Calculus II
Week 4 Lecture

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Definite Integrals

The **definite integral of** f **on** $[a, b]$ is the total signed area of f on $[a, b]$, denoted

$$\int_a^b f(x)\,dx,$$

where a and b are the **bounds of integration.**

The Fundamental Theorem of Calculus I

Let f be continuous on $[a, b]$ and let $F(x) = \int_a^x f(t)\ dt$. Then F is a differentiable function on (a, b), and

$$F'(x) = f(x).$$

Example

Let $F(x) = \int_{-5}^{x} (t^2 + \sin t)\ dt$. *What is* $F'(x)$*?*

F is called an antiderivative of f

The Fundamental Theorem of Calculus II

Let f be continuous on $[a, b]$ and let F be *any* antiderivative of f. Then

$$\int_a^b f(x)\ dx = F(b) - F(a).$$

Properties

1. $$\int_a^a f(x)\ dx = 0$$
2. $$\int_a^b f(x)\ dx + \int_b^c f(x)\ dx = \int_a^c f(x)\ dx$$
3. $$\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$$
4. $$\int_a^b (f(x) \pm g(x))\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$$
5. $$\int_a^b k \cdot f(x)\ dx = k \cdot \int_a^b f(x)\ dx$$

Problems

1. $\displaystyle\int_{-5}^{-2} \frac{x^4 - 1}{x^2 + 1} dx$
2. $\displaystyle\int_{0}^{2} \frac{x^3 dx}{x^4 + 1}$
3. $\displaystyle\int_{0}^{1} \frac{e^{3x} dx}{e^{4x} - 1}$
4. $\displaystyle\int_{1}^{2} \frac{dx}{x(x - h)} \quad h > 0$

Problems

1. $$\int_0^a x \sin x dx$$
2. $$\int_3^4 \frac{x dx}{\sqrt{x^2 - 1}}$$
3. $$\int_0^\pi x(\cos x)^2 dx$$
4. $$\int_4^5 \frac{dx}{x(x-1)(x-2)(x-3)}$$

Average of a function

The average value of a function $f(x)$ over the interval $[a, b]$ is given by

$$f_{avg} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

Mean Value Theorem

If $f(x)$ is is a continuous function on $[a, b]$ then there is a number c in $[a, b]$ such that

$$\int_a^b f(x)\, dx = f(c)(b-a)$$

Example

Example

Determine the number c that satisfies the Mean Value Theorem for Integrals for the function $f(x) = sinx$ *on the interval* $[0, \pi]$