Signals and Systems

Lecture 5. Basic Signals-2

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2019-2020 Fall Semester

Periodicity Condition for DT Sinusoidals

$$x[n] = A\cos(\Omega_o n + \phi)$$

Periodic?

$$x[n] = x[n+N]$$

period $\triangleq$ smallest integer N

$$A\cos[\Omega_o(n+N)+\phi] = A\cos[\Omega_o n + \underbrace{\Omega_o N}_{\text{integer multiple of } 2\pi} + \phi]$$

integer multiple of 2π

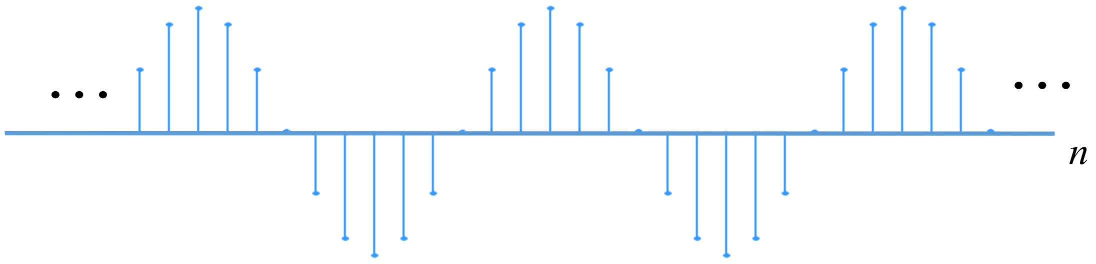
Periodic → $\Omega_o N = 2\pi m$ N, m must be integers

$$N = \frac{2\pi m}{\Omega_o}$$

Periodicity Condition for DT Sinusoidals

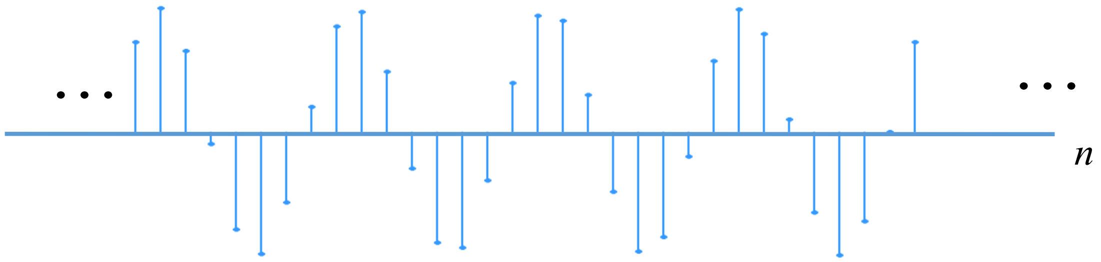
$$\Omega_o = \frac{2\pi}{12}$$

$$\phi = 0$$



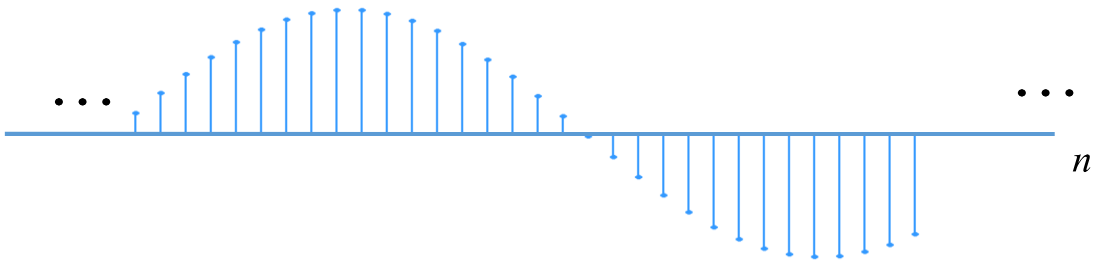
$$\Omega_o = \frac{8\pi}{31}$$

$$\phi = 0$$



$$\Omega_o = \frac{1}{6}$$

$$\phi = 0$$



Frequency ranges for CT and DT Sinusoidals

Continuous Time:

$$x_1(t) = A\cos(\omega_1 t + \phi)$$

$$x_2(t) = A\cos(\omega_2 t + \phi)$$

If $\omega_2 \neq \omega_1$, *then*

$$x_2(t) \neq x_1(t)$$

Discrete Time:

$$x_1[n] = A\cos[\Omega_1 n + \phi]$$

$$x_2[n] = A\cos[\Omega_2 n + \phi]$$

If $\Omega_2 = \Omega_1 + 2\pi m$, *then*

$$x_2[n] = x_1[n]$$

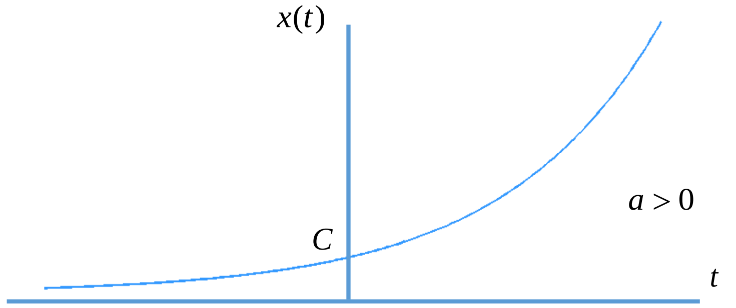
Comparison of CT - DT Sinusoidals

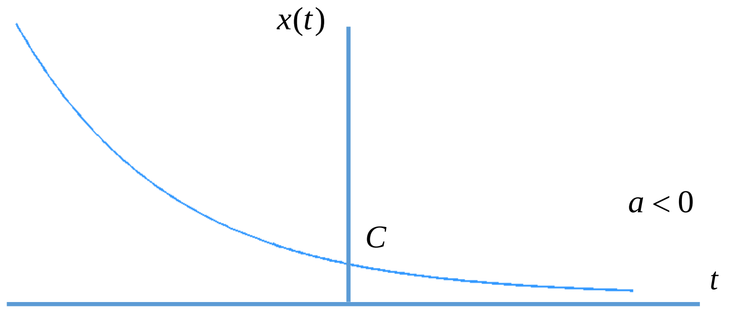
$x(t) = A\cos(\omega_o t + \phi)$	$x[n] = A\cos(\Omega_o n + \phi)$
Periodic for all values of ω_o	Periodic for Ω_o values satisfying $\Omega_o = \frac{2\pi m}{N}$ where *m* and *N* have integer values
Different signals for different ω_o values	The same signals for Ω_o and $\Omega_o + k2\pi$

Real Exponential Signals (CT)

Gerçel Üstel: Sürekli Zaman

$$x(t) = Ce^{at}$$ C and a are real numbers





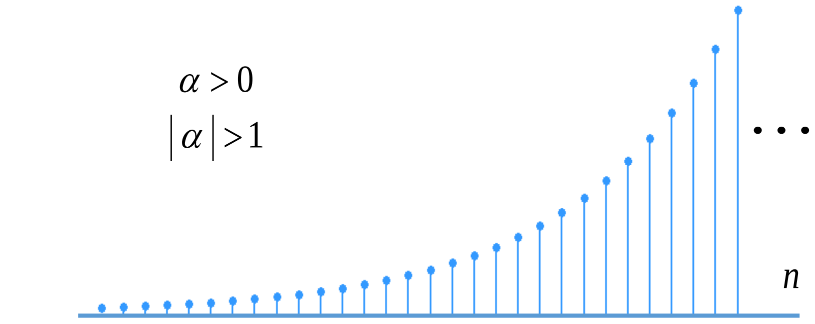
Time Shift → Scale Change

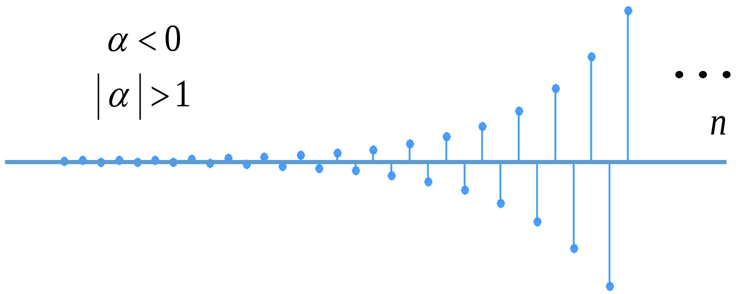
$$Ce^{a(t+t_o)} = Ce^{at_o}e^{at}$$

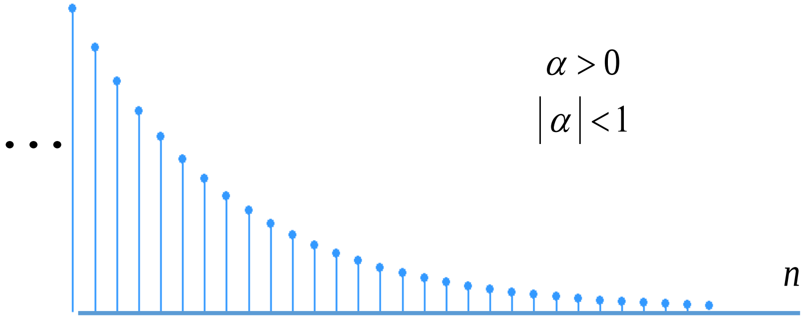
Real Exponential Signals (DT)

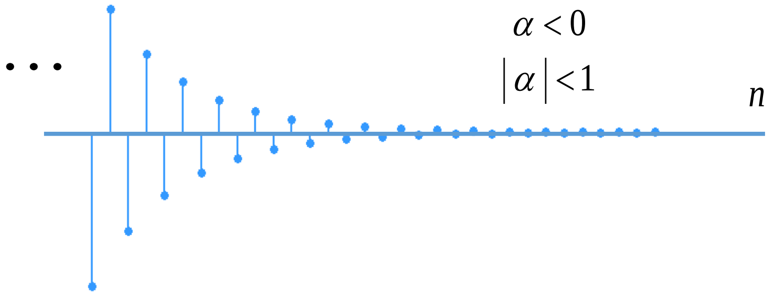
Gerçel Üstel: Ayrık Zaman

$$x[n] = Ce^{\beta n} = C\alpha^n \quad C \text{ and } \alpha \text{ are real numbers}$$









Complex Exponential Signals (CT)

Karmaşık Üstel: Sürekli Zaman

$$x(t) = Ce^{at}$$ C and a are complex numbers

$$C = |C|e^{j\theta} \qquad a = r + j\omega_o$$

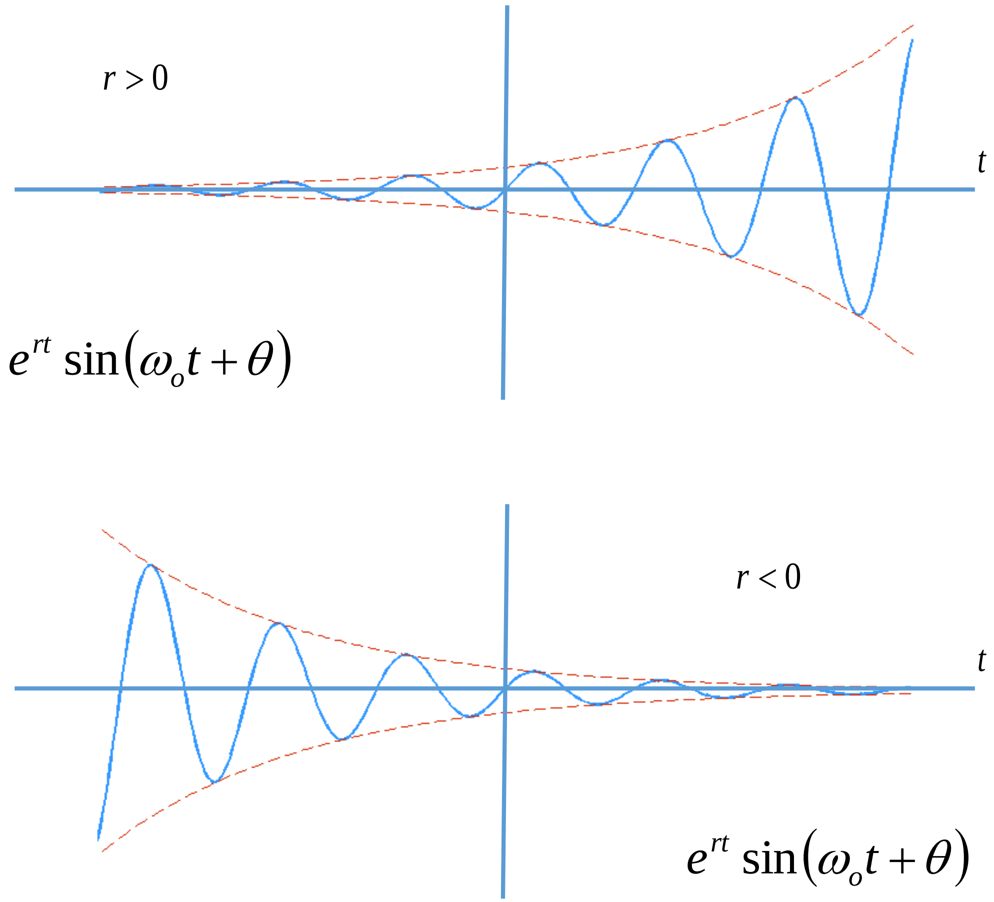
$$x(t) = |C|e^{j\theta}e^{(r+j\omega_o)t}$$
$$= |C|e^{rt}\underbrace{e^{j(\omega_o t+\theta)}}$$

Euler equality: $$e^{jx} = \cos x + j\sin x$$

$$e^{j(\omega_o t+\theta)} = \cos(\omega_o t + \theta) + j\sin(\omega_o t + \theta)$$

$$x(t) = |C|e^{rt}\cos(\omega_o t + \theta) + j|C|e^{rt}\sin(\omega_o t + \theta)$$

Complex Exponential Signals (CT)



Complex Exponential Signals (DT)

Karmaşık üstel: Ayrık Zaman

$$x[n] = C\alpha^n$$

C and α are complex numbers

$$C = |C|e^{j\theta} \qquad \alpha = |\alpha|e^{j\Omega_o}$$

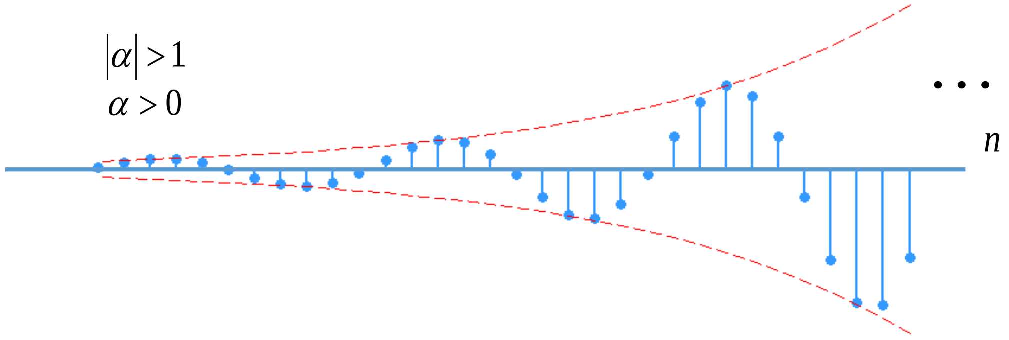
$$x[n] = |C|e^{j\theta}\left(|\alpha|e^{j\Omega_o}\right)^n$$

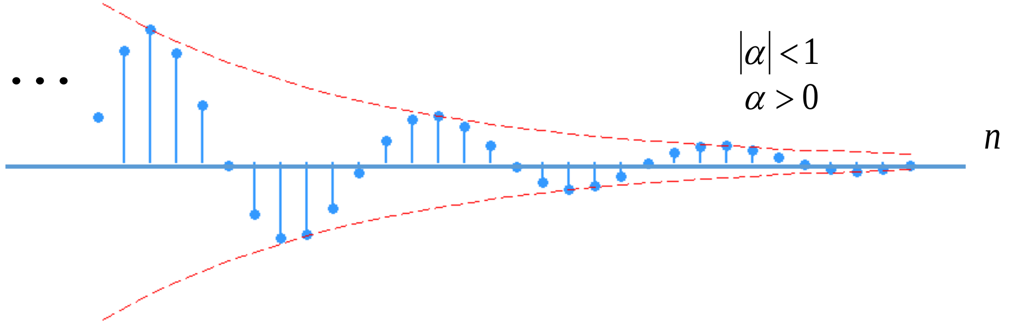
$$= |C||\alpha|^n \underbrace{e^{j(\Omega_o n+\theta)}}$$

Euler equality: $\cos(\Omega_o n + \theta) + j\sin(\Omega_o n + \theta)$

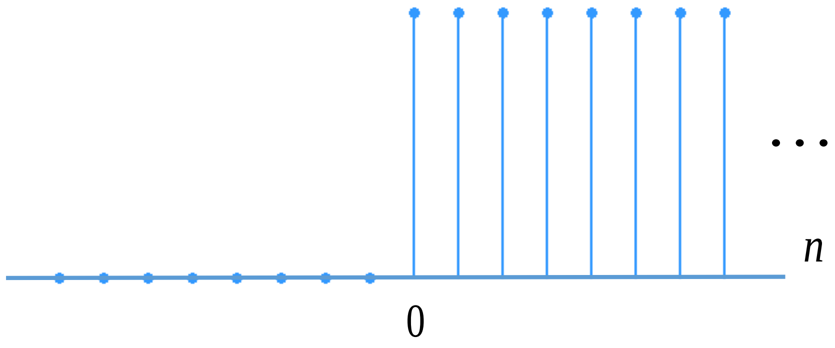
$$x[n] = |C||\alpha|^n \cos(\Omega_o n + \theta) + j|C||\alpha|^n \sin(\Omega_o n + \theta)$$

Complex Exponential Signals (DT)

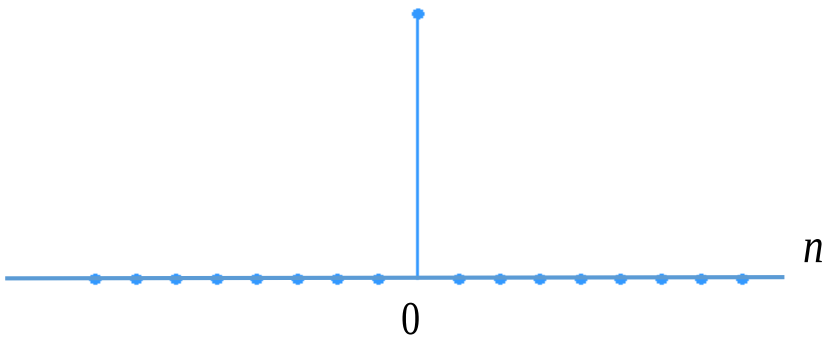




Unit Step and Unit Impulse (DT)

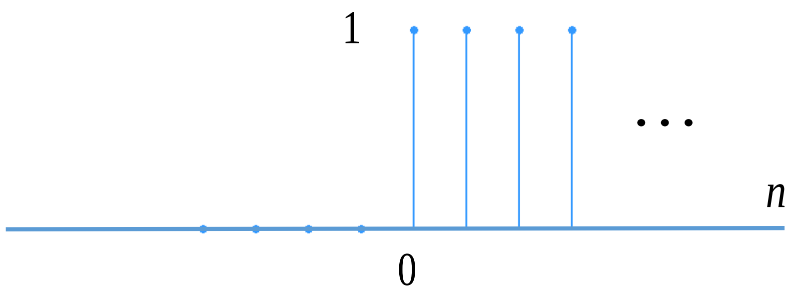


$$u[n]=\begin{cases}1 & n\ge 0\\ 0 & n<0\end{cases}$$

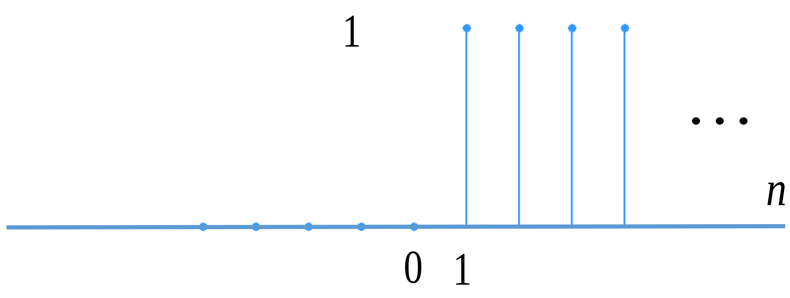


$$\delta[n]=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases}$$

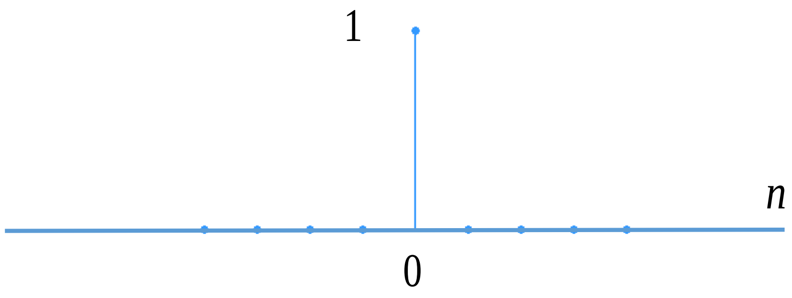
The Relation between Unit Step and Unit Impulse (DT)



$$u[n]$$

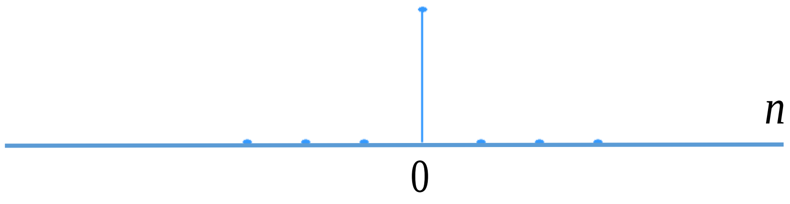


$$u[n-1]$$

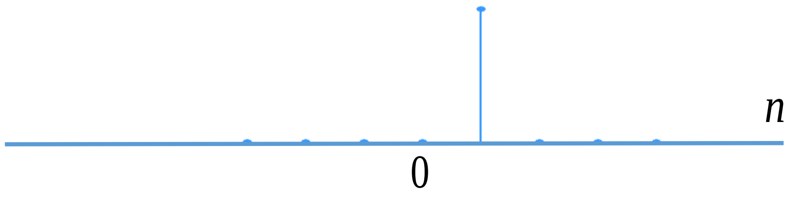


$$\delta[n] = u[n] - u[n-1]$$

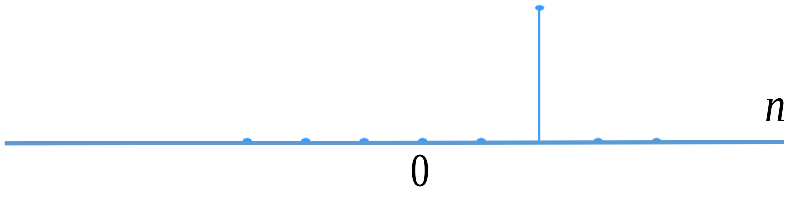
The Relation between Unit Step and Unit Impulse (DT)



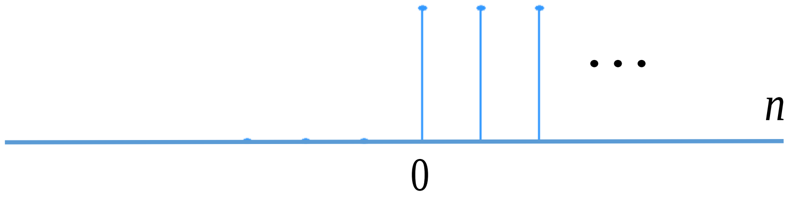
$$\delta[n]$$



$$\delta[n-1]$$



$$\delta[n-2]$$

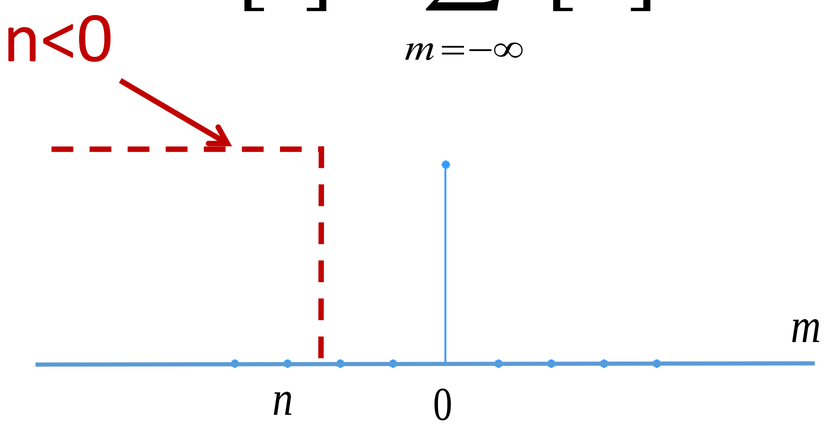


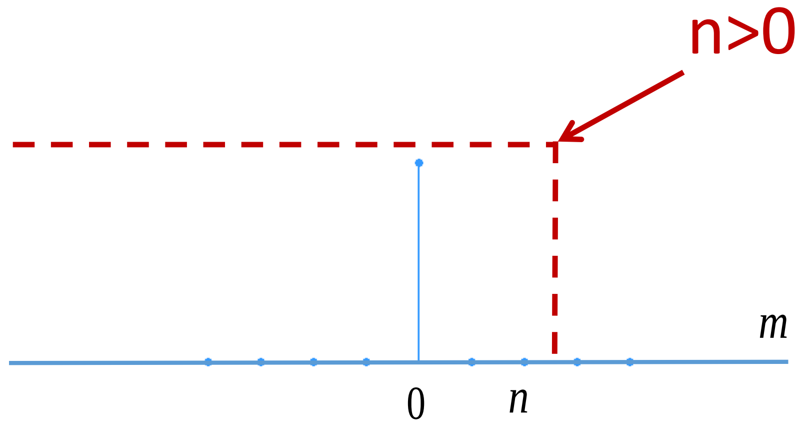
$$u[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\cdots$$

$$u[n]=\sum_{k=0}^{\infty}\delta[n-k]$$

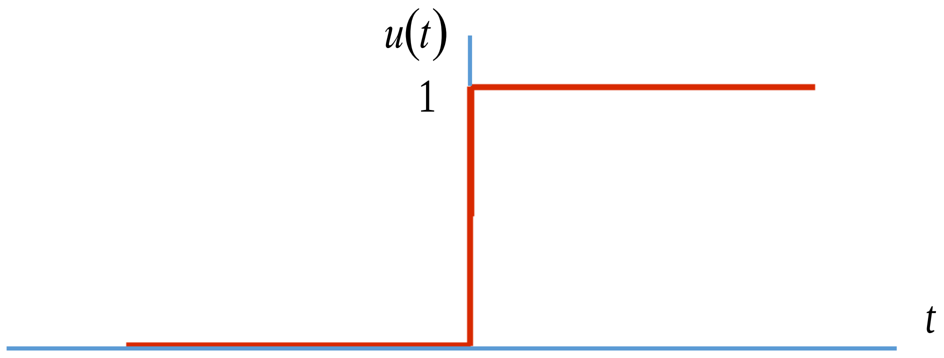
The Relation between Unit Step and Unit Impulse (DT)

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

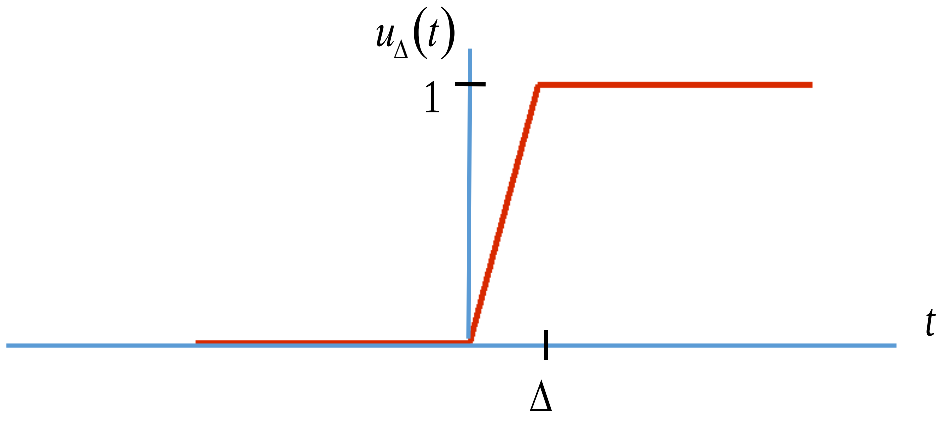




Unit Step (CT)



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



$$\Delta \rightarrow 0$$

$$u(t) = u_\Delta(t)$$

Unit Impulse (CT)

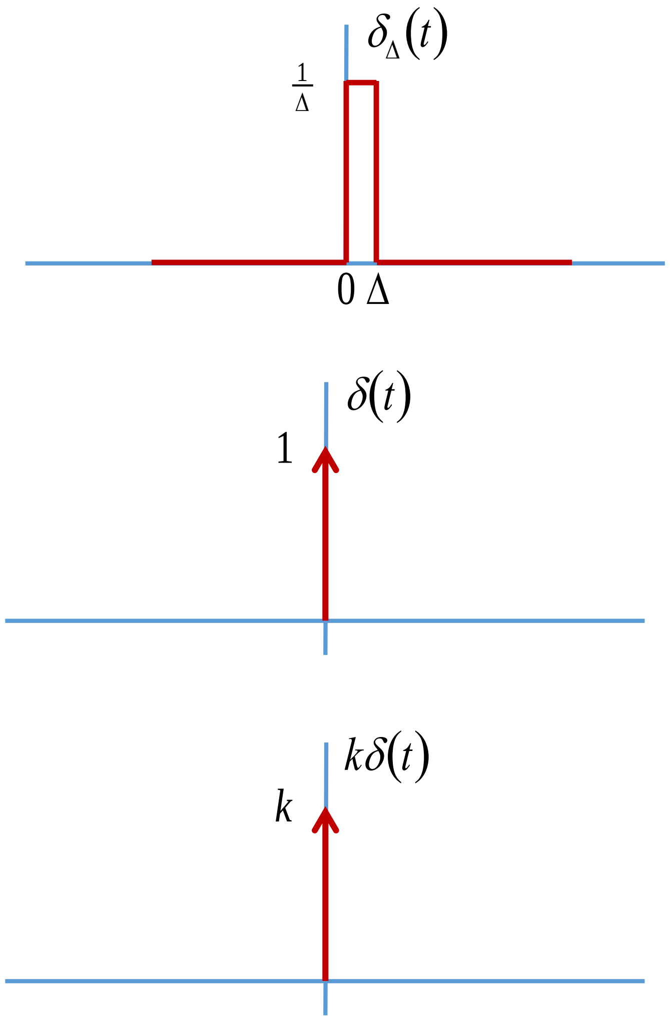
$$\delta(t) = \frac{du(t)}{dt}$$

$$\delta_{\Delta}(t) = \frac{du_{\Delta}(t)}{dt}$$

$$\Delta \rightarrow 0$$

$$\delta(t) = \delta_{\Delta}(t)$$

Unit Impulse (CT)



Area= 1

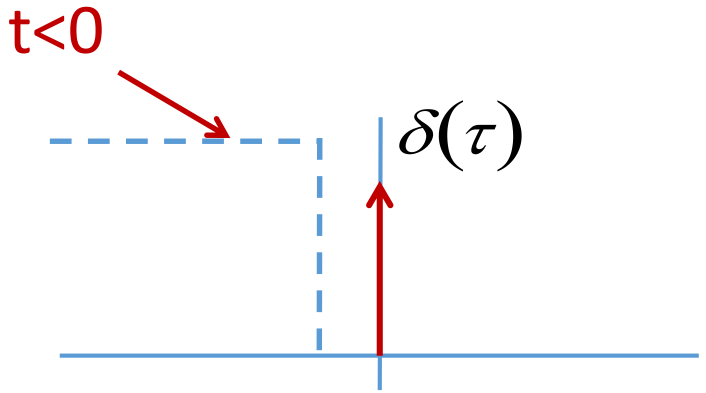
height = "∞"

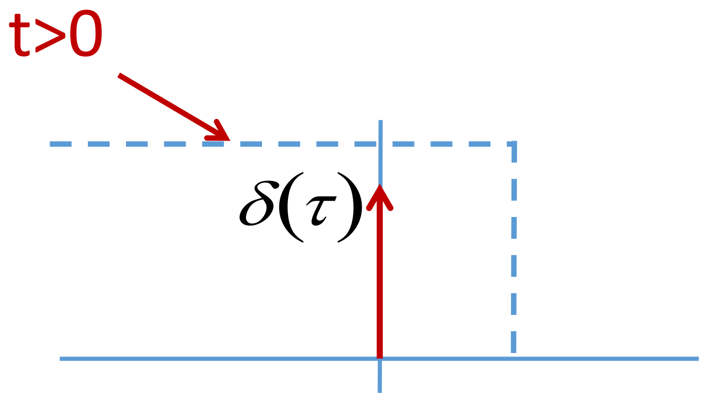
width = "0"

area = 1

The Relation between Unit Step and Unit Impulse (CT)

$$\delta(t) = \frac{du(t)}{dt} \qquad u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$





Summary

- Basic signal operations
- CT ve DT sinusoidal signal, periodicity conditions, differences
- CT/DT and real/complex exponential signals
- Definitions of CT/DT unit step and unit impulse functions
- The relations between CT/DT unit step and unit impulse