BME 211 Circuit Analysis Laboratory

Experiment #8: Low-Pass and

High-Pass Filters

Objective

The objective of this experiment is to understand and analyze two kinds of passive filters, *low-pass filter* and *high-pass filter*. The effect of varying source frequency to the output voltage of these filters will be studied. The cut-off frequency will be determined.

Background

Basically, an electrical filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others. Such circuits are also called frequency-selective circuits. There are numerous applications of filters including radio receivers, television receivers, noise reduction systems and power supply circuits to name just a few.

A filter is a passive filter if it consists of only passive elements R, L and C. It is said to be an active filter if it consists of active elements (such as transistors and opamps) in addition to passive elements.

The signals passed from the input to the output fall within a band of frequencies called the *passband*. Input voltages outside this band have their magnitudes attenuated by the circuit and are thus effectively prevented from reaching the output terminals of the circuit. Frequencies not in a circuit's passband are in its *stopband*. Filters are categorized by the location of the passband.

As shown in Figure 8.1, there are four major types of filters whether passive or active:

1) A *low-pass filter (LPF)* passes signals at frequencies lower than the cut-off frequency from the input to the output.

2) A *high-pass filter (HPF)* passes signals at frequencies higher than the cut-off frequency.

3) A *band-pass filter (BPF)* passes a source voltage to the output only when the source frequency is within the band defined by the two cut-off frequencies.

4) A *band-reject filter (BRF)* (or *band-stop filter, BSF*) passes a source voltage to the output only when the source frequency is outside the band defined by the two cut-off frequencies.

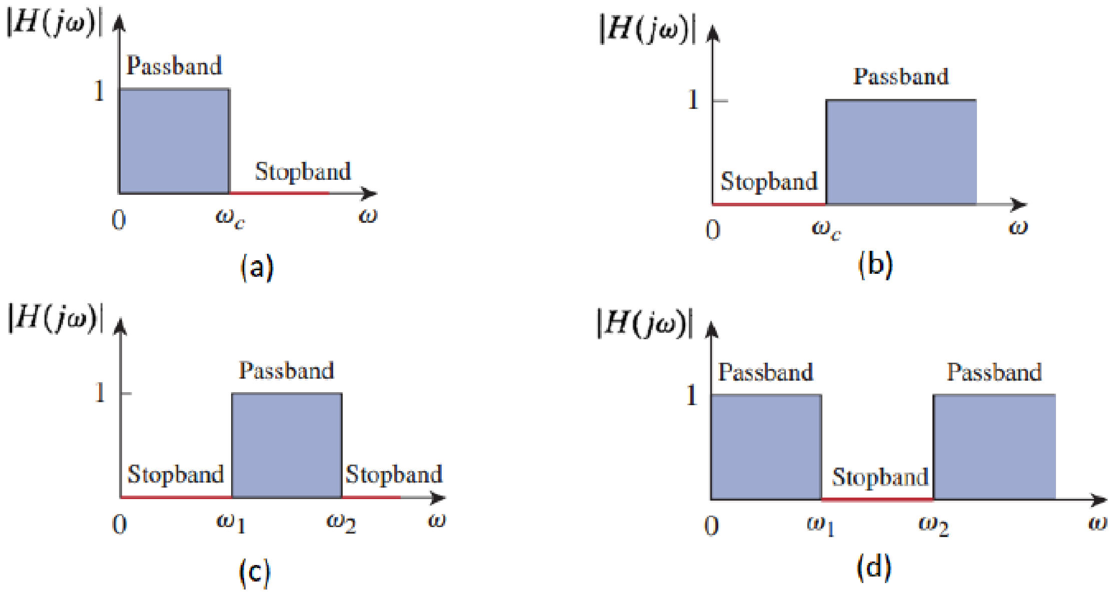

Figure 8.1 Ideal magnitude response plots of four major types of filter circuits: (a) LPF, (b) HPF, (c) BPF, (d) BRF

1. Low-Pass Filter

A typical low-pass filter shown in Figure 8.2 is a series RC circuit. The circuit's input is a sinusoidal voltage source with varying frequency (f, Hz). The circuit's output is defined as the voltage across the capacitor.

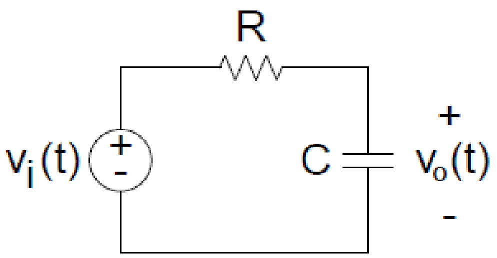

Figure 8.2 A series RC low-pass filter

The transfer function $H(jw)$ is a useful analytic tool for finding the frequency response of a circuit. A frequency response plot shows how a circuit's transfer function changes as the source frequency changes.

The voltage transfer function for the circuit in Figure 8.2 is

$$H(jw) = voltage\ gain = \frac{V_o(jw)}{V_i(jw)} = \frac{1/RC}{jw + 1/RC} \qquad (8.1)$$

The transfer function magnitude is

$$|H(jw)| = \frac{1/RC}{\sqrt{w^2 + (1/RC)^2}} \qquad (8.2)$$

At $w = 2\pi f = 0\, rad/s$, the impedance of the capacitor is infinite and the capacitor acts as an open circuit. The input and output voltages are thus the same.

As the frequency of the voltage source increases, the impedance of the capacitor decreases relative to the impedance of the resistor and the source voltage is now divided between the resistor and the capacitor. The output voltage is thus smaller than the source voltage.

When the frequency of the voltage source is infinite ($w = 2\pi f = \infty$), the impedance of the capacitor is zero and the capacitor acts as a short circuit. The output voltage is thus zero.

Figure 8.3 shows the plot of $|H(jw)|$, along with the ideal characterictic.

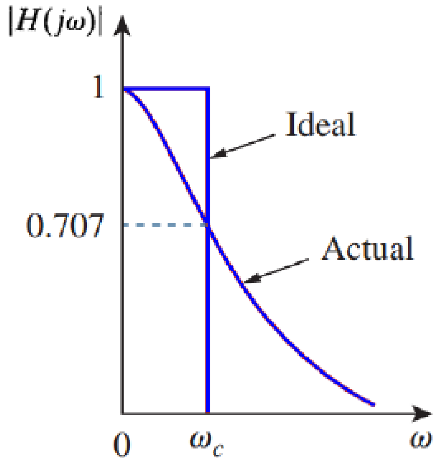

Figure 8.3 Ideal and actual magnitude responses of a low-pass filter

The cut-off frequency is the frequency at which the transfer function drops in magnitude to 70.7% of its maximum value. We can then describe the relationship among the quantities R, C and w_c as follows,

$$|H(jw_c)| = \frac{1}{\sqrt{2}}(1) = \frac{1/RC}{\sqrt{w_c^2 + (1/RC)^2}} \qquad (8.3)$$

Solving this equation for w_c, we get

$$w_c = \frac{1}{RC} \qquad (8.4)$$

At the cut-off frequency w_c, the average power delivered by the circuit is one half the maximum average power. Thus, w_c is also called the *half-power frequency*. Therefore, in the passband, the average power delivered to a load is at least 50% of the maximum average power.

The gain of a system is typically measured in *decibel* (*dB*). The dB value is a logarithmic measurement of the ratio of one variable to another of the same type. For a voltage or current gain G, its dB equivalent is $G_{dB} = 20\ log_{10}G$. At the cut-off frequency, we have

$$20log_{10}\left(1/\sqrt{2}\right) = -3\ dB \qquad (8.5)$$

2. High-Pass Filter

A series RC circuit is shown in Figure 8.4. In contrast to its low-pass counterpart in Figure 8.2, the output voltage here is defined across the resistor, not the capacitor.

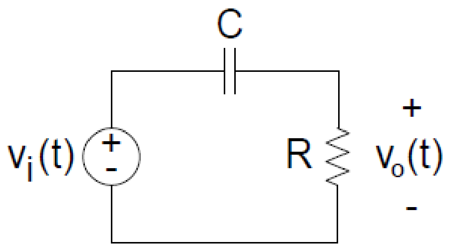

Figure 8.4 A series RC high-pass filter

The voltage transfer function for the circuit in Figure 8.4 is

$$H(jw) = \frac{jw}{jw + 1/RC} \qquad (8.6)$$

The transfer function magnitude is

$$|H(jw)| = \frac{w}{\sqrt{w^2 + (1/RC)^2}} \qquad (8.7)$$

At $w = 2\pi f = 0$, the capacitor behaves like an open circuit, so there is no current flowing in the resistor. In this case, there is no voltage across the resistor and the circuit filters out the low-frequency source voltage before it reaches the circuit's output.

As the frequency of the voltage source increases, the impedance of the capacitor decreases relative to the impedance of the resistor and the source voltage is now divided between the capacitor and the resistor. The output voltage magnitude thus begins to increase.

When the frequency of the voltage source is infinite ($w = 2\pi f = \infty$), the capacitor behaves as a short circuit, and thus there is no voltage across the capacitor. In this case, the input voltage and output voltage are the same.

Figure 8.5 shows the plot of $|H(jw)|$, along with the ideal characterictic.

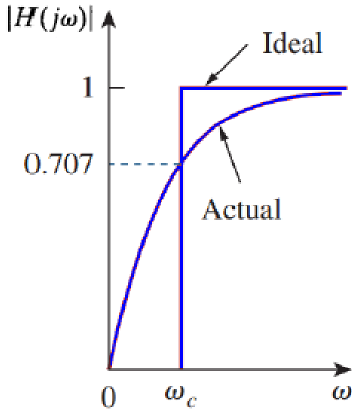

Figure 8.5 Ideal and actual magnitude responses of a high-pass filter

The relationship among the quantities R, C and w_c is described as follows,

$$|H(jw_c)| = \frac{1}{\sqrt{2}}(1) = \frac{w_c}{\sqrt{w_c^2 + (1/RC)^2}} \qquad (8.8)$$

Solving this equation for w_c, we get

$$w_c = \frac{1}{RC} \qquad (8.9)$$

Notice that the cut-off frequency for the series RC circuit has the value $1/RC$, whether the circuit is configured as a low-pass filter in Figure 8.2 or as a high-pass filter in Figure 8.4.

Preliminary Work

1. For **a series RC low-pass filter**, component values are given as $R = 10\ k\Omega$ and $C = 0.1\ \mu F$. The amplitude of the source voltage is $10\ V$ peak-to-peak.
   a) Calculate the cut-off frequency in Hz.
   b) Calculate V_o in magnitude when the source voltage has frequencies of 0.4, 0.8, 1.2, 1.6, 2, 4, 8 kHz.

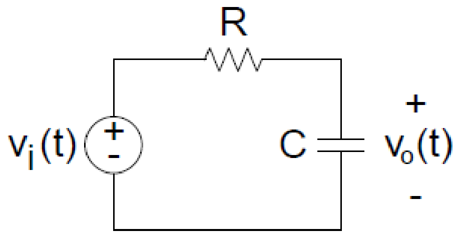

Figure 8.2 A series RC low-pass filter

2. For **a series RC high-pass filter**, component values are given as $R = 10\ k\Omega$ and $C = 0.1\ \mu F$. The amplitude of the source voltage is $10\ V$ peak-to-peak.
   a) Calculate the cut-off frequency in Hz.
   b) Calculate V_o in magnitude when the source voltage has frequencies of 0.4, 0.8, 1.2, 1.6, 2, 4, 8 kHz.

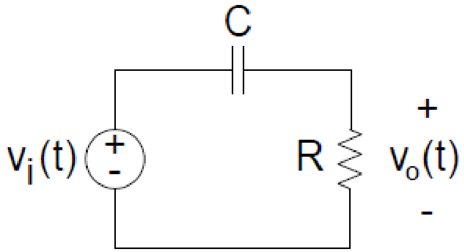

Figure 8.4 A series RC high-pass filter

Procedure

1. Using a resistor of $10\ k\Omega$ and a capacitor of $0.1\ \mu F$, setup the circuit in Figure 8.2.
2. Set V_i to $10\ V$ peak-to-peak at $400\ Hz$ sine wave by using the function generator.
3. Use channel 1 of the oscilloscope to visualize V_i and channel 2 to visualize V_o.
4. Record channel 2 peak-to-peak value.
5. Increase the source frequency as stated in Preliminary Work Q1(b). Record the channel 2 peak-to-peak value each time you increase the frequency. Make sure that V_i is maintaining at $10\ V$ peak-to-peak each time the frequency is increased.
6. Based on your results, plot V_o versus frequency.
7. From the graph, find the cut-off frequency by tracing the frequency where the magnitude of the output voltage is 70.7% of its maximum value.
8. Compare all your results with the Preliminary Work Q1.
9. Using a resistor of $10\ k\Omega$ and a capacitor of $0.1\ \mu F$, setup the circuit in Figure 8.4 and repeat the steps 2-7. Compare all your results with the Preliminary Work Q2.

List of Equipment and Components

Equipment: Function Generator, Oscilloscope

Components: a resistor of $10\ \mathrm{k\Omega}$, a capacitor of $0.1\ \mathrm{\mu F}$

References

[1] James W. Nilsson, Susan A. Riedel, Electric Circuits (Ninth Edition)

[2] Charles K. Alexander, Matthew N. O. Sadiku, Fundamentals of Electric Circuits (Fifth Edition)

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Objective

Results

1. a) Comparision of calculated and measured values.

F (Hz)	50	100	150	200	400	800	1000
$V_{measured}$							

b) V_{out} vs. frequency

Cut-off frequency =

Comment:

2. a) Comparision of calculated and measured values.

F (Hz)	100	150	200	400	800	1000
$V_{measured}$						

b) V_{out} vs. frequency

Cut-off frequency =

Comment: