Calculus II
Week 5 Lecture

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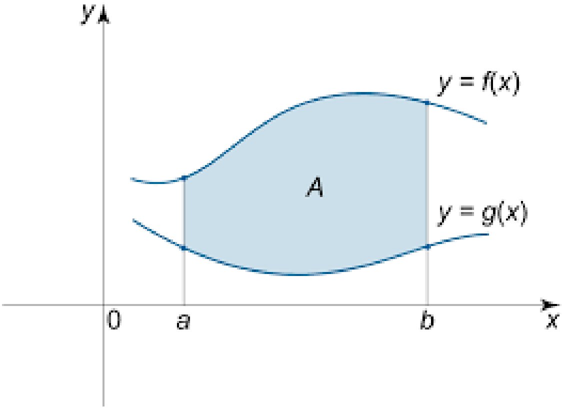
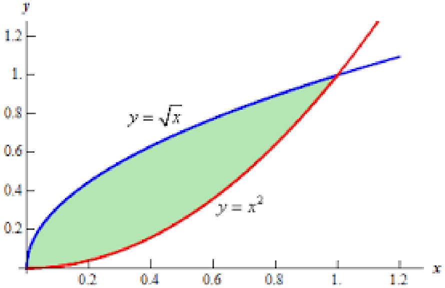
Area Between Curves I



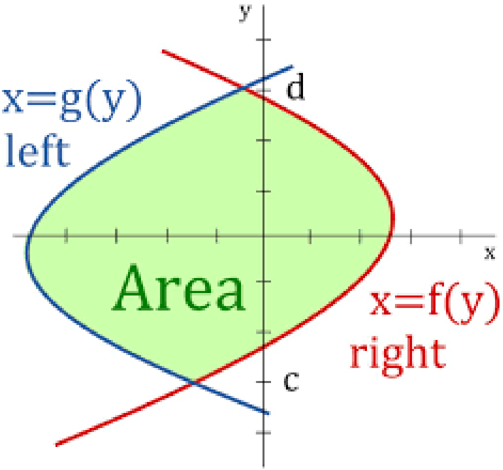
$$A = \int_a^b \left(\begin{array}{c} \text{upper} \\ \text{function} \end{array} \right) - \left(\begin{array}{c} \text{lower} \\ \text{function} \end{array} \right) dx$$

Area Between Curves I



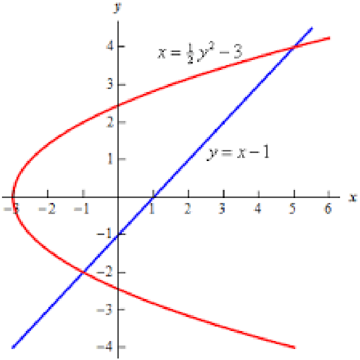
$$A = \int_a^b f(x) - g(x)\, dx$$

Area Between Curves II



$$A = \int_c^d \left(\begin{array}{c} \text{right} \\ \text{function} \end{array} \right) - \left(\begin{array}{c} \text{left} \\ \text{function} \end{array} \right) dy$$

Area Between Curves II



$$A = \int_a^b f(x) - g(x)\ dx$$

Example

Find the area of the region bounded by the curves

$$x = 1, \qquad x = 2, \qquad y = \frac{2}{x^2 - 4x + 5}, \qquad y = \frac{x^2 - 8x + 7}{x^2 - 8x + 16}.$$

Example

Find the area of the region bounded by the curves

$$x = 2, \qquad y = 0, \qquad y = x \ln \frac{x}{2}$$

Method of disk

The volume (V) of a solid generated by revolving the region bounded by $y = f(x)$ and the x-axis on the interval $[a, b]$ about the x-axis is

$$V = \pi \int_a^b f(x)^2 \, dx$$

Method of disk

The volume (V) of a solid generated by revolving the region bounded by $x = f(y)$ and the y-axis on the interval $[c, d]$ about the y-axis is

$$V = \pi \int_c^d f(y)^2 \, dy$$

Method of Washer

The volume (V) of a solid generated by revolving the region bounded by $y = f(x)$ and $y = g(x)$ on the interval $[a, b]$ where $f(x) \geq g(x)$, about the x-axis is

$$V = \pi \int_a^b f(x)^2 - g(x)^2 \, dx$$

Method of Washer

The volume (V) of a solid generated by revolving the region bounded by $x = f(y)$ and $x = g(y)$ on the interval $[c, d]$ where $f(y) \geq g(y)$, about the y-axis is

$$V = \pi \int_c^d f(y)^2 - g(y)^2 \, dy$$

The volume (V) of a solid generated by revolving the region bounded by $y = f(x)$ and the x-axis on the interval $[a, b]$, where $f(x) \geq 0$, about the y-axis is

$$V = 2\pi \int_a^b xf(x)\, dx$$

Cylindrical shell method

The volume (V) of a solid generated by revolving the region bounded by $x = f(y)$ and the y-axis on the interval $[c, d]$, where $f(y) \geq 0$, about the x-axis is

$$V = 2\pi \int_c^d yf\,(y)\;dx$$