Calculus II
Week 6 Lecture

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Improper Integrals: Infinite Limits I

If $\int_a^t f(x)\ dx$ exist for all $t > a$ then

$$\int_a^{\infty} f(x)\ dx = \lim_{t\to\infty} \int_a^t f(x)\ dx$$

provided the limit exists and is finite.

Improper Integrals: Infinite Limits II

If $\int_t^b f(x)\,dx$ exist for all $t < b$ then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t \to -\infty} \int_t^b f(x)\,dx$$

provided the limit exists and is finite.

Improper Integrals: Infinite Limits III

We will call these integrals convergent if the associated limit exists and is a finite number and divergent if the associated limit either doesnt exist or is (plus or minus) infinity.

$$\int_{-\infty}^{\infty} f(x)\ dx = \int_{-\infty}^{c} f(x)\ dx + \int_{c}^{\infty} f(x)\ dx$$

If either of the two integrals is divergent then so is the above integral.

Improper Integrals: Discontinuous Integrand I

If $f(x)$ is is continuous on the interval $[a, b)$ and not continuous at $x = b$ then

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$

provided the limit exists and is finite.

Improper Integrals: Discontinuous Integrand II

If $f(x)$ is is continuous on the interval $(a, b]$ and not continuous at $x = a$ then

$$\int_a^b f(x)\, dx = \lim_{t \to a^+} \int_t^b f(x)\, dx$$

provided the limit exists and is finite.

Improper Integrals: Discontinuous Integrand III

If $f(x)$ is not continuous at $x = c$ for $a < c < b$

$$\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$$

If either of the two integrals is divergent then so is the above integral.

Problems

1. $\int_{-\infty}^{0} \frac{1}{\sqrt{1-x}}\, dx$
2. $\int_{-\infty}^{\infty} x\mathbf{e}^{-x^2}\, dx$
3. $\int_{0}^{\infty} \frac{1}{x^2}\, dx$
4. $\int_{-2}^{3} \frac{1}{x^3}\, dx$

Improper Integrals: Comparison Test

If $f(x) \geq g(x) \geq 0$ on the interval $[a, \infty)$

1. If $\int_a^\infty f(x)\, dx$ converges then so does $\int_a^\infty g(x)\, dx$
2. If $\int_a^\infty g(x)\, dx$ diverges then so does $\int_a^\infty f(x)\, dx$

Example

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$$\int_2^\infty \frac{\cos^2 x}{x^2}\, dx$$

Note that $\frac{\cos^2 x}{x^2} \leq \frac{1}{x^2}$. Since $\int_2^\infty \frac{1}{x^2}\, dx$ converges so does $\int_2^\infty \frac{\cos^2 x}{x^2}\, dx$