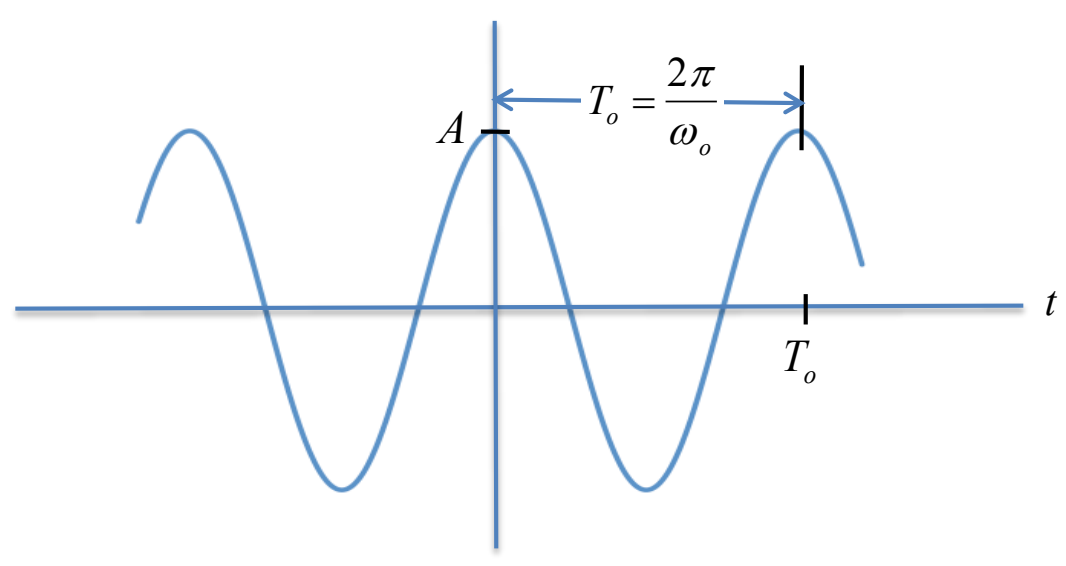
$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow period = \frac{2\pi}{\omega_o}$$

Time Shift $\Leftrightarrow$ Phase shift

$$A\cos[\omega_o(t + t_o) + \phi] = A\cos[\omega_o t + \omega_o t_o + \phi]$$

Continuous Time Sinusoidal Signals

$$\phi = 0 \quad x(t) = A\cos\omega_o t$$

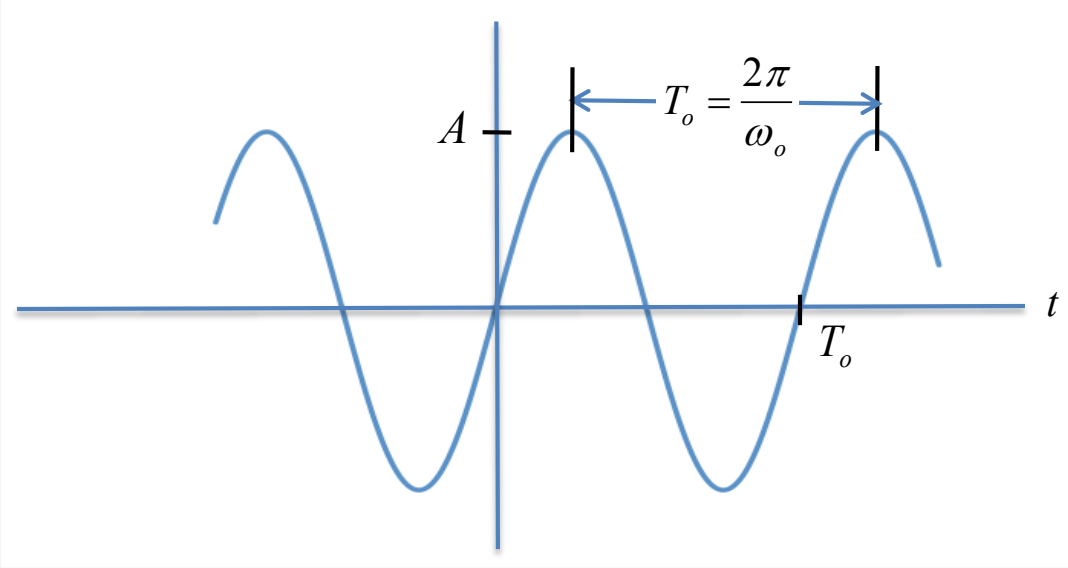


Periodic: $x(t) = x(t+T_o)$

Even: $x(t) = x(-t)$

Continuous Time Sinusoidal Signals

$$\phi = -\frac{\pi}{2} \qquad x(t) = \begin{cases} A\cos\left(\omega_o t - \frac{\pi}{2}\right) \\ A\sin\omega_o t \\ A\cos\left[\omega_o\left(t - \frac{T_o}{4}\right)\right] \end{cases}$$



Periodic: $x(t) = x(t+T_o)$

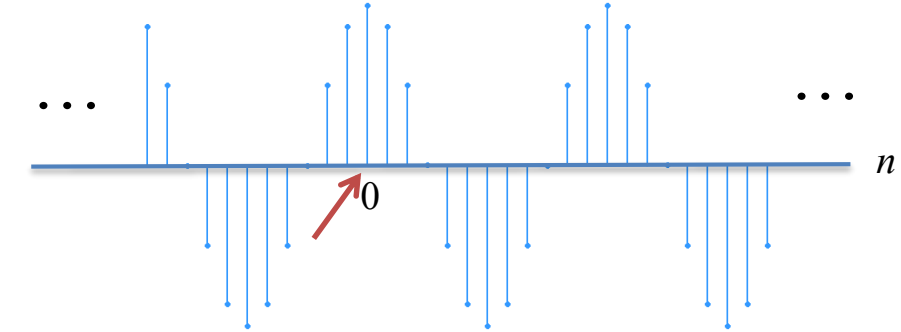
Odd: $x(t) = -x(-t)$

Discrete Time Sinusoidal Signals

$$x[n] = A\cos(\Omega_o n + \phi)$$

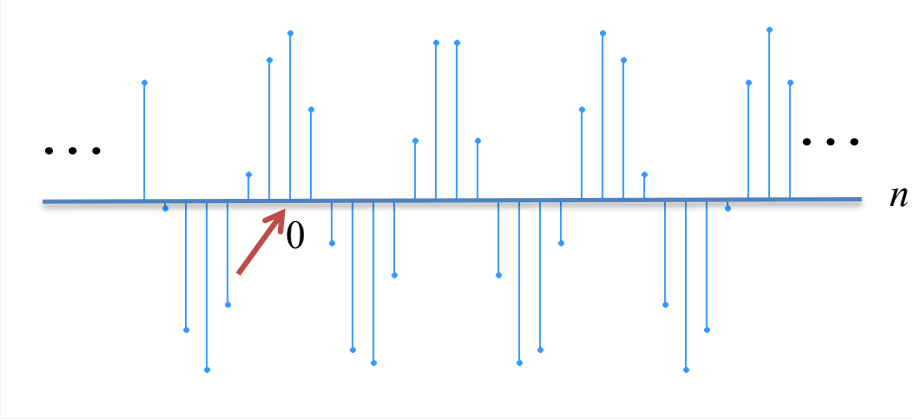
$$\Omega_o = \frac{2\pi}{12}$$

$$\phi = 0$$



$$\Omega_o = \frac{8\pi}{31}$$

$$\phi = 0$$



Discrete Time Sinusoidal Signals

- Time Shift ⟹ phase change

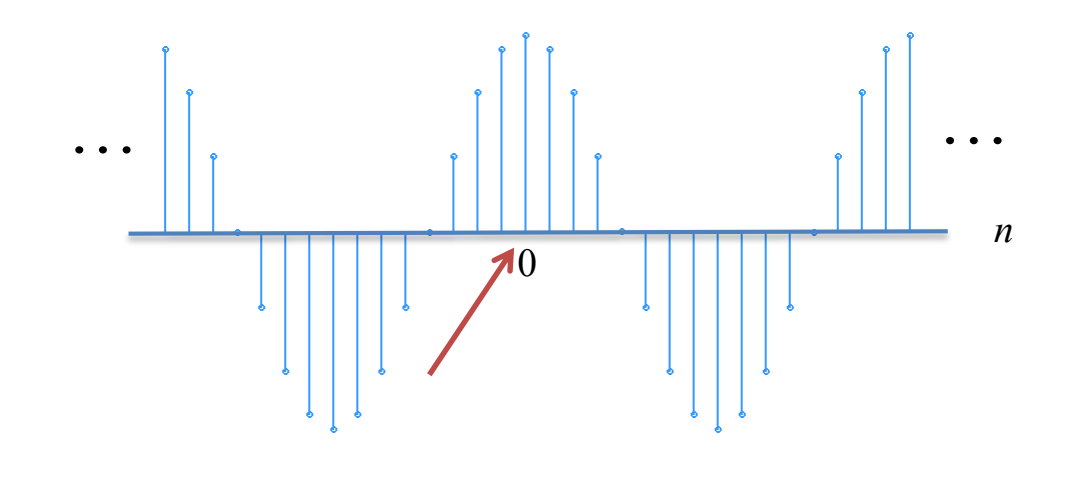
$$A\cos\left[\Omega_o\left(n+n_o\right)\right]=A\cos\left[\Omega_o n+\Omega_o n_o\right]$$

- Time Shift ⟸ phase change?

Discrete Time Sinusoidal Signals

$$\phi = 0 \quad x[n] = A\cos\Omega_o n$$

$$\Omega_o = \frac{\pi}{8}$$

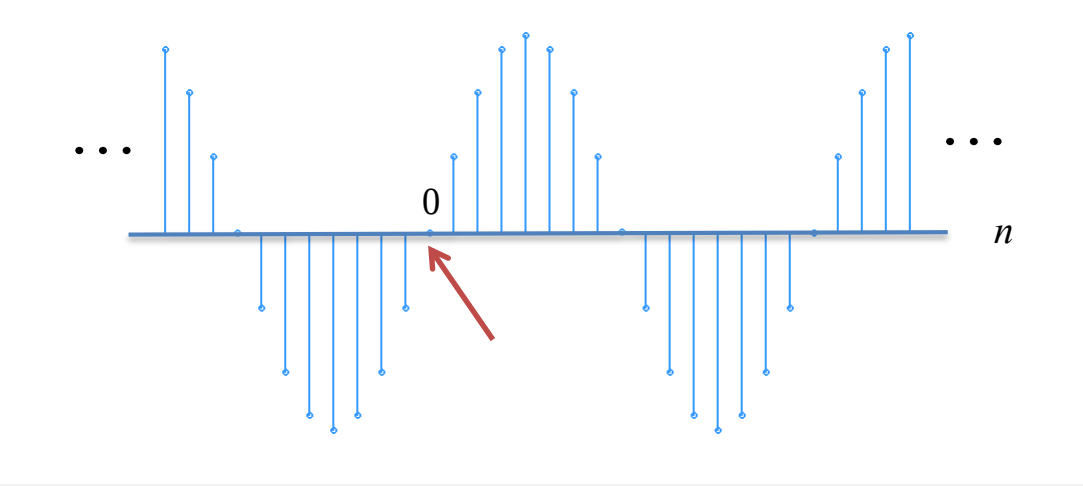


Even: x[n] = x[-n]

Discrete Time Sinusoidal Signals

$$\phi = -\frac{\pi}{2} \qquad x[n] = \begin{cases} A\cos\left(\Omega_o n - \frac{\pi}{2}\right) \\ A\sin\Omega_o n \\ A\cos\left[\Omega_o\left(n - n_o\right)\right] \end{cases}$$

$$\Omega_o = \frac{\pi}{8}$$



odd: x[n] = - x[-n]

Discrete Time Sinusoidal Signals

- Time Shift ⟹ phase change

$$A\cos\left[\Omega_o\left(n+n_o\right)\right]=A\cos\left[\Omega_o n+\Omega_o n_o\right]$$

- Time Shift ⟸ phase change ?

$$A\cos\left[\Omega_o\left(n+n_o\right)\right]\stackrel{?}{=}A\cos\left[\Omega_o n+\phi_o\right]$$