Calculus II
Week 3 Lecture

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Partial Fractions-Type A

Distinct linear factors.

Example

Find

$$\int \frac{5x+3}{x^3-2x^2-3x}\,dx.$$

Partial Fractions-Type B

Repeated linear factors.

Example

Find

$$\int \frac{x}{(x-3)^2}\, dx.$$

Partial Fractions-Type C

Some distinct, some repeated linear factors.

Example

Find

$$\int \frac{3x^2 - 8x + 13}{(x-1)^2(x+3)}\, dx.$$

Partial Fractions-Type D

Single quadratic factor.

Example

Find

$$\int \frac{6x^2 - 3x + 11}{(4x+1)(x^2+1)}\, dx.$$

Partial Fractions-Type E

Repeated quadratic factors.

Example

Find

$$\int \frac{6x^2 - 15x + 22}{(x+3)(x^2+2)^2}\, dx.$$

Reduction Formulas I

Example

$$I_n = \int x^n e^{ax}\ dx.$$

$$\begin{aligned} I_n &= x^n \frac{1}{a} e^{ax} - \int n x^{n-1} \frac{1}{a} e^{ax} dx \\ &= \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx. \end{aligned}$$

Reduction Formulas II

Example

$$S_n = \int x^n \sin x dx.$$

$$\begin{aligned} S_n &= -x^n \cos x + n \int x^{n-1} \cos x dx \\ &= -x^n \cos x + nx^{n-1} \sin x - n(n-1) \int x^{n-2} \sin x dx. \end{aligned}$$

Thus

$$S_n = -x^n \cos x + nx^{n-1} \sin x - n(n-1) S_{n-2}.$$

Reduction Formulas III

Example

$$I_n = \int (\sin x)^n dx$$

$$I_n = \int (\sin x)^{n-1} \sin x dx$$

$$= -(\sin x)^{n-1} \cos x - \int (-\cos x)(n-1)(\sin x)^{n-2} \cos x dx$$

$$= -(\sin x)^{n-1} \cos x + (n-1) \int (\sin x)^{n-2} \cos^2 x dx.$$

We now use $\cos^2 x = 1 - \sin^2 x$, which gives

$$I_n = -(\sin x)^{n-1} \cos x + (n-1) \int sin^{n-2} x - \sin^n x dx$$

$$= -(\sin x)^{n-1} \cos x + (n-1) I_{n-2} - (n-1) I_n.$$