Calculus II
Week 2 Lecture

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Integration by Parts

$$D_x(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

or

$$u(x)v'(x) = u'(x)v(x) - D_x(u(x)v(x))$$

Integration by Parts

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or

$$u(x)v'(x) = u'(x)v(x) - D_x(u(x)v(x))$$

By integrating both sides

$$\int u(x)v'(x)\ dx = u(x)v(x) - \int u'(x)v(x)\ dx$$

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since $dv = v'(x)dx$ and $du = u'(x)dx$ we get

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$$D_x(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

or

$$u(x)v'(x) = u'(x)v(x) - D_x(u(x)v(x))$$

By integrating both sides

$$\int u(x)v'(x)\ dx = u(x)v(x) - \int u'(x)v(x)\ dx$$

since $dv = v'(x)dx$ and $du = u'(x)dx$ we get

$$\int u\ dv = uv - \int v\ du$$

Integration by Parts

Example

Find $\int x \cos(x)\ dx$.

Example

Find $\int \ln(x)\ dx$.

Example

Find $\int e^x \sin(x)\ dx$.

Some Trigonometric Integrals-Type A

$\int \sin^n(x)\, dx$ or $\int \cos^n(x)\, dx$.

Example (n odd)

Find

$$\int \sin^5(x)\, dx.$$

Use Pythagorean Identities.

Example (n even)

Find

$$\int \sin^2(x)\, dx.$$

Use Half-angle Identities.

Some Trigonometric Integrals-Type B

$\int \sin^n(x)\cos^m(x)\ dx.$

Example (n or m odd)

Find

$$\int \sin^3(x)\cos^2(x)\ dx.$$

Use Pythagorean Identities.

Example (Both m and n even)

Find

$$\int \cos^2(x)\sin^2(x)\ dx.$$

Use Half-angle Identities.

Some Trigonometric Integrals-Type C

$\int \sin(nx)\cos(mx)\ dx$, $\int \sin(nx)\sin(mx)\ dx$ or $\int \cos(nx)\cos(mx)\ dx$

Example

Find

$$\int \sin(2x)\cos(3x)\ dx.$$

Use Product Identities.

Some Trigonometric Integrals-Type D

For $\int \tan^n(x)\ dx$ and $\int \cot^n(x)\ dx$ use the identities
$\tan^2(x) = \sec^2(x) - 1$ and $\cot^2(x) = \csc^2(x) - 1$

Example

Find

$$\int \cot^4(x)\ dx.$$

For integrals involved with $\sqrt[n]{ax+b}$ use the substitution $u^n = ax + b$.

Example

Find

$$\int x\sqrt[3]{x-4}\,dx.$$

Rationalizing-Type B

For integrals involved with $\sqrt{a^2 - x^2}$ use the substitution $u = a\sin(t)$.

Example

Find

$$\int \frac{\sqrt{4 - x^2}}{x}\, dx.$$

Rationalizing-Type C

For integrals involved with $\sqrt{a^2 + x^2}$ use the substitution $u = a\tan(t)$.

Example

Find

$$\int \frac{1}{(x^2+4)^{3/2}}\,dx.$$

Rationalizing-Type D

For integrals involved with $\sqrt{x^2 - a^2}$ use the substitution $u = a \sec(t)$.

Example

Find

$$\int \frac{\sqrt{x^2 - 1}}{x^3}\, dx.$$