Signals and Their Properties

Lecture 3

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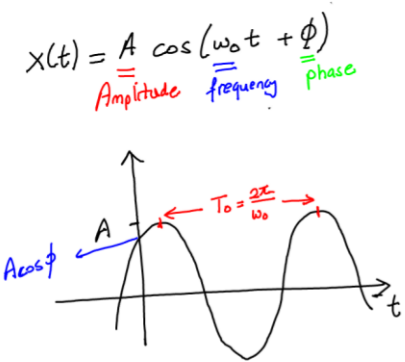
Continuous Time Signals:

Mathematical representation of continous time signal:



Properties of Continous Time Signals - Periodic:

∘ Periodic ⇒ $x(t) = x(t + T_0)$

Since $x(t) = A\cos(\omega_0 t + \phi)$

$$x(t + T_0) = A\cos(\omega_0(t + T_0) + \phi)$$

$$= A\cos(\omega_0 t + \omega_0 T_0 + \phi)$$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Properties of Continous Time Signals - Time Shift vs Phase:

A time shift of a sinusoidal is equivalent to a phase change.

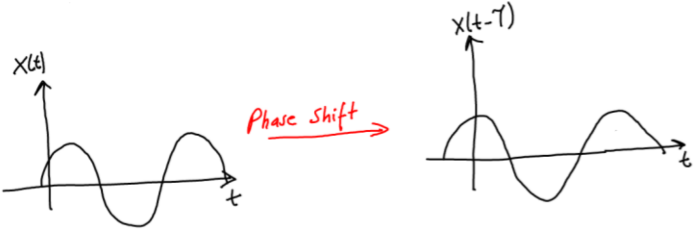
Time Shift $\Longleftrightarrow$ Phase Change

$$A\cos(\omega_0(t+t_0)) = A\cos(\omega_0 t + \underline{\underline{\omega_0 t_0}})$$

A change in phase

$\Delta\phi$

Phase Shift:



A phase shift is equivalent to moving the signal in time.

Even:

A signal is said to be even if we flip it around the origin it looks exactly the same:

$$x(t) = x(-t) \tag{1}$$

Odd:

A signal is said to be even if we flip it around the origin it is exactly the same as the original signals negated version:

$$x(t) = -x(-t) \tag{2}$$

Cosine and Sine Functions

Since,

$$cos(t) = cos(-t) \tag{3}$$

$cos(t)$ is even.

Since,

$$sin(t) = -sin(-t) \tag{4}$$

$sin(t)$ is odd.

You can practice on different functions.

Sine from Cosine

If we apply a phase shift of $-\pi/2$:

$$A\cos\left(\omega_0 t - \frac{\pi}{2}\right) = A\sin(\omega_0 t)$$

$$A\cos\left(\omega_0 t - \frac{\pi}{2}\right) = A\cos\left(\omega_0\left(t - \frac{T_0}{4}\right)\right)$$

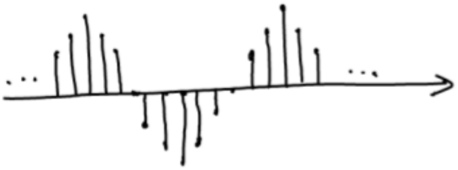
Discrete Time Sinusoidal Signal:

$$x[n] = A\cos(\Omega_0 n + \phi)$$

Amplitude frequency phase

$\Omega_0 = \frac{2\pi}{N}$

$\phi = 0$



Properties of Discrete Time Sinusoidal:

In continous time: A time shift $\iff$ phase change.

In discerete time: A time shift $\Rightarrow$ phase change.

In discrete time a phase change does not necessarily corresponds to a time shift.

$cos[n]$ has even symmetry.

$sin[n]$ has odd symmetry.

Mathematical Representation:

$$Acos(\Omega_0(n+n_0)) = A(cos(\Omega_0 n + \Omega_0 n_0) \tag{5}$$

$\Omega_0 n_0$ is the phase change, $\Delta\phi$.

What if we shift cosine by $\frac{\pi}{2}$:

$$Acos(\Omega_0 n - \frac{\pi}{2}) = Asin(\Omega_0 n) \tag{6}$$

$$Acos(\Omega_0 n - \frac{\pi}{2}) = Acos(\Omega_0(n - n_0)) \tag{7}$$

where n_0 is $\frac{T_0}{4}$

Phase Change vs. Time Change:

Phase change $\overset{?}{\Rightarrow}$ time change:

It is not necessarily true.

$$A cos(\Omega_0(n+n_0)) \overset{?}{=} A cos(\Omega_0 n+\phi) \quad (8)$$

$$\Omega_0 n+\Omega_0 n_0 = \phi \quad (9)$$

Since n_0 must be an integer, this is not satisfied for every value of ϕ.

The Issue of Periodicity

Are all discrete time sinusoidals periodic?

All continuous-time sinusoidals are periodic.

Discrite-time sinusoidals are not always periodic.

Proof:

To have a periodic signal: $x[n] = x[n + N]$.

$$Acos(\Omega_0(n + N) + \phi) = cost(\Omega_0 n + \Omega_0 N + \phi) \qquad (10)$$

It is periodic if: $\Omega_0 N = 2\pi m$

which means: $N = \frac{2\pi m}{\Omega_0}$

Since N should be an integer, this is not satisfied for all values of Ω_0.

Property of Discrete-Time Sinusoidals:

Let $x_1[n]$ and $x_2[n]$ be:

$$x_1[n] = Acos(\Omega_1 n + \phi) \tag{11}$$
$$x_2[n] = Acos(\Omega_2 n + \phi) \tag{12}$$

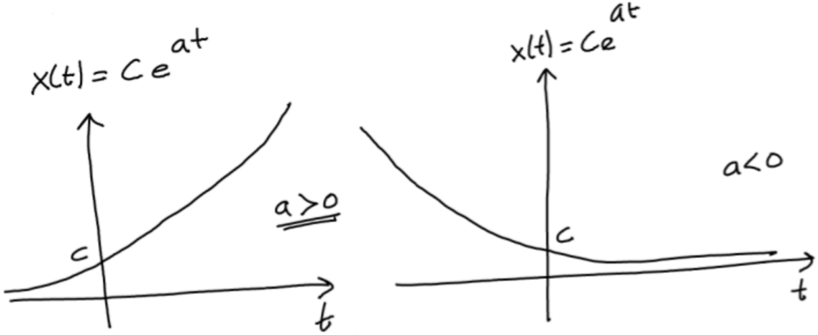
if $\Omega_2 = \Omega_1 + 2\pi$

$$x_2[n] = Acos(\Omega_1 n + 2\pi n + \phi) \tag{13}$$

Since n is always an integer:

$$Acos(\Omega_1 n + 2\pi n + \phi) = Acos(\Omega_1 n + \phi) \tag{14}$$
$$x_2[n] = x_1[n] \tag{15}$$

The Class of Real & Complex Exponentials:



Continuous-Time Complex Exponential

$x(t) = Ce^{at}$

C and a are complex numbers:

$$C = |C|e^{j\Theta} \quad (16)$$

$$a = r + j\omega_0 \quad (17)$$

$$x(t) = |C|e^{j\Theta}e^{(r+j\omega_0)t} \quad (18)$$

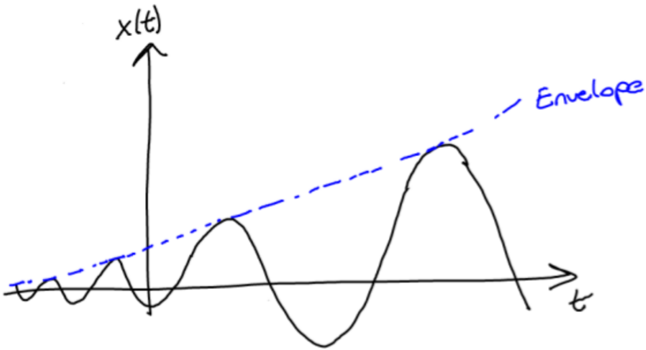
$$= |C|e^{rt}e^{j(\omega_0 t+\Theta)} \quad (19)$$

Using Euler's Identity:

$e^{j\pi} = cos(\pi) + jsin(\pi)$

$$e^{j(\omega_0 t+\Theta)} = cos(\omega_0 t + \Theta) + jsin(\omega_0 t + \Theta) \quad (20)$$

$$x(t) = |C|e^{rt}cos(\omega_0 t + \Theta) + j|C|e^{rt}sin(\omega_0 t + \Theta) \quad (21)$$


x(t)
Envelope
t