Signals and Systems

Lecture 7. Systems-2

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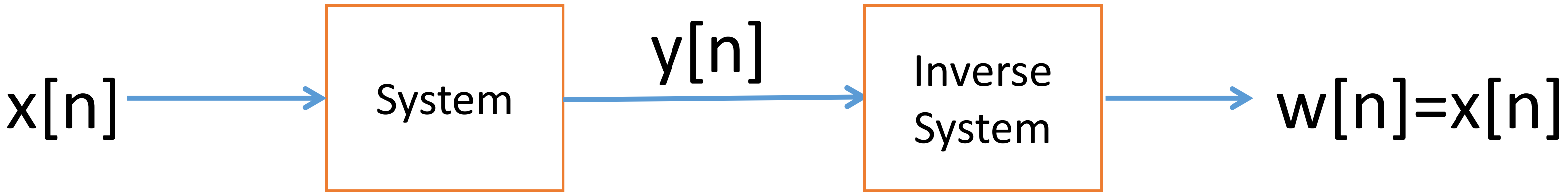
Fall Semester

Invertibility/Inverse System

- A system is *invertible* if distinct inputs lead to distinct output.
- If a system is invertible, then an *inverse system* exists such that when cascaded with the original system yields an output equal to the input of the first system.



Causality/Causal systems

- A system is *causal* if for every t_o (n_o), the output signal y(t) (y[n]) depends on values of the input x(t) (y[n]) at only present (t_o (n_o)) and earlier times.
- *Nonanticipative*; the system output does not use (anticipate) future values of the input.

Causality

example: backward difference

$$y[n] = x[n-1] - x[n]$$

causal?

example: forward difference

$$y[n] = x[n+1] - x[n]$$

causal?

example: moving average

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$$

For which values of M_1 and M_2 is the system causal?

Stability/Stable Systems

- A system is said to be ***stable*** in BIBO sense *(bounded-input-bounded output, BIBO)* if a bounded input generates a bounded output.

(stability can be defined in other ways as well)

Mathematically;

A system is said to be BIBO stable,

If there exists a B_x satisfying $|x(t)| \leq B_x < \infty$

there must be a B_y such that: $|y(t)| \leq B_y < \infty$

note: the same definition and condition is valid for discrete time

Stability

example: ideal delay

$$y[n] = x[n - n_d]$$

stable?

example : compressor

$$y[n] = x[Mn]$$

stable?

example : square

$$y(t) = \{x(t)\}^2$$

stable?

example : moving average

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$$

stable?

Stability

example: accumulator

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Stable?

solution: homework

(*hint: examine the output for the input x[n]=u[n]*)

Linearity/Linear systems

- A system is *linear* if it satisfies superposition property.

$$x(t) \rightarrow y(t) \qquad x_1(t) \rightarrow y_1(t) \qquad x_2(t) \rightarrow y_2(t)$$

Additive Property:

$$T\{x_1(t) + x_2(t)\} = T\{x_1(t)\} + T\{x_2(t)\} = y_1(t) + y_2(t)$$

Scalability Property:

$$T\{ax(t)\} = aT\{x(t)\} = ay(t)$$

Linearity

Additivity and scalability combined:

$$T\{ax_1(t)+bx_2(t)\}=aT\{x_1(t)\}+bT\{x_2(t)\}=ay_1(t)+by_2(t)$$

Generalized superposition property:

$$T\{a_1x_1(t)+a_2x_2(t)+\cdots+a_kx_k(t)\}$$
$$=a_1T\{x_1(t)\}+a_2T\{x_2(t)\}+\cdots+a_kT\{x_k(t)\}$$
$$=a_1y_1(t)+a_2y_2(t)+\cdots+a_ky_k(t)$$

$$x(t)=\sum_k a_kx_k(t) \quad \Rightarrow \quad y(t)=\sum_k a_ky_k(t)$$

Linearity

example: integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

linear?

example :

$$y[n] = 2x[n] + 3$$

linear?

example : square operator

$$y[n] = x^2[n]$$

linear?

Time invariance / Time invariant systems

- A system is *time invariant* is a time shift results in an identical time shift in the output system.
- *i.e.* The behaviour and characteristics of the system are fixed over time.

Mathematically, *if*:

$$x(t) \rightarrow y(t)$$

then,

$$x(t - t_o) \rightarrow y(t - t_o)$$

Time invariant systems

example: $y(t) = (\sin t)x(t)$ Time invariant?

1. replace $x(t)$ with $x(t\text{-}t_o)$

$$x(t) \to (\sin t)x(t)$$

$$x(t - t_o) \to (\sin t)x(t - t_o) \quad (*)$$

2. in $y(t)$, replace t with $(t\text{-}t_o)$

$$y(t - t_o) \to (\sin(t - t_o))x(t - t_o) \quad (**)$$

If (*) and (**) are not equal, the system is not time invariant.

Summary

- Definition of a System
- Interconnection of Systems *(Series, Feedback, etc.)*
- Examples of Systems
- Properties of Systems

  Memory (Bellek)
  Invertibility (Tersi Alınabilirlik)
  Causality (Nedensellik)
  Stability (Kararlılık)
  Linearity (Doğrusallık)
  Time invariance (Zaman Değişmezlik)

- *Time Invariant (LTI) Systems*

  Doğrusal Zamanda Değişmeyen Sistemler