For #1-4, find the area of the plane region.

1. $y = x^2 - 1$ over the interval $[-1, 1]$.

2. $y = x^2 + 1$, $x - y = 2$, and the lines $x = -2$, $x = 2$

3. $x = y^3 + 2y^2 - 3y$, where y is between -3 and 1

4. $y = x + 3$, $x = -y^2 + 3$

For #5-8, find the volume of the solid of revolution using the disk method. You do not need to solve the integral, just set it up.

5. bounded by $y = \frac{1}{2}x^2 + 2$, x-axis, lines $x = -1$, $x = 2$, rotated about the x-axis

6. bounded by $x = y^2 - 4$, x-axis, y-axis, the line $y = 2$ rotated about the y-axis

7. bounded by $y = x^2$, the line $y = 4$, rotated about the line $y = 4$

8. bounded by $y = x^2$, the line $y = 4$, rotated about the line $y = 5$ (use washer method)

For #9-15, find the volume of the solid of revolution using the washer method. You do not need to solve the integral, just set it up.

9. bounded by $y = x^2$, $y = 4 - x^2$, rotated about x-axis

10. bounded by $y = -2x^2 + 2$, $y = -x^2 + 1$, rotated about x-axis

11. bounded by $x = \sqrt{25 - y^2}$, line $x = 3$, rotated about y-axis

12. bounded by $x+y = 1$, $x-y = -1$, line $x = 2$, rotated about y-axis (use shell method instead)

13. bounded by $y = x^3$, $y = 4x$, rotated about the line $y = 8$

14. bounded by $y = x^{2/3}$, $y = x^2$, rotated about $y = -1$

15. bounded by $x + y = 3$, $y + x^2 = 3$, rotated about the line $x = 2$

For #16-21, find the volume of the solid of revolution using the shell method. You do not need to solve the integral, just set it up.

16. bounded by $y = \sqrt{x}$, x-axis, the line $x = 4$, rotated about y-axis

17. bounded by $y^3 = x$, y-axis, the line $y = 3$, rotated about x-axis

18. bounded by $y = \sqrt{x + 4}$, x-axis, y-axis, rotated about x-axis

19. bounded by $2x - y = 12$, $x - 2y = 3$, the line $x = 4$, rotated about y-axis

20. bounded by $y = x^2$, the line $y = 4$, rotated about the line $x = -3$

21. bounded by $y = x^2$, the line $y = 4$, rotated about the line $y = 5$

Solutions:

1. 4/3

2. 52/3

3. 71/6

4. 125/6

5. $\pi \int_{-1}^{2} (\frac{1}{2}x^2 + 2)^2 dx$

6. $\pi \int_{0}^{2} (y^2 - 4)^2 dy$

7. $\pi \int_{-2}^{2} (4 - x^2)^2 dx$

8. $\pi \int_{-2}^{2} [(5 - x^2)^2 - 1] dx$

9. $\pi \int_{-\sqrt{2}}^{\sqrt{2}} [(4 - x^2)^2 - x^4] dx$

10. $\pi \int_{-1}^{1} [(-2x^2 + 2)^2 - (-x^2 + 1)^2] dx$

11. $\pi \int_{-4}^{4} [16 - y^2] dy$

12.

13. $\pi \int_{0}^{2} [(8 - x^3)^2 - (8 - 4x)^2] dx$

14. $\pi \int_{0}^{1} [(x^{2/3} + 1)^2 - (x^2 + 1)^2] dx$

15. $\pi \int_{2}^{3} [(y - 1)^2 - (2 - \sqrt{3 - y})^2] dy$

16. $2\pi \int_{0}^{4} x^{3/2} dx$

17. $2\pi \int_{0}^{3} (3y - y^4) dy$

18. $2\pi \int_{0}^{2} (4y - y^3) dy$

19. $2\pi \int_{4}^{7} x[(\frac{1}{2}x - \frac{3}{2}) - (2x - 12)] dx$

20. $2\pi \int_{-2}^{2} (x + 3)(4 - x^2) dx$

21. $4\pi \int_{0}^{4} (5 - y)\sqrt{y} dy$

1. $\int \dfrac{2}{\sqrt{9-4x^2}}\,dx$

2. $\int \dfrac{12x^{1/2}}{\sqrt{1-64x^3}}\,dx$

3. $\int \dfrac{e^x}{\sqrt{2-e^{2x}}}\,dx$

4. $\int \dfrac{5}{x^2+2}\,dx$

5. $\int \dfrac{2x}{9+x^4}\,dx$

6. $\displaystyle\int \frac{x+2}{x^2+1}\,dx$

7. $\displaystyle\int \frac{2}{x^2+4x+13}\,dx$

8. $\displaystyle\int \frac{x^2+3x}{x-2}\,dx$

9. $\displaystyle\int \frac{x^2+7x+11}{x^2+5x+6}\,dx$

10. $\displaystyle\int \cos^3(x)\,dx$

11. $\int \sin^6(x)dx$

12. $\int \sin^4(x)\cos^2(x)dx$

13. $\int \cos^3(x)\csc^2(x)dx$

14. $\displaystyle\int \sin{(2x)}\sin{(5x)}dx$

15. $\displaystyle\int \sin{(3x)}\cos{(4x)}dx$

16. $\displaystyle\int \cos{(x)}\cos{(2x)}dx$

17. $\displaystyle\int x\sqrt[3]{x+9}dx$

18. $\displaystyle\int \frac{x}{\sqrt[5]{3x+2}}dx$

19. $\displaystyle\int \frac{1}{\sqrt{x}+4}\,dx$

20. $\displaystyle\int \frac{1}{x\sqrt{4-x^2}}\,dx$

21. $\displaystyle\int \frac{1}{x\sqrt{9+x^2}}\,dx$

22. $\displaystyle\int \frac{x^3}{\sqrt{9x^2+49}}\,dx$

23. $\displaystyle\int \frac{1}{\sqrt{x^2 + 8x + 25}}dx$

24. $\displaystyle\int x\sqrt{x^2 - 9}dx$

1. $\sin^{-1}(\frac{2}{3}x) + C$

2. $\sin^{-1}(8x^{3/2}) + C$

3. $\sin^{-1}(\frac{e^x}{\sqrt{2}}) + C$

4. $\frac{5}{\sqrt{2}} \tan^{-1}(\frac{x}{\sqrt{2}}) + C$

5. $\frac{1}{3} \tan^{-1}(\frac{x^2}{3}) + C$

6. $\frac{1}{2} \ln|x^2 + 1| + 2\tan^{-1}(x) + C$

7. $\frac{2}{3} \tan^{-1}(\frac{x+2}{3}) + C$

8. $\frac{x^2}{2} + 4x + 4\ln|x - 2| + C$

9. $x + \ln|x^2 + 5x + 6| + C$

10. $\sin(x) - \frac{1}{3}\sin^3(x) + C$

11. $\frac{1}{4}\sin^3(2x) - \frac{3}{64}\sin(4x) - \frac{1}{4}\sin(2x) + \frac{5}{16}x + C$

12. $-\frac{1}{12}\sin^3(2x) - \frac{1}{64}\sin(4x) + \frac{3}{16}\sin(2x) + \frac{1}{16}x + C$

13. $\frac{1}{3}\sin^3(x) - \sin(x) + C$

14. $\frac{1}{14}\sin(7x) - \frac{1}{6}\sin(3x) + C$

15. $\frac{1}{2}\cos(x) - \frac{1}{14}\cos(7x) + C$

16. $\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x) + C$

17. $\frac{3}{7}(x+9)^{7/3} - \frac{27}{4}(x+9)^{4/3} + C$

18. $\dfrac{5}{81}(3x+2)^{9/5} - \dfrac{5}{18}(3x+2)^{4/3} + C$

19. $2\sqrt{2} - 8\ln|\sqrt{x}+4| + C$

20. $\dfrac{1}{2}\ln\left|\dfrac{2}{x} - \dfrac{\sqrt{4-x^2}}{x}\right| + C$

21. $\dfrac{1}{3}\ln\left|\dfrac{\sqrt{x^2+9}}{x} - \dfrac{3}{x}\right| + C$

22. $\dfrac{49}{81}\sqrt{9x^2+49} - \dfrac{1}{243}(9x^2+49)^{3/2} + C$

23. $\ln\left|\dfrac{\sqrt{x^2+8x+25}}{3} + \dfrac{x+4}{3}\right| + C$

24. $\dfrac{1}{3}(x^2-9)^{3/2} + C$

For #1-18, evaluate the following integrals using integration by parts.

1. $\int xe^{-x}dx$

2. $\int x^2 e^{3x}dx$

3. $\int x\ln(x)dx$

4. $\int x^2 \ln(x)dx$

5. $\displaystyle\int x \cos{(5x)}dx$

6. $\displaystyle\int x \sec(x) \tan(x)dx$

7. $\displaystyle\int x^2 \cos(x)dx$

8. $\displaystyle\int \tan^{-1}(x)dx$

9. $\displaystyle\int \sqrt{x}\ln(x)dx$

10. $\int x \csc^2(x)dx$

11. $\int e^{-x} \sin(x)dx$

12. $\int \sin(x) \ln(\cos(x))dx$

13. $\int x(2x + 3)^{99}dx$

14. $\int (x + 1)^{10}(x + 2)dx$

15. $\int (\ln x)^2 dx$

16. $\int e^{4x} \sin(5x) dx$

17. $\int x^3 \sinh(x) dx$

18. $\int \cos(\sqrt{x}) dx$ – hard

For #19-27, evaluate the following integrals using partial fraction decomposition.

19. $\int \dfrac{5x-12}{x(x-4)}\,dx$

20. $\int \dfrac{x+16}{x^2+2x-8}\,dx$

21. $\int \dfrac{4x^2+13x-9}{x^3+2x^2-3x}\,dx$

22. $\int \dfrac{6x-1}{(x-1)^2}\,dx$

23. $\displaystyle\int \frac{2x^2 - 25x - 33}{(x+1)^2(x-5)}\,dx$

24. $\displaystyle\int \frac{3x^3 - 18x^2 + 29x - 4}{(x+1)(x-2)^3}\,dx$

25. $\displaystyle\int \frac{x^2 + 3x + 1}{x^4 + 5x^2 + 4}\,dx$

26. $\displaystyle\int \frac{x^2 - x - 21}{2x^3 - x^2 + 8x - 4}\,dx$

27. $\displaystyle\int \frac{2x^3 + 10x}{(x^2 + 1)^2}\,dx$

1. $-xe^{-x} - e^{-x} + C$

2. $\dfrac{1}{3}x^2 e^{3x} - \dfrac{2}{9}xe^{3x} + \dfrac{2}{27}e^{3x} + C$

3. $\dfrac{1}{2}x^2 \ln(x) - \dfrac{1}{4}x^2 + C$

4. $\dfrac{1}{3}x^3 \ln(x) - \dfrac{1}{9}x^3 + C$

5. $\dfrac{1}{5}x \sin(5x) + \dfrac{1}{25}\cos(5x) + C$

6. $x\sec(x) - \ln|\sec(x) + \tan(x)| + C$

7. $x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + C$

8. $x\tan^{-1}(x) - \dfrac{1}{2}\ln|1 + x^2| + C$

9. $\dfrac{2}{3}x^{3/2}\ln|x| - \dfrac{4}{9}x^{3/2} + C$

10. $-x\cot(x) + \ln|\sin(x)| + C$

11. $-\dfrac{1}{2}e^{-x}(\sin(x) + \cos(x)) + C$

12. $\cos(x)(1 - \ln|\cos(x)|) + C$

13. $\dfrac{x}{200}(2x + 3)^{100} - \dfrac{1}{40400}(2x + 3)^{101} + C$

14. $\dfrac{(x + 2)}{11}(x + 1)^{11} - \dfrac{1}{132}(x + 1)^{12} + C$

15. $x(\ln x)^2 - 2x\ln x + 2x + C$

16. $\dfrac{1}{41}e^{4x}(4\sin(5x) - 5\cos(5x)) + C$

17. $x^3 \cosh(x) - 3x^2 \sinh(x) + 6x\cosh(x) - 6\sinh(x) + C$

18. $2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right) + C$

19. $3 \ln |x| + 2 \ln |x - 4| + C$

20. $3 \ln |x - 2| - 2 \ln |x + 4| + C$

21. $\ln \left| \dfrac{x^3 (x - 1)^2}{x + 3} \right| + C$

22. $6 \ln |x - 1| + \dfrac{5}{x - 1} + C$

23. $5 \ln |x + 1| - \dfrac{1}{x + 1} - 3 \ln |x - 5| + C$

24. $\ln \left[(x + 1)^2 |x - 2| \right] + \dfrac{3}{x - 2} - \dfrac{1}{(x - 2)^2} + C$

25. $-\dfrac{1}{2} \ln(x^2 + 4) + \dfrac{1}{2} \tan^{-1}(\dfrac{x}{2}) + \dfrac{1}{2} \ln(x^2 + 1) + C$

26. $\dfrac{3}{2} \ln(x^2 + 4) + \dfrac{1}{2} \tan^{-1}(\dfrac{x}{2}) - \dfrac{5}{2} \ln |2x - 1| + C$

27. $\ln(x^2 + 1) - \dfrac{4}{x^2 + 1} + C$