

Name-Surname :

14.11.2019

Number :

1	2	3	4	5	Total

MTH311 ALGEBRA I MIDTERM EXAM

1. Let $G = \left\{ \begin{bmatrix} 0 & a \\ 0 & a \end{bmatrix} \mid a \in \mathbb{Z} \right\}$. Is G a group under matrix multiplication? Explain it.

2. Find the multiplicative inverse of the element $\overline{23} \in \mathbb{Z}_{26}$, if it exists.

3. Let $G = \langle a \rangle$ be a cyclic group of order 30.

a) Find the generators of G .

b) Determine the subgroups of G .

c) Describe the elements of order 5 in G .

4. a) State the Lagrange's Theorem.

b) Let G be a noncyclic group of order 25. Find the order of the subgroup $\langle a \rangle$, where $e \neq a \in G$.
(Use the Lagrange's Theorem).

5. Indicate whether each of the following statements is **True (T)**, or **False (F)**.

a) Every group can be written as a union of its cyclic subgroups.....

b) There exists only one group of prime order (up to isomorphism).....

c) Every abelian group is cyclic.....

d) If all subgroups of a group G are cyclic, then the group is cyclic.....

e) Any two groups of three elements are isomorphic.

f) All generators of \mathbb{Z}_{20} are prime numbers.....

g) Every infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$

h) If the center $M(G) = G$, then G is abelian.....

i) Order of the product of finite order elements of a group is finite.....

j) I would like to have three bonus points, please.....

Good Luck!

Bibliography

- [1] J. B. Fraleigh, A First Course In Abstract Algebra, Addison Wesley. (7th Edition).
- [2] D. S. Malik, J. M. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw Hill, 1997.