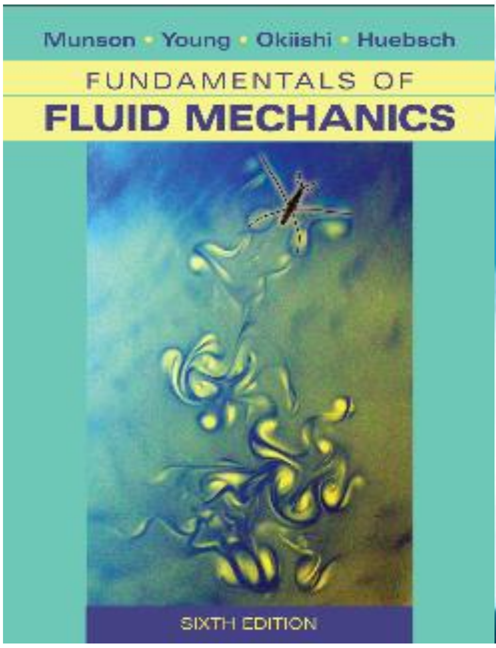
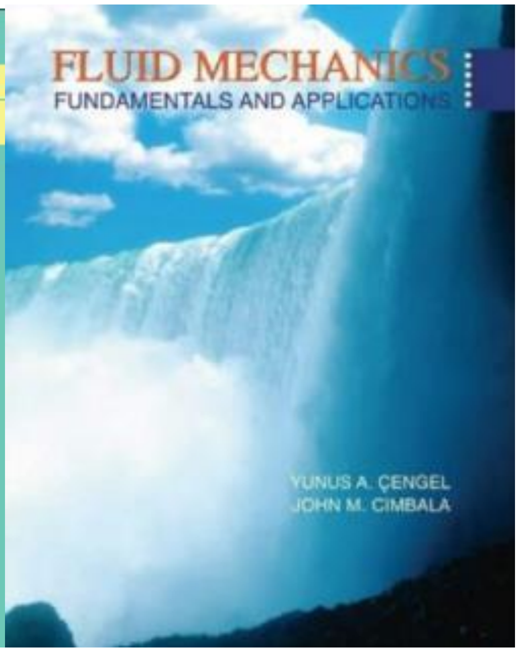
# FLUID MECHANICS



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# 2. PROPERTIES OF FLUIDS

## 2.5.2. Compression and Expansion of Gases

When gases are compressed (or expanded), the relationship between pressure and density depends on the nature of the process. If the compression or expansion takes place under constant temperature conditions (***isothermal process***), from ideal gas formula.

$$\frac{P}{\rho} = constant$$

If the compression or expansion is frictionless and no heat is exchanged with the surroundings (***isentropic process***), then

$$\frac{P}{\rho^k} = constant$$

Where $k$ is the ratio of the specific heat at constant pressure, $c_p$, to the specific heat at constant volume, $c_p$

$$k = \frac{c_p}{c_v}$$

The two specific heats are related to the gas constant, $R$, through the equation,

$$R = c_p - c_v$$

As was the case for the ideal gas law, the pressure in the above must be expressed as an absolute pressure. The pressure–density variations for isothermal and isentropic conditions are illustrated in the Fig.2.9.

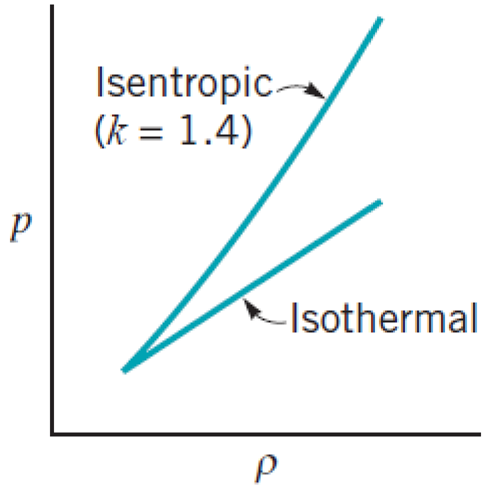



**Figure 2.9. The relationship between pressure and density for gases**

With explicit equations relating pressure and density, the bulk modulus for gases can be determined. It follows that for an ***isothermal process***;

$$E_v = P$$

and for an ***isentropic process***,

$$E_v = kP$$

Note that in both cases the bulk modulus varies directly with pressure. It is thus clear that in dealing with gases, greater attention will need to be given to the effect of compressibility on fluid behavior.

**Example:** A cubic meter of air at an absolute pressure of 101325 Pa is compressed isentropically to 0.5 $m^3$ by the tire pump, k=1.4. What is the final pressure?

**Solution:** For an isentropic compression

$$\frac{P_0}{\rho_0^k} = \frac{P}{\rho^k} \rightarrow P = \frac{\rho^k}{\rho_0^k} P_0 = (\frac{\rho}{\rho_0})^k P_0$$

As the volume, is reduced by one-half, the density must double, since the mass $m = \rho\forall$ , of the gas remains constant. Thus with $k$= 1.40 for air

$$P = (2)^{1.4} \times 101\,325 = 267398.3 \text{ Pa}$$

### 2.5.3. Speed of Sound

The ***speed of sound (c)*** is the distance traveled per unit of time by a sound wave propagating through an elastic medium. The speed of sound in a fluid medium (gas or liquid) is used as a relative measure of speed itself.

The speed of sound is related to changes in pressure and density of the fluid medium through the equation

$$c = \sqrt{\frac{dP}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

The pressure–density relationship used in the above equation is that for an isentropic process.

For gases undergoing an isentropic process, $E_v = kP$ and $c = \sqrt{\frac{kP}{\rho}}$ and making use of the ideal gas law, it follows that $c = \sqrt{kRT}$

Thus, for ideal gases the speed of sound is proportional to the square root of the absolute temperature.

For gases undergoing an ***isothermal process***, $E_v = P$ and $c = \sqrt{\frac{P}{\rho}}$ .

The speed of an object divided by the speed of sound in the fluid is called the Mach number (Ma).

$$Ma = \frac{V}{c} = V\sqrt{\frac{\rho}{E_v}}$$

Where; *Ma* is the Mach number, *V* is the velocity of the source relative to the medium, and *c* is the speed of sound in the medium.

a) Subsonic conditions occur for Mach numbers less than one, **Ma < 1**
b) As the speed of the object approaches the speed of sound, the flight Mach number is nearly equal to one, **Ma = 1**, and the flow is said to be transonic.
c) Supersonic conditions occur for Mach numbers greater than one, **1 < M a< 3**.
d) For speeds greater than five times the speed of sound, **M a> 5**, the flow is said to be hypersonic.

***Example:*** A jet aircraft flies at a speed of 500.4 km/h at an altitude of 11000 m, where the temperature is 216 K and the specific heat ratio is *k*= 1.4. Determine the ratio of the speed of the aircraft, *V*, to that of the speed of sound, *c*, at the specified altitude using isentropic process.

***Solution:*** Undergoing an isentropic process, the speed of sound can be calculated as

$$c = \sqrt{kRT} = \sqrt{1.4\times 286.9\times 216} = 294.5\, m/s$$

$Ma = \frac{V}{c} = \frac{500.4/3.6}{294.5} = 0.472$ The flow is said to be subsonic (Ma<1)

### 2.5.4. Surface Tension

At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface which cause the surface to behave as if it were a "skin" or "membrane" stretched over the fluid mass. Although such a skin is not actually present, this conceptual analogy allows us to explain several commonly observed phenomena.

For example, a steel needle or a razor blade will float on water if placed gently on the surface because the tension developed in the hypothetical skin supports it. Small droplets of mercury will form into spheres when placed on a smooth surface because the cohesive forces in the surface tend to hold all the molecules together in a compact shape. Similarly, discrete bubbles will form in a liquid.

These various types of surface phenomena are due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface. Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally. However, molecules along the surface are subjected to a net force toward the interior.

The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin or membrane. A tensile force may be considered to be acting in the plane of the surface along any line in the surface.

*The intensity of the molecular attraction per unit length along any line in the surface is called the* ***surface tension*** and is designated by the Greek symbol $\sigma$ (sigma). For a given liquid the surface tension depends on temperature as well as the other fluid it is in contact with at the interface. The dimensions of surface tension are $FL^{-1}$ with SI units of N/m. As indicated by the Fig.2.10., the value of the surface tension decreases as the temperature increases.

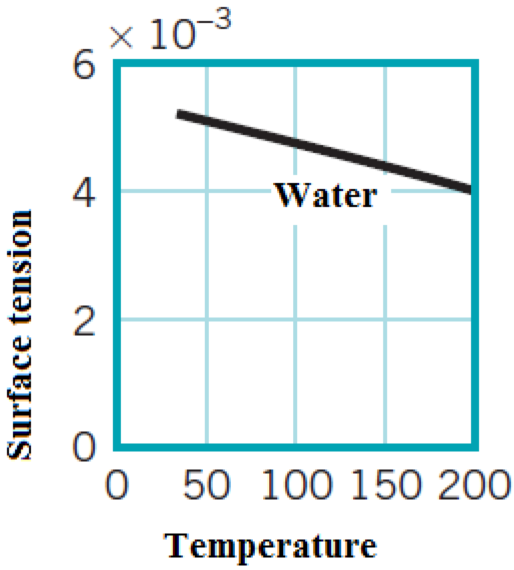



**Figure 2.10. The relationship between temperature and surface tension**

The pressure inside a drop of fluid can be calculated using the free-body diagram in the below Fig.2.11. If the spherical drop is cut in half (as shown), the force developed around the edge due to surface tension is $2\pi R\sigma$. This force must be balanced by the pressure difference, $\Delta P$, between the internal pressure, $P_i$ , and the external pressure, $P_e$, acting over the circular area, $\pi R^2$,Thus,

$$2\pi R\sigma = \nabla P\pi R^2 \text{ or } \nabla P = P_i - P_e = \frac{2\sigma}{R}$$

It is apparent from this result that the pressure inside the drop is greater than the pressure surrounding the drop.

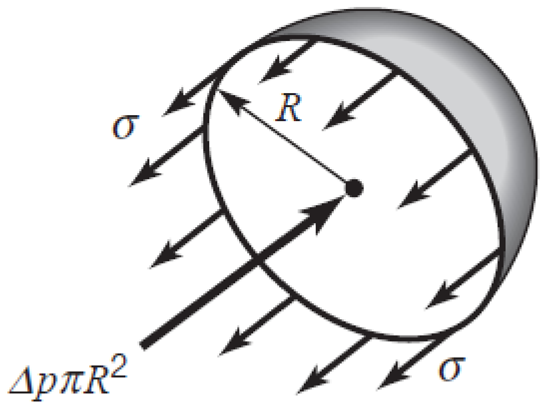



**Figure 2.11. Forces acting on one-half of a liquid drop.**

Among common phenomena associated with surface tension is the rise (or fall) of a liquid in a capillary tube. If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube, as is illustrated in Fig.2.12*a*. In this situation we have a liquid–gas–solid interface.

For the case illustrated there is an attraction (***adhesion***) between the wall of the tube and liquid molecules which is strong enough to overcome the mutual attraction (***cohesion***) of the molecules and pull them up the wall. Hence, the liquid is said to *wet* the solid surface.

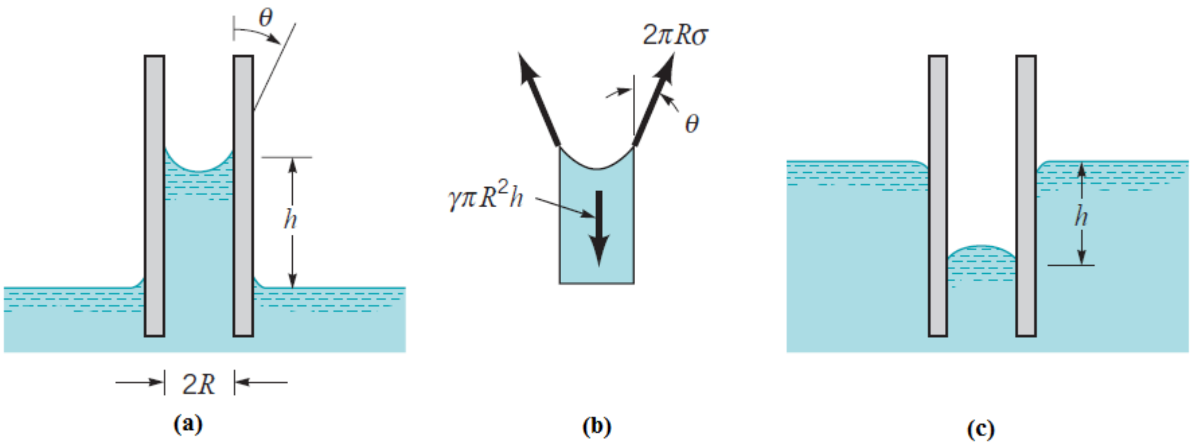


**Figure2.12. Effect of capillary action in small tubes. (*a*) Rise of column for a liquid that wets the tube. (*b*) Free-body diagram for calculating column height. (*c*) Depression of column for a nonwetting liquid.**

The height, *h*, is governed by the value of the surface tension, $\sigma$, the tube radius, *R*, the specific weight of the liquid, $\gamma$, and the *angle of contact*, $\theta$, between the fluid and tube.

From the freebody diagram of Fig.2.12$b$ we see that the vertical force due to the surface tension is equal to $2\pi R\sigma cos\theta$ and the weight is $\gamma\pi hR^2$and these two forces must balance for equilibrium. Thus,

$$\gamma\pi hR^2 = 2\pi R\sigma cos\theta$$

so that the height is given by the relationship

$$h = \frac{2\sigma cos\theta}{\gamma R}$$

The angle of contact is a function of both the liquid and the surface. For water in contact with clean glass $\theta \cong 0$. It is clear from above equation that the height is inversely proportional to the tube radius, and therefore, as indicated by the figure, the rise of a liquid in a tube as a result of capillary action becomes increasingly pronounced as the tube radius is decreased.

If adhesion of molecules to the solid surface is weak compared to the cohesion between molecules, the liquid will not wet the surface and the level in a tube placed in a nonwetting liquid will actually be depressed, as shown in Fig.*c*. Mercury is a good example of a nonwetting liquid when it is in contact with a glass tube. For nonwetting liquids the angle of contact is greater than 90°, and for mercury in contact with clean glass $\theta = 130°$.

Surface tension effects play a role in many fluid mechanics problems, including the movement of liquids through soil and other porous media, flow of thin films, formation of drops and bubbles, and the breakup of liquid jets. For example, surface tension is a main factor in the formation of drops from a leaking faucet. Surface phenomena associated with liquid–gas, liquid–liquid, and liquid–gas–solid interfaces are exceedingly complex, and a more detailed and rigorous discussion of them is beyond the scope of this text. Fortunately, in many fluid mechanics problems, surface phenomena, as characterized by surface tension, are not important, since inertial, gravitational, and viscous forces are much more dominant.

## Chapter Summary

Some of the important equations in this chapter.

Specific weight: $\gamma = \rho g$

Specific gravity: $SG = \frac{\rho}{\rho_{water+4C^{\circ}}}$

Ideal gas law: $\rho = \frac{P}{RT}$

Newtonian fluid shear stress: $\tau = \mu \frac{du}{dy} = \mu \frac{V}{y}$

Bulk modulus: $E_v = -\frac{dP}{d\forall/\forall} = -\forall \frac{dP}{d\forall} = -\forall_0 \frac{(P-P_{0})}{(\forall-\forall_0)}$; $E_v = \frac{dP}{d\rho/\rho} = \rho \frac{(P-P_{0})}{(\rho-\rho_0)}$

Speed of sound for ideal gas: $c = \sqrt{kRT}$

Mach number: $Ma = \frac{V}{c} = V\sqrt{\frac{\rho}{E_v}}$

Capillary rise in tube: $h = \frac{2\sigma cos\theta}{\gamma R}$

## EXAMPLES

**Example:** Define internal, external, and open-channel flows.

**Solution:** *External flow* is the flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe. The flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces. The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface, such as the flow of water in rivers and irrigation ditches.

**Example:** Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

**Solution:** A fluid flow during which the density of the fluid remains nearly constant is called *incompressible flow*. A flow in which density varies significantly is called *compressible flow*. A fluid whose density is practically independent of pressure (such as a liquid) is commonly referred to as an "incompressible fluid," although it is more proper to refer to incompressible *flow*.

The flow of compressible fluid (such as air) does not necessarily need to be treated as compressible since the density of a compressible fluid may still remain nearly constant during flow – especially flow at low speeds.

**Example:** What is the no-slip condition? What causes it?

**Solution:** A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the *viscosity* of the fluid.

**Example:** What is a boundary layer? What causes a boundary layer to develop?

**Solution:** When a fluid stream encounters a solid surface that is at rest, the fluid velocity assumes a value of zero at that surface. The velocity then varies from zero at the surface to the freestream value sufficiently far from the surface. The region of flow in which the velocity gradients are significant and frictional effects are important is called the *boundary layer*. The development of a boundary layer is caused by the *no-slip condition*.

**Example:** What is a steady-flow process?

**Solution:** A process is said to be *steady* if it involves no changes with time anywhere within the system or at the system boundaries.

**Example:** Define stress, normal stress, shear stress, and pressure.

**Solution:** *Stress* is defined as force per unit area, and is determined by dividing the force by the area upon which it acts. The normal component of a force acting on a surface per unit area is called the *normal stress*, and the tangential component of a force acting on a surface per unit area is called *shear stress*. In a fluid at rest, the normal stress is called *pressure*.

**Example:** What is the difference between kg-mass and kg force?

**Solution:** The unit *kilogram* (kg) is the mass unit in the SI system, and it is sometimes called *kg-mass*, whereas *kg-force* (kgf) is a force unit. One kg-force is the force required to accelerate a 1-kg mass by 9.807 $m/s^2$. In other words, the weight of 1-kg mass at sea level on earth is 1 kg-force.

**Example:** What is the net force acting on a car cruising at a constant velocity of 70 km/h (*a*) on a level road and (*b*) on an uphill road?

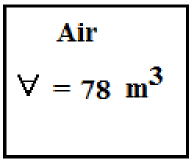
**Solution:** There is no acceleration, thus the net force is zero in both cases. By Newton's second law, the force on an object is directly proportional to its acceleration. If there is zero acceleration, there must be zero net force

**Example:** A plastic tank that has a volüme of 0.4 m$^3$ is filled with water. The mass of the tank is 5 kg. The weight of the combined system is to be determined. The density of water is given to be ρ= 1000 kg/m$^3$.

**Solution:**
$m_w = \rho\forall = 1000 \times 0.4 = 400\ kg \quad m_{tot} = m_w + m_{tank} = 400 + 5 = 45\ kg$

Thus $W = mg = 45 \times 9.81 = 441.45\ N$

**Example-**Determine the mass and the weight of the air contained in a room whose volume is 78 m$^3$. Assume the density of the air is 1.16 kg/m$^3$




**Solution:** The mass and weight of the air in the room are to be determined. The density of air is constant throughout the room. The density of air is given to be ρ= 1.16 kg/m$^3$.
$m = \rho\forall = 1.16 \times 78 = 90.48\ kg$
$W = mg = 90.48 \times 9.81 = 887.61\ N$

**Example:** At 45° latitude, the gravitational acceleration as a function of elevation *z* above sea level is given by *g*=*a*-*bz*, here *a* = 9.807 m/s$^2$ and *b* = $3.32 \times 10^{-6}$ s$^{-2}$. Determine the height above sea level where the weight of an object will decrease by 2 percent.

**Solution:** The variation of gravitational acceleration above sea level is given as a function of ltitude. The height at which the weight of a body decreases by 2% is to be determined.

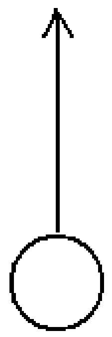
$W = mg = m(9.807 - 3.32 \times 10^{-6} z$

In our case; $W = 0.98W_s = 0.98mg_s = 0.98m(9.807)$

Substituting,

$0.98(9.807) = (9.807 - 3.32 \times 10^{-6} z)$ $\quad z$ =59078 m

**Example:** A 6-kg rock is thrown upward with a force of 200 N at a location where the local gravitational acceleration is 9.80 m/s$^2$. Determine the acceleration of the rock, in m/s$^2$.




**Solution:** $W = mg = 6 \times 9.80 = 58.8\ N$ Then the net force that acts on the rock is

$$F_{net} = F_{up} - F_{down} = 200 - 58.8 = 141.2\ N$$

From Newton's second law, the acceleration of the rock becomes

$$a = \frac{F}{m} = \frac{141.2}{6} = 23.53\ m/s^2$$

**Example:** The value of the gravitational acceleration g decreases with elevation from 9.807 m/s$^2$ at sea level to 9.767 m/s$^2$ at an altitude of 13.000 m, where large passenger planes cruise. Determine the percent reduction in the weight of an airplane cruising at 13.000 m relative to its weight at sea level.

**Solution:** Gravitational acceleration g and thus the weight of bodies decreases with increasing elevation. The percent reduction in the weight of an airplane cruising at 13.000 m is to be determined.

The gravitational acceleration g is 9.807 m/s$^2$ at sea level and 9.767 m/s$^2$ at an altitude of 13.000 m. Weight is proportional to the gravitational acceleration g, and thus the percent reduction in weight is equivalent to the percent reduction in the gravitational acceleration, which is determined from % Reduction in weight % Reduction in g= $g = \frac{\nabla g}{g} 100 = \frac{9.807-9.767}{9.807} 100 = 0.41\%$ . Therefore, the airplane and the people in it will weigh 0.41% less

at 13.000 m altitude.

**Example:** What is the difference between intensive and extensive properties?

**Solution:** Intensive properties do not depend on the size (extent) of the system but extensive properties do depend on the size (extent) of the system. An example of an intensive property is temperature. An example of an extensive property is mass.

**Example:** A spherical balloon with a diameter of 8 m is filled with helium at 25°C and 220 kPa. Determine the mole number and the mass of the helium in the balloon.

**Solution:** At specified conditions, helium behaves as an ideal gas. The universal gas constant is Ru= 8.314 kPa.m$^3$/kmol. K. The molar mass of helium is 4.0 kg/kmol. The volume of the sphere is

$$\forall = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi 4^3 = 268.1\, m^3$$

Assuming ideal gas behavior, the number of moles of He is determined from

$$n = \frac{PV}{R_u T} = \frac{220\times 268.1}{8.314\times 298} = 23.81\, kmol$$

Then the mass of He is determined from

$$m = nM = 23.81\times 4 = 95.24\, kg$$

or $P = \frac{m}{\forall}\mathrm{RT} = \frac{m}{\forall}\frac{R_u}{M}T \rightarrow m = \frac{P\forall M}{R_u T} = \frac{220000\times 268.1\times 4}{8314\times 298}$=95.23 kg

**Example:** The pressure in an automobile tire depends on the temperature of the air in the tire. When the air temperature is 20°C, the pressure gage reads 200 kPa. If the volume of the tire is 0.020 m$^3$, determine the pressure rise in the tire when the air temperature in the tire rises to 60°C. Also, determine the amount of air that must be bled off to restore pressure to its original value at this temperature. Assume the atmospheric pressure to be 101 kPa. The gas constant of air is R= 0.287 kPa m$^3$/kg·K.


**Solution:** At specified conditions, air behaves as an ideal gas. The volume of the tire remains constant. Initially, the absolute pressure in the tire is

$$P_1 = P_g + P_{atm} = 200 + 101 = 301\ kPa$$

Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire is determined from

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow P_2 = P_1 \frac{T_2}{T_1} = 200 \frac{333}{293} = 227.30\ kPa$$

Thus the pressure rise is $\Delta P = P_2 - P_1 = 227.30 - 200 = 27.30\ kPa$

The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1 \forall}{RT_1} = \frac{301 \times 0.020}{0.287 \times 293} = 0.072\ kg \qquad m_2 = \frac{P_2 \forall}{RT_2} = \frac{227.30 \times 0.020}{0.287 \times 333} = 0.048\ kg$$
$$\Delta m = m_1 - m_2 = 0.072 - 0.048 = 0.024\ kg$$

Notice that absolute rather than gage pressure must be used in calculations with the ideal gas law.

**Example:** In a piping system, the water temperature remains under 40°C. Determine the minimum pressure allowed in the system to avoid cavitation.

**Solution:** The minimum pressure in a piping system to avoid cavitation is to be determined. The vapor pressure of water at 40°C is 7.38 kPa

$$P_{min} = P_{sat\ at+°C} = 7.376\ kPa$$

Therefore, the pressure should be maintained above 7.376 kPa everywhere in flow. Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

| Temperature - $t$ - ($^oC$) | Absolute Vapor Pressure - $p_v$ - ($10^3$ Pa, N/m$^2$) |
|---|---|
| 0 | 0.6105 |
| 5 | 0.8722 |
| 10 | 1.228 |
| 20 | 2.338 |
| 30 | 4.243 |
| 40 | 7.376 |
| 50 | 12.33 |
| 60 | 19.92 |
| 70 | 31.16 |
| 80 | 47.34 |
| 90 | 70.10 |
| 100 | 101.3 |

**Example:** What is surface tension? What is it caused by? Why is the surface tension also called surface energy?

**Solution:** The magnitude of the pulling force at the surface of a liquid per unit length is called *surface tension* $\sigma s$. It is caused by the attractive forces between the molecules. The surface tension is also surface energy (per unit area) since it represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount. Surface tension is the cause of some very interesting phenomena such as capillary rise and insects that can walk on water.

**Example:** Consider a soap bubble. Is the pressure inside the bubble higher or lower than the pressure outside?

**Solution:** The pressure inside a soap bubble is greater than the pressure outside, as evidenced by the stretch of the soap film.

**Example:** What is the capillary effect? What is it caused by? How is it affected by the contact angle?

**Solution:** The *capillary effect* is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. It is caused by the net effect of the *cohesive forces* (the forces between like molecules, like water) and *adhesive forces* (the forces between unlike molecules, like water and glass). The capillary effect is proportional to the cosine of the *contact angle*, which is the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

**Example:** What is a Newtonian fluid? Is water a Newtonian fluid?

**Solution:** Fluids whose shear stress is linearly proportional to the velocity gradient (shear strain) are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.

**Example:** Consider two identical small glass balls dropped into two identical containers, one filled with water and the other with oil. Which ball will reach the bottom of the container first? Why?

**Solution:** When two identical small glass balls are dropped into two identical containers, one filled with water and the other with oil, the ball dropped in water will reach the bottom of the container first because of the much lower viscosity of water relative to oil.

**Example:** How does the dynamic viscosity of (*a*) liquids and (*b*) gases vary with temperature?

**Solution:** (*a*) The dynamic viscosity of liquids decreases with temperature. (*b*) The dynamic viscosity of gases increases with temperature. A good way to remember this is that a car engine is much harder to start in the winter because the oil in the engine has a higher viscosity at low temperatures.

**Example:** How does the kinematic viscosity of (*a*) liquids and (*b*) gases vary with temperature?

**Solution:** (*a*) For *liquids*, the kinematic viscosity decreases with temperature. (*b*) For *gases*, the kinematic viscosity increases with temperature.

**Example:** The analysis of a propeller that operates in water at 20°C shows that the pressure at the tips of the propeller drops to 2 kPa at high speeds. Determine if there is a danger of cavitation for this propeller.

**Solution:** The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation. The vapor pressure of water at 20°C is 2.338 kPa. To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is $P_{min} = P_{saturation\ at 20°\mathrm{C}} = 2.338\ kPa$

The minimum pressure in the pump is 2 kPa, which is less than the vapor pressure. Therefore, there is danger of cavitation in the pump.

**Example:** A pump is used to transport water to a higher reservoir. If the water temperature is 25°C, determine the lowest pressure that can exist in the pump without cavitation.

**Solution:** The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation. The vapor pressure of water at 25°C is 3.17 kPa To avoid cavitation, the pressure anywhere in the system should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{min} = P_{saturation\ at 25°C} = 3.17\ kPa$$

Therefore, the lowest pressure that can exist in the pump is 3.17 kPa

**Example:** Water at 1 atm pressure is compressed to 800 atm pressure isothermally. Determine the increase in the density of water. Take the isothermal compressibility of water to be $4.80 \times 10^{-5}\ atm$.

**Solution:** Water at a given temperature and pressure is compressed to a high pressure isothermally. The increase in the density of water is to be determined. a) The isothermal compressibility is constant in the given pressure range. b) An approximate analysis is performed by replacing differential changes by finite changes. The density of water at 20°C and 1 atm pressure is $\rho_1 = 998\ kg/m^3$. The isothermal compressibility of water is given to be $\propto = 4.80 \times 10^{-5}\ atm$.

When differential quantities are replaced by differences and the properties $\alpha$ and $\beta$are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

The change in density due to a change of pressure from 1 atm to 800 atm at constant temperature is

$$\Delta\rho = \alpha\rho\Delta P = 4.80 \times 10^{-5} \times 998 \times (800 - 1) = 38.3\ kg/m^3$$

**Example:** A liquid compressed in a cylinder has a volume of 1100 cm$^3$ at 110 MPa and a volume of 1000 cm$^3$ at 220 MPa. What is the bulk modulus of elasticity?

**Solution:**

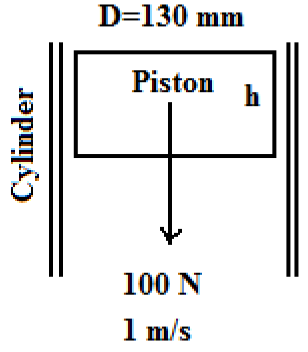
$$E_v = -\frac{dP}{\frac{d\forall}{\forall}} = -\forall\frac{dP}{d\forall} = -1100\frac{(220-110)}{(1000-1100)} = 1210\, MPa$$

**Example:** A piston of weight 100 N slides in a lubricated pipe at a speed of 1 m/s, as shown in Fig. The clearance between piston and pipe is 0.3 mm. The diameter and height of piston are 130 mm and 150 mm, respectively. What is the viscosity of the oil?

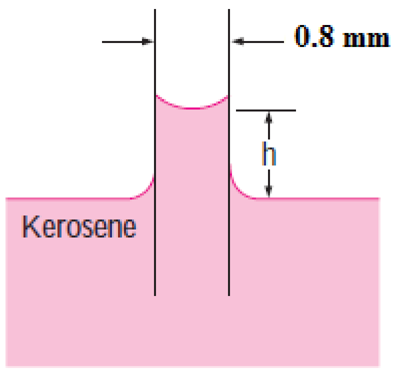
**Solution:**

$$\tau = \mu\frac{du}{dy} = \mu\left(\frac{v}{y}\right)$$

$$\mu = \tau\frac{y}{v} = \frac{F}{A}\frac{y}{v} = \frac{100\times0.0003}{\pi\times0.130\times0.150\times1} = 0.4897\, Pas$$




**Example:** A 0.8 mm diameter glass tube is inserted into kerosene at 20 °C. The contact angle of kerosene with a glass surface is 26°. Determine the capillary rise of kerosene in the tube in Fig. The surface tension of kerosene-glass at 20 °C , $\sigma = 0.028\, N/m$. The density of kerosene is 820.1 kg/m$^3$.




**Solution:** There are no impurities in the kerosene, and no contamination on the surfaces of the glass tube. The kerosene is open to the atmospheric air. The contact angle of kerosene with the glass surface is given to be 26°.

Capillary rise in tube: $h = \frac{2\sigma cos\theta}{\rho g R} = \frac{2\times0.028\times cos26}{8045\times0.0004} = 0.0156\, m = 15.6\, mm$

**Example:** A 2-mm-diameter tube is inserted into an unknown liquid whose density is 950 kg/m$^3$, and it is observed that the liquid rises 6 mm in the tube, making a contact angle of 16°. Determine the surface tension of the liquid.

**Solution:** $\sigma = \frac{\rho g R h}{2cos\theta} = \frac{950\times9.81\times0.001\times0.006}{2cos16} = 0.029\ N/m$

**Example:** Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube. What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than $h$=1.5 mm? For water at 20 °C, $\sigma = 0.0728\ N/m$, and $\gamma = 9810\ N/m^3$, $\theta = 0$

**Solution:**

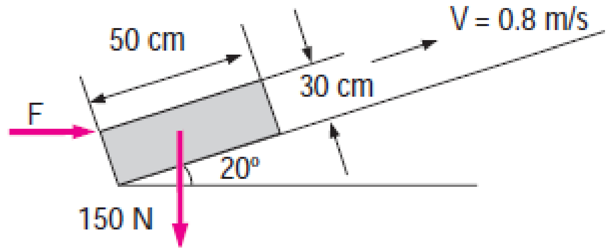
$$h = \frac{2\sigma cos\theta}{\rho g R} \rightarrow R = \frac{2\sigma cos\theta}{h\rho g} = \frac{2\times0.0728\times\cos 0}{0.0015\times9810} = 0.009895\ m = 9.895\ mm$$

$$2R = 19.79\ mm$$

**Example:** What is viscosity? What is the cause of it in liquids and in gases? Do liquids or gases have higher dynamic viscosities?

**Solution:** *Viscosity* is a measure of the "stickiness" or "resistance to deformation" of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. In general, liquids have higher dynamic viscosities than gases. The ratio of viscosity μ to density ρ often appears in the equations of fluid mechanics, and is defined as the *kinematic viscosity*, $\nu = \mu/\rho$

**Example:** A 50-cm× 30-cm × 20-cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. (*a*) Determine the force *F* that needs to be applied in the horizontal direction. (*b*) If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa. s is applied between the block and inclined surface, determine the percent reduction in the required force.

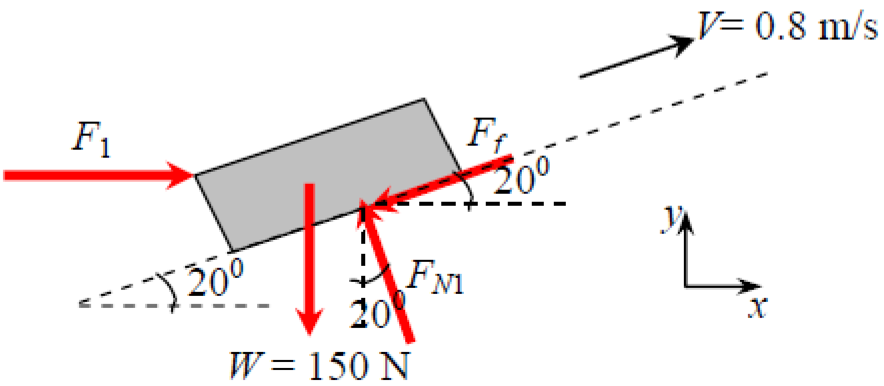



**Solution:** (*a*) The velocity of the block is constant, and thus itsacceleration and the net force acting on it are zero. A free body diagram of the block is given. Then the force balance gives

$$\sum F_x = 0: \quad F_1 - F_f \cos 20° - F_{N1} \sin 20° = 0 \quad (1)$$

$$\sum F_y = 0: \quad F_{N1} \cos 20° - F_f \sin 20° - W = 0 \quad (2)$$

$$\text{Friction force:} \quad F_f = fF_{N1} \quad (3)$$




Substituting Eq. (3) into Eq. (2) and solving for $FN1$ gives

$$F_{N1} = \frac{W}{\cos 20° - f \sin 20°} = \frac{150\text{ N}}{\cos 20° - 0.27 \sin 20°} = 177.0\text{ N}$$

Then from Eq. (1):

$$F_1 = F_f \cos 20° + F_{N1} \sin 20° = (0.27 \times 177\text{ N}) \cos 20° + (177\text{ N}) \sin 20° = \mathbf{105.5\ N}$$

(*b*) In this case, the friction force is replaced by the shear force applied on the bottom surface of the block due to the oil. Because of the no-slip condition, the oil film sticks to the inclined surface at the bottom and the lower surface of the block at the top. Then the shear force is expressed as

$$
\begin{aligned}
F_{shear} &= \tau_w A_s \\
&= \mu A_s \frac{V}{h} \\
&= \left(0.012\ \mathrm{N\cdot s/m^2}\right)\left(0.5\times 0.2\ \mathrm{m^2}\right)\frac{0.8\ \mathrm{m/s}}{4\times 10^{-4}\ \mathrm{m}} \\
&= 2.4\ \mathrm{N}
\end{aligned}
$$

Replacing the friction force by the shear force in part (*a*),

$$\sum F_x = 0: \quad F_2 - F_{shear}\cos 20° - F_{N2}\sin 20° = 0 \qquad (4)$$

$$\sum F_y = 0: \quad F_{N2}\cos 20° - F_{shear}\sin 20° - W = 0 \qquad (5)$$

Eq. (5) gives $F_{N2} = (F_{shear}\sin 20° + W)/\cos 20° = [(2.4\ \mathrm{N})\sin 20° + (150\ \mathrm{N})]/\cos 20° = 160.5\ \mathrm{N}$

Substituting into Eq. (4), the required horizontal force is determined to be

$$F_2 = F_{shear}\cos 20° + F_{N2}\sin 20° = (2.4\ \mathrm{N})\cos 20° + (160.5\ \mathrm{N})\sin 20° = 57.2\ \mathrm{N}$$

Then, our final result is expressed as Percentage reduction in required force =

$$\frac{F_1 - F_2}{F_1}\times 100\% = \frac{105.5 - 57.2}{105.5}\times 100\% = \mathbf{45.8\%}$$

Note that the force required to push the block on the inclined surface reduces significantly by oiling the surface.