Continuous Time Fourier Transform

Lecture 12

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Signals & Systems, 2019-2020 Fall

Content of This Lecture:

In this lecture, we will learn how to take the Fourier transform of continuous time signals which we can use for aperiodic signals.

We will learn about some important properties of the Fourier transform.

We will learn the Fourier transform of some fundamental signals.

Continuous Time Fourier Transform:

Fourier series cannot be applied directly to aperiodic signals.

With some approximations, Fourier transform can be applied to aperiodic signals as follows:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{1}$$

where $X(\omega)$ is the Fourier transform of the continuous time signal $x(t)$. In other words, $X(\omega)$ is the frequency domain representation or spectrum of the time-domain signal, $x(t)$.

Inverse Fourier Transform:

It is possible to transform $X(\omega)$ back to time domain using inverse Fourier transform as follows:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} dw \tag{2}$$

This is also represented as:

$$x(t) = \mathcal{F}^{-1} X(\omega) \tag{3}$$

Properties of Fourier Transform - 1:

Linearity:

$$ax(t) + by(t) \leftrightarrow aX(\omega) + bY(\omega)$$

Time Shift:

$$x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$$

Properties of Fourier Transform - 2:

Time and Frequency Scaling:

$$x(at) \leftrightarrow \frac{1}{|a|} X(\frac{\omega}{a})$$

Conjugation and Conjugate Symmetry:

$$x^*(t) \leftrightarrow X^*(-\omega)$$

Properties of Fourier Transform - 3:

Differentiation and Integration:

$$\frac{d}{dt}x(t) \leftrightarrow j\omega X(\omega)$$

$$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$$

Properties of Fourier Transform - 4:

Duality:

$$X(t) \leftrightarrow 2\pi x(-\omega) \tag{4}$$

Parseval's Relation:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \tag{5}$$

Properties of Fourier Transform - 5:

Convolution Property:

$$x(t) * y(t) \leftrightarrow X(\omega)Y(\omega) \tag{6}$$

Multiplication Property:

$$x(t)y(t) \leftrightarrow \frac{1}{2\pi}X(\omega) * Y(\omega) \tag{7}$$

Fourier Transform of Some Fundamental Signals - 1:

$$e^{j\omega_0 t} \rightarrow 2\pi\delta(\omega - \omega_0) \tag{8}$$

$$cos(\omega_0 t) \rightarrow \pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)) \tag{9}$$

$$sin(\omega_0 t) \rightarrow \frac{\pi}{j}(\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) \tag{10}$$

Fourier Transform of Some Fundamental Signals - 2:

$$x(t) = 1 \rightarrow 2\pi\delta(\omega) \tag{11}$$

$$\delta(t) \rightarrow 1 \tag{12}$$

Fourier Transform of Some Fundamental Signals - 3:

$$u(t) \rightarrow \frac{1}{j\omega} + \pi\delta(\omega) \tag{13}$$

$$\delta(t - to) \rightarrow e^{-j\omega t_0} \tag{14}$$