Discrete Time Fourier Transform

Lecture 13

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Content of This Lecture:

In this lecture, we will learn how to take the Fourier transform of discrete time signals which we can use for aperiodic signals.

We will learn about some important properties of the Fourier transform.

We will learn the Fourier transform of some fundamental signals.

Discrete Time Fourier Transform:

Fourier series cannot be applied directly to aperiodic signals.

With some approximations, Fourier transform can be applied to aperiodic signals as follows:

$$X(\Omega) = \sum_{-\infty}^{\infty} x[n] e^{-j\Omega n} \qquad (1)$$

where $X(\Omega)$ is the Fourier transform of the discrete time signal $x[n]$. In other words, $X(\Omega)$ is the frequency domain representation or spectrum of the time-domain signal, $x[n]$.

Inverse Fourier Transform:

It is possible to transform $X(\Omega)$ back to time domain using inverse Fourier transform as follows:

$$x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(\Omega) e^{j\Omega n} d\Omega \qquad (2)$$

This is also represented as:

$$x[n] = \mathcal{F}^{-1} X(\Omega) \qquad (3)$$

Properties of Fourier Transform - 1:

Linearity:

$$ax[n] + by[n] \leftrightarrow aX(\Omega) + bY(\Omega)$$

Time Shift:

$$x[n - n_0] \leftrightarrow e^{-j\Omega n_0} X(\Omega)$$

Properties of Fourier Transform - 2:

Periodicity:

$$X(\Omega) = X(\Omega + 2\pi)$$

Conjugation and Conjugate Symmetry:

$$x^*[n] \leftrightarrow X^*(-\Omega)$$

Properties of Fourier Transform - 3:

Differencing and Accumulation:

$$x[n] - x[n-1] \leftrightarrow (1 - e^{j\Omega})X(\Omega)$$

$$\sum_{m=-\infty}^{\infty} x[m] \leftrightarrow \frac{1}{1 - e^{j\Omega}} X(\Omega) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$$

Properties of Fourier Transform - 4:

Time Reversal:

$$x[-n] \leftrightarrow X(-\Omega) \tag{4}$$

Parseval's Relation:

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{<2\pi>} |X(\Omega)|^2 d\Omega \tag{5}$$

Properties of Fourier Transform - 5:

Convolution Property:

$$x[n] * y[n] \leftrightarrow X(\Omega)Y(\Omega) \tag{6}$$

Multiplication Property:

$$x[n]y[n] \leftrightarrow \frac{1}{2\pi}\int_{2\pi} X(\theta) * Y(\Omega - \theta)d\theta \tag{7}$$

Fourier Transform of Some Fundamental Signals - 1:

$$e^{j\Omega_0 n} \rightarrow 2\pi \sum_{l=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi l) \tag{8}$$

$$cos(\Omega_0 n) \rightarrow \pi(\sum_{l=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi l) + \delta(\Omega + \Omega_0 - 2\pi l)) \tag{9}$$

$$sin(\Omega_0 n) \rightarrow \frac{\pi}{j}(\sum_{l=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi l) - \delta(\Omega + \Omega_0 - 2\pi l)) \tag{10}$$