For #1-11, evaluate the following limits using L'Hopitals Rule. Make sure you have 0/0 or ∞/∞ before using L'Hopital.

1. $\displaystyle \lim_{x\to 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$

2. $\displaystyle \lim_{x\to 0} \frac{\sin(x) - x}{\tan(x) - x}$

3. $\displaystyle \lim_{x\to 0} \frac{x - \sin(x)}{x^3}$

4. $\displaystyle \lim_{x\to 0^+} \frac{\ln(x)}{\cot(x)}$

5. $\displaystyle \lim_{x\to \infty} \frac{x^2}{\ln(x)}$

6. $\displaystyle \lim_{x\to 0} \frac{x\cos(x) + e^{-x}}{x^2}$

7. $\lim_{x\to\infty} \frac{x\ln(x)}{x+\ln(x)}$

8. $\lim_{x\to(\pi/2)^-} (1+\cos(x))^{\tan(x)}$

9. $\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$

10. $\lim_{x\to\infty} [\ln(4x+3) - \ln(3x+4)]$

11. $\lim_{x\to 0^+} x\ln(x)$

For #12-21, evaluate the following improper integrals, or state that they diverge.

12. $\int_1^\infty \frac{1}{x^{4/3}}\,dx$

13. $\displaystyle\int_0^\infty e^{-2x}dx$

14. $\displaystyle\int_0^\infty \frac{\cos(x)}{1+\sin^2(x)}dx$

15. $\displaystyle\int_1^\infty \frac{\ln(x)}{x}dx$

16. $\displaystyle\int_3^\infty \frac{1}{x^2-1}dx$

17. $\displaystyle\int_0^{\pi/2} \tan(x)dx$

18. $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

19. $\int_0^1 x \ln(x) dx$

20. $\int_0^2 \frac{x}{x^2 - 1} dx$

21. $\int_{1/e}^e \frac{1}{x(\ln x)^2} dx$

1. 15/13
2. -1/2
3. 1/6
4. 0
5. ∞
6. DNE
7. ∞
8. e
9. 0
10. ln(4/3)
11. 0
12. 3
13. 1/2
14. diverges
15. diverges
16. $-\ln(\sqrt{1/2})$
17. diverges
18. $2e - 2$

19. $-1/4$

20. diverges

21. diverges

For #1-7, determine whether the sequence converges or diverges. If it converges, find its limit.

1. $\{\frac{4n+1}{2n^2-1}\}$

2. $\{\frac{4n^3+5n+1}{2n^3-n^2+5}\}$

3. $\{\frac{\cos(n)}{n}\}$

4. $\{\frac{e^n}{n^4}\}$

5. $\{2^{-n}\sin(n)\}$

6. $\{\frac{\tan^{-1}(n)}{n}\}$

7. $\{n^{1/n}\}$

For #8-13, determine whether the following series converge or diverge. If they converge, what is their sum?

8. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{2k+1}$

9. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{5^k} + \frac{1}{k}\right)$

10. $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k}$

11. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

12. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{7}{k(k+1)} + \frac{2}{3^{k-1}}\right)$

13. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}+\sqrt{k}}$ (Hint: rationalize the denominator)

The function $f(x)$ determined by the n-th terms of the following series satisfy the hypotheses of the integral test; i.e. they are positive, continuous, and decreasing for every x $\geq$ (lower index). For #14-21, use the integral test to determine whether these series converge or diverge.

14. $\sum_{k=1}^{\infty} \frac{1}{(3+2k)^2}$

15. $\sum_{k=3}^{\infty} \frac{\ln k}{k}$

16. $\sum_{k=1}^{\infty} k^2 e^{-k^3}$

17. $\sum_{k=4}^{\infty} \left(\frac{1}{k-3} - \frac{1}{k}\right)$

18. $\sum_{k=1}^{\infty} \frac{arctan(n)}{1+n^2}$

19. $\sum_{k=2}^{\infty} \frac{1}{n(\ln n)^2}$

20. $\sum_{k=1}^{\infty} \frac{1}{1+16k^2}$

1. conv, 0
2. conv, 2
3. conv, 0
4. div
5. conv, 0
6. conv, 0
7. conv, 1
8. div
9. div
10. div
11. conv, 1
12. conv, 10
13. div
14. conv
15. div
16. conv
17. conv
18. conv

19. conv

20. conv

For #1-6, use the limit comparison tests to determine whether the following series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n+7}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n^5+4n^3+1}{2n^8+n^4+2}$

6. $\sum_{n=1}^{\infty} \frac{arctan(n)}{n}$

For #7-16, use the ratio test to determine whether the following series converge or diverge, or indicate that the test is inconclusive.

7. $\sum_{n=1}^{\infty} \frac{3n+1}{2^n}$

8. $\sum_{n=1}^{\infty} \frac{3^n}{n^2+4}$

9. $\sum_{n=1}^{\infty} \frac{5^n}{n(3^{n+1})}$

10. $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n(n+1)}$

11. $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

12. $\sum_{n=1}^{\infty} \frac{n^{10}+10}{n!}$

13. $\sum_{n=1}^{\infty} \frac{n!}{e^n}$

14. $\sum_{n=1}^{\infty} \frac{n!}{(n+1)^5}$

15. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

16. $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n}$

For #17-19, use the root test to determine whether the following series converge or diverge, or indicate that the test is inconclusive.

17. $\sum_{n=1}^{\infty} \frac{1}{n^n}$

18. $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$

19. $\sum_{n=2}^{\infty} \frac{5^{n+1}}{(\ln(n))^n}$

For #20-23, use the alternating series test to determine whether the following series converge or diverge.

20. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2+7}$

21. $\sum_{n=1}^{\infty} (-1)^{n-1} n 5^{-n}$

22. $\sum_{n=1}^{\infty}(-1)^n(1+e^{-n})$

23. $\sum_{n=1}^{\infty}(-1)^n\frac{e^{2n}+1}{e^{2n}-1}$

For #24-30, use any method to determine the convergence or divergence of the following series.

24. $\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^n n!$

25. $\sum_{n=1}^{\infty}\frac{arctan(n)}{n^2}$

26. $\sum_{n=1}^{\infty}\frac{2}{n^3+e^n}$

27. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

28. $\sum_{n=1}^{\infty} 3^{1/n}$

29. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{3n+4}$

30. $\sum_{n=1}^{\infty} \frac{99^n(n^5+2)}{n^2 10^{2n}}$

1. div
2. div
3. div
4. conv
5. conv
6. div
7. conv
8. div
9. div
10. conv
11. conv
12. conv
13. conv
14. div
15. conv
16. div
17. conv
18. conv

19. conv

20. conv

21. conv

22. div

23. div

24. conv

25. conv

26. conv

27. div

28. conv

29. div

30. conv

For #1-8, find the power series expansions for the following functions. Feel free to use any known power series derived in class.

1. $f(x) = \dfrac{1}{1-x^2}$

2. $f(x) = \dfrac{x^2}{1-x^2}$

3. $f(x) = xe^{3x}$

4. $f(x) = x^2 \ln(1+x^2)$

5. $f(x) = \sinh(-5x)$

6. $f(x) = \int_0^x \ln(1+t)dt$

7. $f(x) = \dfrac{e^{-x}}{1-x}$

8. $f(x) = \int_0^x \dfrac{e^t}{1-t}dt$

For #9-15, find the Maclaurin expansions of the following functions. Feel free to use any known power series derived in class.

9. $f(x) = x\sin(3x)$

10. $f(x) = x^2\sin(x)$

11. $f(x) = \cos(-2x)$

12. $f(x) = \cos^2(x)$ – try a trig identity

13. $f(x) = \sin^2(x)$ – try a trig identity

14. $f(x) = 10^x$

15. $f(x) = \ln(3 + x)$

For #16-19, use third order Taylor polynomials $P_3(x)$, centered at an appropriate $x = a$, to evaluate the following numbers.

16. $\sin(91^o)$ – must first convert to radians

17. $\ln(1.12)$

18. $\sqrt{16.7}$

19. $\sqrt[3]{26}$

1. $1 + x^2 + x^4 + x^6 + ...$

2. $x^2 + x^4 + x^6 + x^8 + ...$

3. $x + 3x^2 + \frac{9x^3}{2!} + \frac{27x^4}{3!} + ...$

4. $x^4 - \frac{x^6}{2} + \frac{x^8}{3} - \frac{x^{10}}{4} + ...$

5. $-5x - \frac{(-5x)^3}{3!} - \frac{(-5x)^5}{5!} - \frac{(-5x)^7}{7!} - ...$

6. $\frac{x^2}{2(1)} - \frac{x^3}{3(2)} + \frac{x^4}{4(3)} - \frac{x^5}{5(4)} + ...$

7. $1 + \frac{1}{2!}x^2 + (-\frac{1}{3!} + \frac{1}{2!})x^3 + (\frac{1}{4!} - \frac{1}{3!} + \frac{1}{2!})x^4 + ...$

8. $x + x^2 + \frac{5}{6}x^3 + \frac{2}{3}x^4 + \frac{13}{24}x^5 + ...$

9. $3x^2 - \frac{3^3x^4}{3!} + \frac{3^5x^6}{5!} - \frac{3^7x^8}{7!} + ...$

10. $x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \frac{x^9}{7!} + ...$

11. $1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + ...$

12. $1 - \frac{(2x)^2}{2(2!)} + \frac{(2x)^4}{2(4!)} - \frac{(2x)^6}{2(6!)} + ...$

13. $\frac{(2x)^2}{2(2!)} - \frac{(2x)^4}{2(4!)} + \frac{(2x)^6}{2(6!)} + ...$

14. $1 + x(\ln 10) + x^2\frac{(\ln 10)^2}{2!} + x^3\frac{(\ln 10)^3}{3!} + ...$

15. $\ln(3) + \frac{1}{3}x - \frac{1}{2(3^2)}x^2 + \frac{1}{3(3^3)}x^3 - \frac{1}{4(3^4)}x^4 + ...$

16. .999848

17. .113376

18. 4.0865

19. 2.9625