1. Total Area
   –One Curve: use sign table to find where region is above/below x-axis
   –Between Two Curves: find intersection points, look at slices

2. Solids of Revolution
   –disks: $\triangle V = \pi(\text{radius})^2 \triangle w$
   –washers: $\triangle V = \pi\Big((\text{out rad})^2 - (\text{in rad})^2\Big)\triangle w$
   –shells: $\triangle V = 2\pi(\text{radius})(\text{height})\triangle w$
   –disks, washers: slices are perpendicular to axis
   –shells: slices are parallel to axis
   –radius is measured with respect to the axis, it is not necessarily the function itself

3. Arclength
   – $L = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$
   – $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx$
   – $L = \int_a^b \sqrt{1 + (\frac{dx}{dy})^2} dy$

4. Spring Problems
   – Hooke's Law: $F(x) = kx$
   – Work: $W = \int_a^b F(x)dx$
   – need to know k first

5. Center of Mass / Centroids
   – discrete, 1 dimension:
   $$x = \frac{M}{m} = \frac{\sum_{i=1}^n x_i m_i}{\sum_{i=1}^n m_i}$$
   – continuous, 1 dimension:
   $$x = \frac{M}{m} = \frac{\int_a^b x\delta(x)dx}{\int_a^b \delta(x)dx}$$
   – discrete, 2 dimension:
   $$x = \frac{M_y}{m} = \frac{\sum_{i=1}^n x_i m_i}{\sum_{i=1}^n m_i}, \qquad y = \frac{M_x}{m} = \frac{\sum_{i=1}^n y_i m_i}{\sum_{i=1}^n m_i}$$
   – continuous, 2 dimension: If this is on the test, I will give you the equations for M_x, M_y, and m, but I will not label them.

You will not have to do a pumping water problem on the test, but know how to set up integrals for solids of revolution. Also know how to do the substitution method for integration (the stuff before the manipulating the integrand part of Sectino 8.1).

Integration: Which method do I use?

1. Try substitution first.
   – might need to manipulate the integrand; i.e. cleverly multiply by 1 or add 0
   – number on top, quadratic on bottom? use completing the square to get inverse tangent

2. Try integration by parts.
   – What is u? ILATE
   – Keep in mind that u goes down (derivative) and dv goes up (integral).

3. If it is a rational expression, try Partial Fraction Decomposition.
   – linear denominator, numerator is A
   – quadratic denominator, numerator is Ax+B
   – need a term for every power of everything in denominator

$$\frac{4x^3 - 5x + 1}{x^2(x^2+3)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+3} + \frac{Ex+F}{(x^2+3)^2} + \frac{Gx+H}{(x^2+3)^3}$$

4. Is it a trig to a power? $\sin^n(x)$, $\cos^m(x)$, $\sin^n(x)\cos^m(x)$
   – powers even: use double angle formulas to get all first order terms
   – at least one power odd: break one off ($\sin^{\text{even }\#}(x)\sin(x)$) and use $\sin^2(x) + \cos(x) = 1$

5. If it is $\sin(nx)\cos(mx)$, $n \neq m$, use trig product formulas.

6. If substitution and integration by parts didn't work, and it has a radical in it:
   – linear ($\sqrt[n]{ax+b}$): instead of $u = ax + b$, try $u = (ax+b)^{1/n}$ (or something similar)
   – quadratic ($\sqrt{a^2 - x^2}, \sqrt{x^2 - a^2}, \sqrt{a^2 + x^2}$):
   recall that $\sin^2(x)+\cos^2(x) = 1$ and $1+\tan^2(x) = \sec^2(x)$, use these to determine substitution
   Ex. $\sqrt{a^2 - x^2}$, know that $1 - \sin^2(x) = \cos^2(x)$, so use $x = a\sin(t)$ Then

$$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\sin^2(t)} = \sqrt{a^2(1 - \sin^2(t))} = a\sqrt{\cos^2(t)} = a\cos(t)$$

7. L'Hopital's Rule and Indeterminate Forms
   – must be of form $0/0$ or ∞/∞ to use L'Hopital's Rule
   – $\infty - \infty$ or $0 \times \infty$, use algebra to get as a fraction of the form $0/0$ or ∞/∞
   – exponentials: 0^0, ∞^0, 1^∞, use logarithms to get rid of exponents; find limit of $\ln(y)$, then desired limit is $e^{\lim \ln(y)}$

8. Improper Integrals
   – infinite endpoint, look at limit as endpoint approaches ∞, $-\infty$
   – unbounded discontinuity, look at limit as endpoint approaches it
   – if unbounded discontinuity between endpoints, break up into two integrals with discontinuity as an endpoint

1. Sequences
   – discrete ordered listing of numbers following a specific pattern, $\{a_n\}_{n=1}^{\infty}$ where a_n =something
   – converges if $\lim_{n\to\infty} a_n$ equals a finite number (need not be 0), converges to this limit
   – squeeze theorem: trap a_n between lower/upper bound, if limits of outsides are same, limit of middle must be same
   Example: $\{a_n\}_{n=1}^{\infty}$, where $a_n = \dfrac{\sin(n)}{n}$
   $-1 \leq \sin(n) \leq 1$

   $$\frac{-1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}$$
   $$\lim_{n\to\infty} \frac{-1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n} \Rightarrow 0 \leq \lim_{n\to\infty} \frac{\sin(n)}{n} \leq 0 \Rightarrow \lim_{n\to\infty} \frac{\sin(n)}{n} = 0$$

2. Positive Series – similar to sequence, but add the numbers together
   – Nth term test: $\lim\limits_{n\to\infty} a_n \neq 0$, series diverges

   – Collapsing Sum: Look at S_n to evaluate, most terms sum out. $\sum\limits_{n=1}^{\infty} a_n = \lim\limits_{n\to\infty} S_n = S$

   –Geometric Series: $\sum\limits_{n=1}^{\infty} ar^n = \dfrac{a}{1-r}$, $|r| < 1$, diverges otherwise

   – p-series: $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^p}$ converges if $p > 1$, diverges if $0 < p \leq 1$

   – Integral Test: Improper integral converges or diverges with series

   – Ordinary Comparison Test: compare series $\sum a_n$ to a known p-series $\sum b_n$
   $\sum a_n < \sum b_n$, $\sum b_n$ converges, then $\sum a_n$ converges
   $\sum a_n > \sum b_n$, $\sum b_n$ diverges, then $\sum a_n$ diverges

   – Limit Comparison Test: Compare series $\sum a_n$ to a known p-series $\sum b_n$
   If $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = L$, series converge or diverge together.
   Useful if a_n is a fraction of polynomials (or radicals).

   –Ratio Test: look at ratio of two consecutive terms $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \rho$
   If $\rho < 1$, converges.
   If $\rho > 1$, diverges.
   If $\rho = 1$, inconclusive.
   Useful if series includes $n!$, r^n, n^n.

   –Root Test: look at nth root of nth term $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \rho$
   same conditions on ρ as ratio test
   Useful if includes r^n, n^n

3. Series with Negative Terms

   – Alternating Series Test: $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ converges if

   a) $\lim_{n\to\infty} a_n = 0$
   b) $a_n > a_{n+1}$ (terms are decreasing)
   If series converges, it is AT LEAST conditionally convergent.

   Absolutely convergent - does $\sum |a_n|$ converge?

4. Power Series - Know the following series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + ...$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + ...$$

   Know how to alter these to get new series.
   Know that you can add, subtract, multiply, divide, differentiate, integrate, plug in x^2 in place of x, multiply by 2, multiply by x, etc...

5. Other Series: Maclaurin Series – Taylor Series with $a = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + ...$$

   Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + ...$$

6. Convergence Set: – Usually find with absolute ratio test
   – Find all x where series converges; in case of ratio test, need limit less than 1.
   – Check endpoints seperately.