BME 211 Circuit Analysis Laboratory

Experiment #10: Resonance Circuits,

Band-Pass and Band-Reject Filters

Objective

The objective of this experiment is to examine some of circuits constructed using resistors, capacitors and inductors for inputs with different frequency values and understand the concept of resonance using series and parallel resonance circuits. One other objective is to analyze band-pass and band-reject filter circuits.

Background

In previous experiments, we observed that in a circuit which contains only inductive component, the voltage ***leads*** current by 90° but in a circuit containing only capacitance, the voltage ***lags*** behind the current by 90°. Thus, the phase angle between the inductive and capacitive voltages is 180° and the two voltages oppose each other. In circuits where inductors and capacitors are used together, the two more or less oppose each other. If the inductive effect is greater than the capacitive effect, then the circuit will act like an inductor in overall and the voltage will lead the current. On the other hand, if the capacitive effect is greater than the inductive effect, the circuit will behave like a capacitor and the voltage will lag behind the current. If, however, the inductive effect of the coil is exactly equal to the capacitive effect of the capacitor, the two effect will cancel each other out and the voltage and current will be in phase. Such a circuit is called a resonance circuit and the phenomenon is called ***resonance***. There are two types of resonant circuits which are series resonant circuits and parallel resonant circuits. These resonant circuits are useful for constructing band-pass and band-reject filters.

1. Series Resonance Circuits

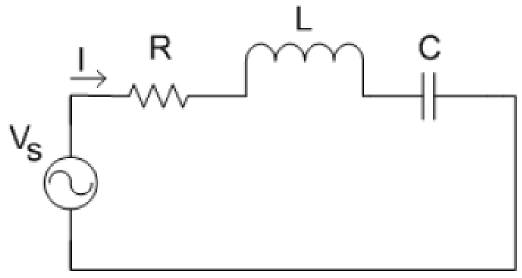

Figure 10.1 Series resonant circuit

A pure inductance or coil (without resistance) is not obtainable in practice. Any coil will have some resistance depending upon the size of the wire used. Even the capacitance and the connecting leads in an LC circuit will have resistance [1]. So, any resonant LC circuit is actually an RLC resonant circuit which is shown in Figure 10.1. The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (10.1)$$

If the circuit is resonant to the frequency of the applied voltage V_s, then $X_L = X_C$ and the impedance of the circuit $Z = \sqrt{R^2 + 0^2} = R$. The voltage across L and C are equal and opposite in phase and they cancel out leaving only the resistance R in the circuit [1].

The circuit is resistive and the current I is given as below

$$I = \frac{V_s}{Z} = \frac{V_s}{R} \qquad (10.2)$$

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance, $X_L = X_C$.

$$X_L = X_C \rightarrow \omega L = \frac{1}{\omega C} \rightarrow \omega^2 = \frac{1}{LC} \rightarrow \omega_s = \frac{1}{\sqrt{LC}}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \qquad (10.3)$$

A certain combination of L and C resonates at a particular frequency when the capacitive reactance becomes equal to the inductive reactance. At this frequency, the impedance of the circuit is minimum and the current flowing is the maximum. When the frequency is lower than the frequency of resonance, the capacitive reactance is greater than the inductive reactance. The circuit will then behave like a capacitive circuit with current leading voltage. If the frequency of the applied voltage is higher than the resonant frequency, the inductive reactance is greater than the capacitive reactance and the circuit behaves like a coil and the voltage will lead the current. In either case, the impedance of the circuit will be minimum only at resonance. It will be higher both above and below the resonant frequency. Accordingly, the current is maximum at resonant frequency and is less at frequencies higher and lower than the resonant frequency. The variation of impedance with frequency and the variation of current with frequency is shown in Figure 10.2. Such curves are called resonance curves [1].

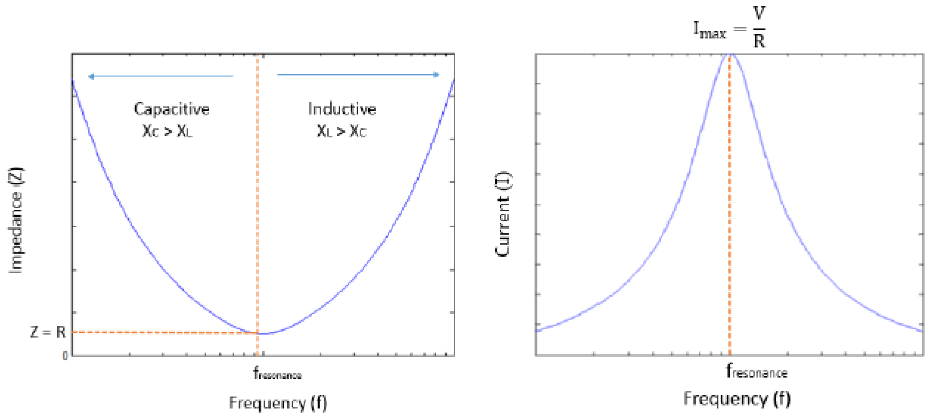

Figure 10.2 Resonance curves of series resonant circuit

2. Parallel Resonance Circuits

The same voltage appears across both L and C in a parallel resonant circuit which is shown in Figure 10.3. The parallel resonant frequency formula is the same as the formula of the series resonant frequency,

$$\omega_p = \frac{1}{\sqrt{LC}} \qquad (10.4)$$

$$f_p = \frac{1}{2\pi\sqrt{LC}}$$

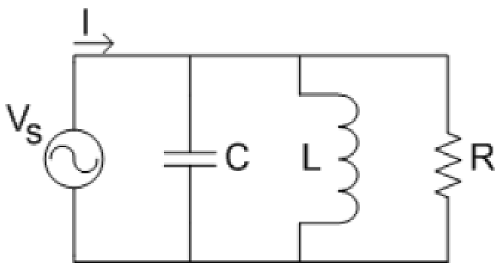

Figure 10.3 Parallel resonant circuit

The current of the circuit is

$$I = \sqrt{{I_R}^2 + (I_L - I_C)^2} \qquad (10.5)$$

At resonance, inductive and capacitive branch currents are equal ($I_L = I_C$) and are 180° out of phase so, the current of the circuit is

$$I = \sqrt{{I_R}^2 + 0^2} = I_R$$

The admittance of a parallel circuit is

$$Y = G + B_L + B_C \rightarrow Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \qquad (10.6)$$

$$Y = \frac{1}{R} + \frac{1}{j2\pi f L} + j2\pi f C$$

The variation of impedance with frequency and the variation of current with frequency is shown in Figure 10.4. The behaviour of a parallel resonant circuit is exactly opposite to that of a series resonant circuit. Whereas the impedance of a series resonant circuit is minimum at resonance and the line current high, the impedance of a parallel resonant circuit is maximum at resonance and the line current minimum [1].

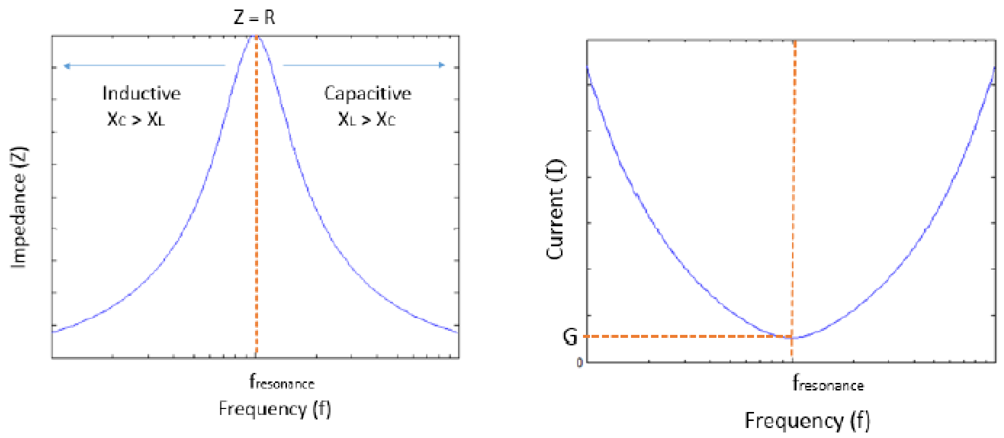

Figure 10.4 Resonance curves of parallel resonant circuit

3. Band-Pass Filters

A band-pass filter passes a band of frequencies ($w_{c1} < w < w_{c2}$) centered on w, the resonant (or center) frequency. The series RLC resonant circuit provides a band-pass filter when the output is taken off the resistor as shown in Figure 10.5. The parallel RLC resonant circuit in Figure 10.6 is also a band-pass filter circuit.

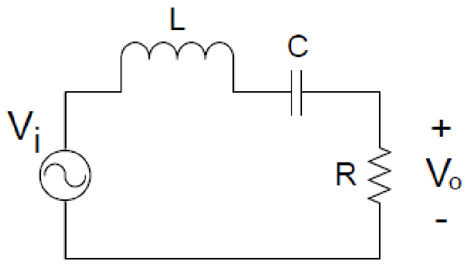

Figure 10.5 A series RLC band-pass filter

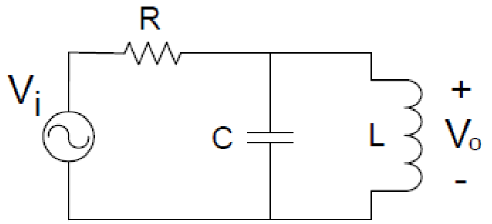

Figure 10.6 A parallel RLC band-pass filter

The transfer function magnitude versus frequency plot for a band-pass filter circuit is given in Figure 10.7.

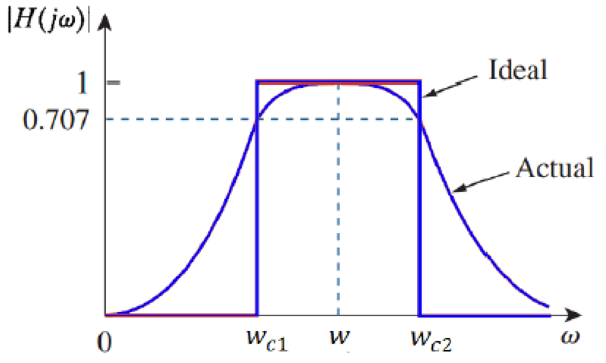

Figure 10.7 Ideal and actual magnitude responses of a band-pass filter

The band-pass filters have two cut-off frequencies, w_{c1} and w_{c2}, which identify the passband. Recall that the cut-off frequency is the frequency at which the transfer function drops in magnitude to 70.7% of its maximum value, $|H(jw_{c1,c2})| = 1/\sqrt{2}$.

The resonant frequency is the geometric center of the passband, that is, $w = \sqrt{w_{c1}w_{c2}}$. The magnitude of the transfer function is a maximum at the resonant frequency.

There are two other important parameters that characterize a band-pass filter. The first parameter is the bandwidth, β, which is the width of the passband. It is defined as the difference between the two cut-off frequencies. Because $w_{c2} > w_{c1}$,

$$\beta = w_{c2} - w_{c1} \qquad (10.7)$$

The second parameter is the quality factor, Q, which is the ratio of the resonant frequency to the bandwidth,

$$Q = \frac{w}{\beta} \qquad (10.8)$$

The quality factor describes the shape of the magnitude plot, independent of frequency. As illustrated in Figure 10.8, the higher the value of Q, the more selective the circuit is but the smaller the bandwidth. The selectivity of an RLC circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies. If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonant circuit must be high. If the band of frequencies is wide, the quality factor must be low.

Table 10.1 presents a summary of the characteristics of the series and parallel band-pass filters.

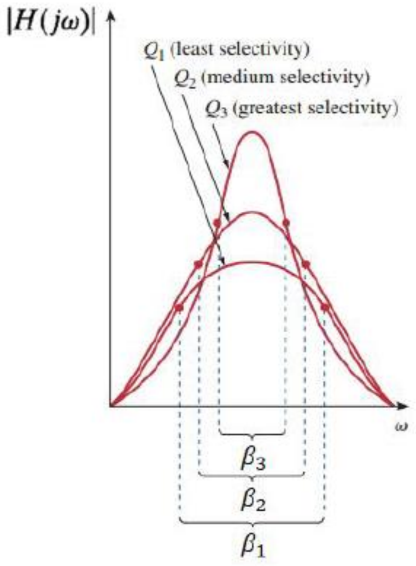

Figure 10.8 Relationship between bandwidth and quality factor

Table 10.1 Summary of the characteristics of band-pass filters

	Series RLC Band-Pass Filter (Figure 10.5)	**Parallel RLC Band-Pass Filter (Figure 10.6)**
Transfer Function, $H(jw) = V_o/V_i$	$\dfrac{\left(\frac{R}{L}\right)jw}{(jw)^2+\left(\frac{R}{L}\right)jw+\frac{1}{LC}}$	$\dfrac{\left(\frac{1}{RC}\right)jw}{(jw)^2+\left(\frac{1}{RC}\right)jw+\frac{1}{LC}}$
Transfer Function Magnitude, $\lvert H(jw)\rvert$	$\dfrac{\frac{R}{L}w}{\sqrt{\left(\frac{1}{LC}-w^2\right)^2+\left(\frac{R}{L}w\right)^2}}$	$\dfrac{\frac{1}{RC}w}{\sqrt{\left(\frac{1}{LC}-w^2\right)^2+\left(\frac{1}{RC}w\right)^2}}$
Resonant (Center) Frequency, w	$\sqrt{\frac{1}{LC}}$	$\sqrt{\frac{1}{LC}}$
Cut-off Frequencies, w_{c1}, w_{c2}	$\mp\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$	$\mp\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$
Bandwidth, β	$\frac{R}{L}$	$\frac{1}{RC}$
Quality Factor, Q	$\sqrt{\frac{L}{CR^2}}$	$\sqrt{\frac{R^2C}{L}}$

4. Band-Reject Filters

A band-reject filter passes source voltages outside the band between the two cut-off frequencies to the output (passband), and attenuates source voltages before they reach the output at frequencies between the two cut-off frequencies (stopband). Band-pass filters and band-reject filters thus perform complementary functions in the frequency domain.

Figure 10.9 shows a series RLC resonant circuit. Although the circuit components and connections are identical to those in the series RLC band-pass filter in Figure 10.5, the circuit in Figure 10.9 has an important difference: the output voltage is now defined across the inductor-capacitor pair.

Another configuration that produces a band-reject filter is a parallel RLC resonant circuit as shown in Figure 10.10.

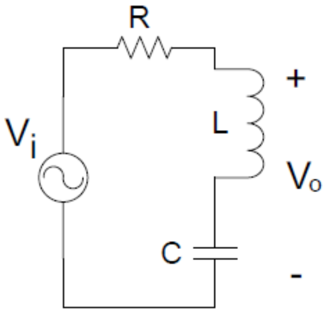

Figure 10.9 A series RLC band-reject filter

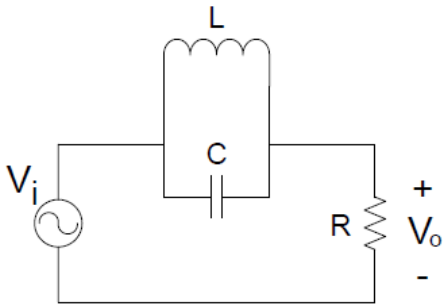

Figure 10.10 A parallel RLC band-reject filter

The transfer function magnitude versus frequency plot for a band-reject filter circuit is given in Figure 10.11.

Band-reject filters are characterized by the same parameters as band-pass filters: the two cut-off frequencies, the resonant frequency, the bandwidth and the quality factor.

Table 10.2 gives a summary of the characteristics of the series and parallel band-reject filters.

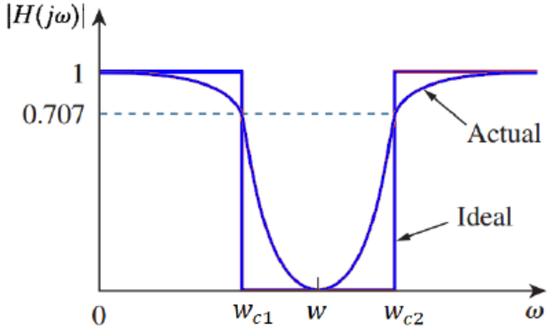

Figure 10.11 Ideal and actual magnitude responses of a band-reject filter

Table 10.2 Summary of the characteristics of band-reject filters

	Series RLC Band-Reject Filter (Figure 10.9)	**Parallel RLC Band-Reject Filter (Figure 10.10)**
Transfer Function, $H(jw) = V_o/V_i$	$\dfrac{(jw)^2 + \frac{1}{LC}}{(jw)^2 + \left(\frac{R}{L}\right)jw + \frac{1}{LC}}$	$\dfrac{(jw)^2 + \frac{1}{LC}}{(jw)^2 + \left(\frac{1}{RC}\right)jw + \frac{1}{LC}}$
Transfer Function Magnitude, $\lvert H(jw)\rvert$	$\dfrac{\left\lvert\frac{1}{LC} - w^2\right\rvert}{\sqrt{\left(\frac{1}{LC} - w^2\right)^2 + \left(\frac{R}{L}w\right)^2}}$	$\dfrac{\left\lvert\frac{1}{LC} - w^2\right\rvert}{\sqrt{\left(\frac{1}{LC} - w^2\right)^2 + \left(\frac{1}{RC}w\right)^2}}$
Resonant (Center) Frequency, w	$\sqrt{\frac{1}{LC}}$	$\sqrt{\frac{1}{LC}}$
Cut-off Frequencies, w_{c1}, w_{c2}	$\mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$	$\mp\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$
Bandwidth, β	$\frac{R}{L}$	$\frac{1}{RC}$
Quality Factor, Q	$\sqrt{\frac{L}{CR^2}}$	$\sqrt{\frac{R^2C}{L}}$

Preliminary Work

1) For the series RLC circuit in Figure 10.1, if $V_s = 50\angle 0^\circ\ mV$, $R = 10\ \Omega$ and $X_L = 30\ \Omega$,

a) Find the value of X_C for resonance.
b) Determine the total impedance of the circuit at resonance.
c) Find the magnitude of the current I at resonance.
d) Calculate the voltages V_R, V_L and V_C at resonance. How are V_L and V_C related? How does V_R compare to the applied voltage V_s?

2) For the parallel RLC circuit in Figure 10.3, if $I = 2\angle 0^\circ\ mA$, $R = 2\ k\Omega$, $C = 10\ \text{nF}$ and $L = 0.1\ mH$,

a) Determine the resonant frequency f_p.
b) Find the voltage V_c at resonance.
c) Determine the currents I_L and I_C at resonance.

3) For the parallel RLC band-pass filter in Figure 10.6, if $R = 1\ k\Omega$, $C = 220\ nF$ and $L = 120\ \mu H$, calculate the values of w, w_{c1}, w_{c2}, β and Q. How can you increase the bandwidth (β) of this circuit?

4) For the series RLC band-reject filter in Figure 10.9, if $R = 100\ \Omega$, $C = 10\ \text{nF}$ and $L = 1\ mH$, calculate the values of w, w_{c1}, w_{c2}, β and Q. How can you decrease the bandwidth (β) of this circuit?

Procedure

1) Using a resistor of $1\ k\Omega$, a capacitor of $220\ nF$ and an inductor of $100\ \mu H$, setup the circuit in Figure 10.12.

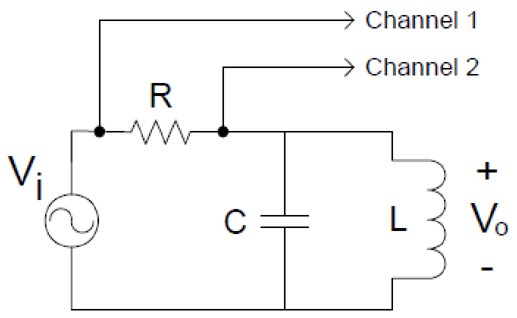

Figure 10.12

a) Set V_i to $10\ V$ peak-to-peak at $10\ kHz$ sine wave by using the function generator.
b) Connect the oscilloscope probes as shown in the figure.
c) Increase the source frequency and observe the input and output voltages. Make sure that V_i is maintaining at $10\ V$ peak-to-peak each time the frequency is increased:
   i. Find the resonant frequency at which the magnitude of the output voltage is maximum, and the input and output voltages are in phase. Compare your result with the Preliminary Work Q3. Record the frequency and magnitude values.
   ii. Find the lower and upper cut-off frequencies at which the magnitude of the output voltage is 70.7% of its maximum value. Compare your results with the Preliminary Work Q3. Record the frequency and magnitude values.
   iii. Record the magnitude of the output voltage for three different frequencies smaller than the lower cut-off frequency.
   iv. Record the magnitude of the output voltage for three different frequencies larger than the upper cut-off frequency.
d) Based on your results in c.i-iv, plot V_o versus frequency.

2) Using a resistor of 100 Ω, a capacitor of 10 nF and an inductor of 1 mH, setup the circuit in Figure 10.13.

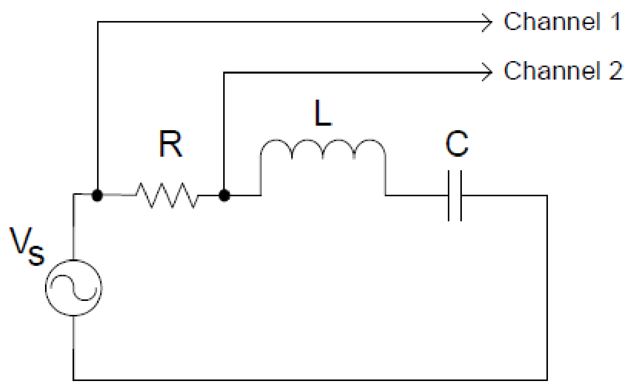

Figure 10.13

a) Set V_i to 10 V peak-to-peak at 15 kHz sine wave by using the function generator.

b) Connect the oscilloscope probes as shown in the figure.

c) Increase the source frequency and observe the input and output voltages. Make sure that V_i is maintaining at 10 V peak-to-peak each time the frequency is increased:

   i. Find the resonant frequency at which the magnitude of the output voltage is minimum. Compare your result with the Preliminary Work Q4. Record the frequency and magnitude values.

   ii. Find the lower and upper cut-off frequencies at which the magnitude of the output voltage is 70.7% of its maximum value. Compare your results with the Preliminary Work Q4. Record the frequency and magnitude values.

   iii. Record the magnitude of the output voltage for three different frequencies smaller than the lower cut-off frequency.

   iv. Record the magnitude of the output voltage for three different frequencies larger than the upper cut-off frequency.

d) Based on your results in c.i-iv, plot V_o versus frequency.

BME 211 Report #10

Resonance Circuits, Band-Pass and Band-Reject Filters

Objective

Results

1. Comparision of calculated - measured values and plotting *Vo vs. freguency* graph.

	fc	*lcf*	*ucf*
Measured			
$V_{o\ Measured}$			

Smaller than lcf (1.–3.) *Larger than ucf* (4.–6.)

	1.	*2.*	*3.*	*4.*	*5.*	*6.*
frequency						
Vo						

2. Comparision of calculated - measured values and plotting *Vo vs. freguency* graph.

	fc	*lcf*	*ucf*
Measured			
$V_{o\ Measured}$			

	Smaller than lcf			*Larger than ucf*		
	1.	*2.*	*3.*	*4.*	*5.*	*6.*
frequency						
V_o						

Comments: