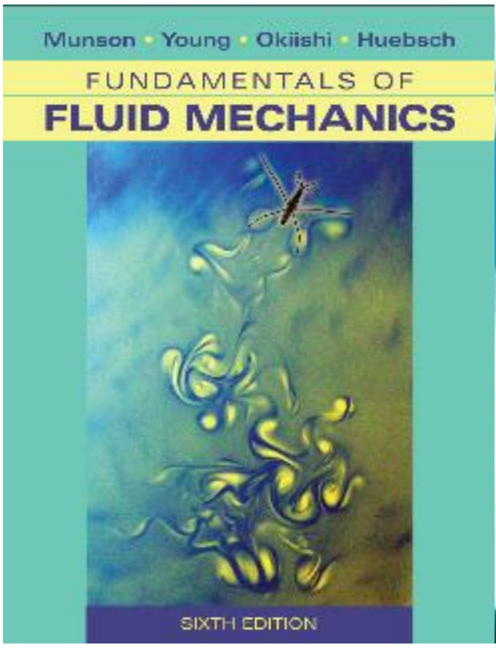
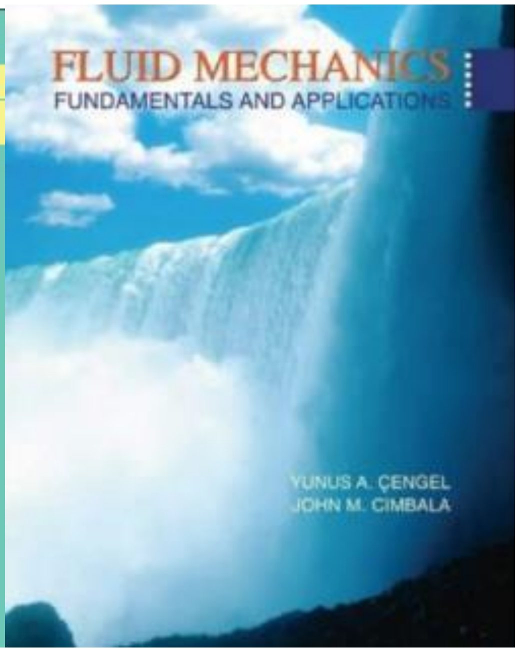
FLUID MECHANICS





PROF. DR. METİN GÜNER
COMPILER

ANKARA UNIVERSITY
FACULTY OF AGRICULTURE
DEPARTMENT OF AGRICULTURAL MACHINERY AND TECHNOLOGIES ENGINEERING

3-FLUID STATICS

3.1.Pressure at a Point

Pressure is the ***compressive force*** per unit area, and it gives the impression of being a vector. However, pressure at any point in a fluid is the same in all directions. That is, it has magnitude but not a specific direction, and thus it is a scalar quantity. This can be demonstrated by considering a small wedge-shaped fluid element of unit length (into the page) in equilibrium, as shown in Fig.3.3. The mean pressures at the three surfaces are P_s P_y, and P_z, and the force acting on a surface is the product of mean pressure and the surface area.

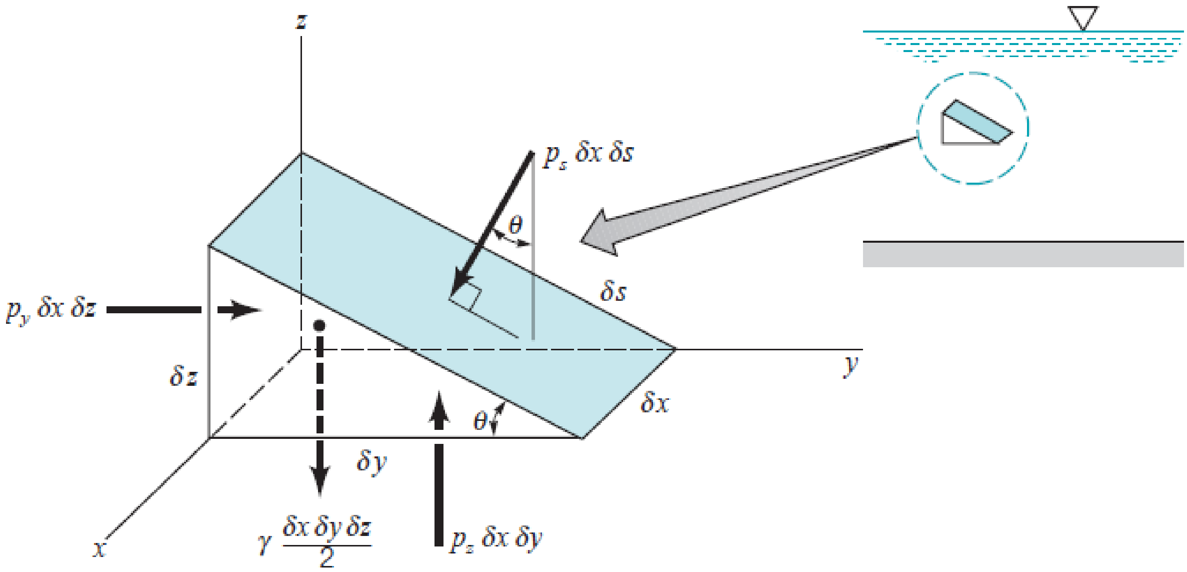


Figure 3.3. Forces on an arbitrary wedge-shaped element of fluid. Where and P_s P_y, and P_z, are the average pressures on the faces, and γ and ρ are the fluid specific weight and density, respectively, and a_y, a_z are the accelerations.

$$P_s = P_y = P_z$$

The pressure at a point in a fluid has the same magnitude in all directions. It can be shown in the absence of shear forces that this result is applicable to fluids in motion as well as fluids at rest. *The pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present*. This important result is known as ***Pascal's law,*** named in honor of Blaise Pascal 11623– 16622, a French mathematician who made important contributions in the field of hydrostatics. Thus, as shown by the Fig.3.4. at the junction of the side and bottom of the beaker, the pressure is the same on the side as it is on the bottom.

Figure 3.4. The pressure at a point in a fluid at rest is independent of direction.

3.2. Basic Equation for Pressure Field

Although we have answered the question of how the pressure at a point varies with direction, we are now faced with an equally important question—how does the pressure in a fluid in which there are no shearing stresses vary from point to point? To answer this question consider a small rectangular element of fluid removed from some arbitrary position within the mass of fluid of interest as illustrated in Fig.. There are two types of forces acting on this element: ***surface forces*** due to the pressure, and a ***body force*** equal to the weight of the element. Other possible types of body forces, such as those due to magnetic fields, will not be considered in this text.

The resultant surface force acting on a small fluid element depends only on the pressure gradient if there are no shearing stresses present.

$$-\hat{\nabla}\mathrm{P} - \gamma\hat{k} = \rho\hat{a}$$

$$\hat{\nabla}(\,) = \frac{\partial(\,)}{\partial x}\hat{\imath} + \frac{\partial(\,)}{\partial y}\hat{\jmath} + \frac{\partial(\,)}{\partial z}\hat{k}$$

∇ is the *gradient* or "del" vector operator. $\nabla P$is the pressure gradient (N/m^3), γ is the specific weight (N/m^3). ρ is the density (kg/m^3), and a is the acceleration of the element (m/s^2). $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along the coordinate axes shown in Fig.3.5

The above equation is the general equation of motion for a fluid in which there are no shearing stresses. We will use this equation when we consider the pressure distribution in a moving fluid. For the present, however, we will restrict our attention to the special case of a fluid at rest.

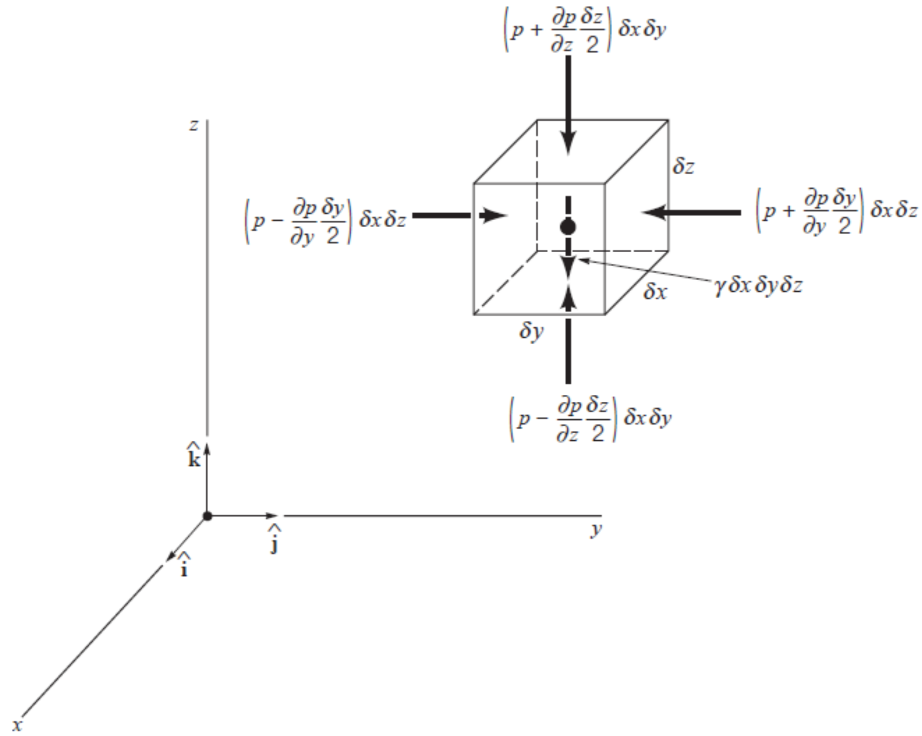


Figure 3.5. Surface and body forces acting on small fluid element.

3.3. Pressure Variation in a Fluid Rest

For a fluid at rest $\hat{a} = 0$ and the equation $-\hat{\nabla}\mathrm{P} - \gamma\hat{k} = \rho\hat{a}$ reduces to

$-\hat{\nabla}\mathrm{P} - \gamma\hat{k} = 0$ or in component form,

$$\frac{\partial P}{\partial x} = 0 \quad \frac{\partial P}{\partial y} = 0 \quad \frac{\partial P}{\partial z} = 0$$

These equations show that the pressure does not depend on *x* or *y*. Thus, as we move from point to point in a horizontal plane (any plane parallel to the *x*–*y* plane), the pressure does not change. Since P depends only on *z*, the last of Equation can be written as the ordinary differential equation.

$$\frac{dP}{dz} = -\gamma \rightarrow \; dP = -\gamma dz = -\gamma \int_1^1 dz$$

This is the ***fundamental equation*** for fluids at rest and can be used to determine how pressure changes with elevation. This equation and the Figure 3.6 indicate that the pressure gradient in the vertical direction is negative; that is, the pressure decreases as we move upward in a fluid at rest. There is no requirement that be a constant. Thus, it is valid for fluids with constant specific weight, such as liquids, as well as fluids whose specific weight may vary with elevation, such as air or other gases. However, to proceed with the integration of the above equation it is necessary to stipulate how the specific weight varies with *z*.

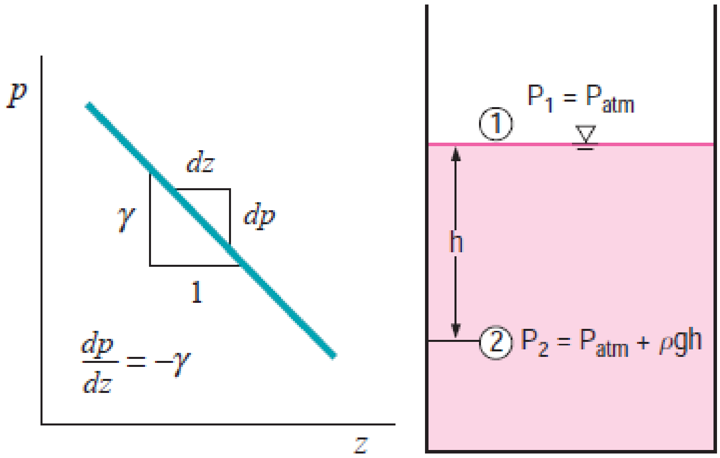


Figure 3.6. For liquids or gases at rest, the pressure gradient in the vertical direction at any point in a fluid depends only on the specific weight of the fluid at that point.

Pressure in a fluid at rest is independent of the shape or cross section of the container (Fig.3.7). It changes with the vertical distance, but remains constant in other directions. Therefore, the pressure is the same at all points on a horizontal plane in a given fluid.

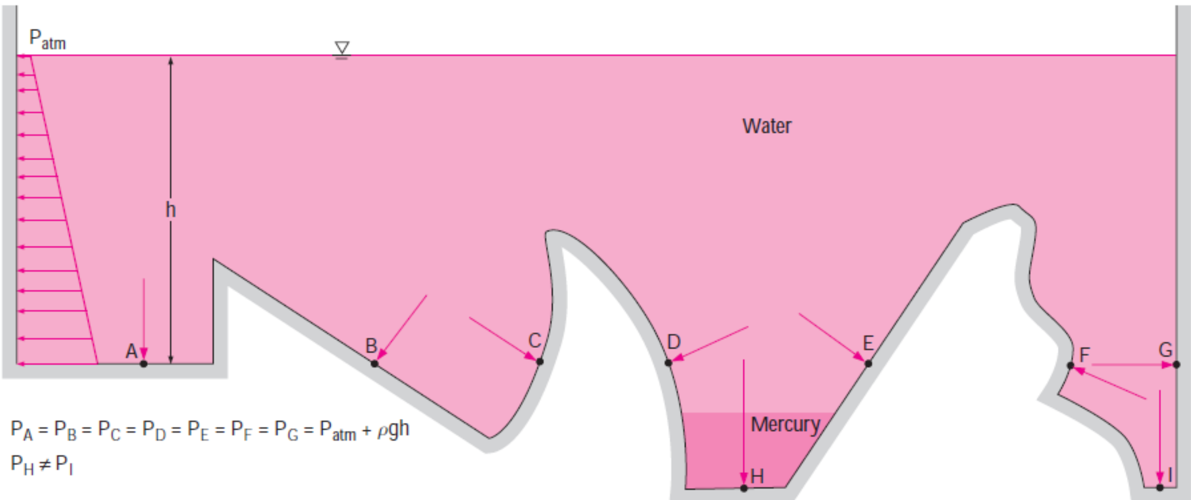


Figure 3.7. The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

3.3.1. Incompressible Fluid

Since the specific weight is equal to the product of fluid density and acceleration of gravity ($\gamma = \rho g$),changes in γ are caused either by a change in ρ or g. For most engineering applications the variation in g is negligible, so our main concern is with the possible variation in the fluid density. In general, a fluid with constant density is called an ***incompressible fluid.*** For liquids the variation in density is usually negligible, even over large vertical distances, so that the assumption of constant specific weight when dealing with liquids is a good one. For this instance, the above equation can be directly integrated

$$\frac{dP}{dz} = -\gamma \rightarrow \int_{P_1}^{P_2} dP = -\gamma dz = -\gamma \int_{z_1}^{z_2} dz$$

$P_2 - P_1 = -\gamma(z_2 - z_1)$ or $P_1 - P_2 = \gamma(z_2 - z_1)$

Where; p_1 and p_2 are pressures at the vertical elevations z_1 and z_2 as is illustrated in the below figure.

The above equation can be written in the compact form

$$P_1 - P_2 = \gamma h$$
$$P_1 = P_2 + \gamma h$$

Where; h is the distance, $(z_2 - z_1)$, which is the depth of fluid measured downward from the location of P_2. This type of pressure distribution is commonly called a ***hydrostatic distribution,*** and last equation shows that in an incompressible fluid at rest the pressure varies linearly with depth. The pressure must increase with depth to “hold up” the fluid above it.

It can also be observed that the pressure difference between two points can be specified by the distance h since

$$h = \frac{P_1 - P_2}{\gamma}$$

In this case h is called the ***pressure head*** and is interpreted as the height of a column of fluid of specific weight γ required to give a pressure difference $P_1 - P_2$ For example, a pressure difference of 49050 Pa can be specified in terms of pressure head as h_{water} =49050/9810= 5 m of water $\left(\gamma_{water} = 9810 \frac{N}{m^3}\right)$ or $h_{Hg} = \frac{49050}{133416} = 0.3676\, m$ ($\gamma_{Hg} = 133416\, N/m^3$).
When one works with liquids there is often a free surface it is convenient to use this surface as a reference plane. The reference pressure P_0 would correspond to

the pressure acting on the free surface (which would frequently be atmospheric pressure), and thus if we let $P_2 = P_0$ in the above equation. It follows that the pressure p at any depth h below the free surface is given by the equation:

$$P = \gamma h + P_0$$

The pressure in a homogeneous, incompressible fluid at rest depends on the depth of the fluid relative to some reference plane, and it is *not* influenced by the *size* or *shape* of the tank or container in which the fluid is held. Thus the pressure is the same at all points along the line *AB* even though the containers may have the very irregular shapes shown in the Fig. 3.8. The actual value of the pressure along *AB* depends only on the depth, h, the surface pressure, p_0, and the specific weight, γ,of the liquid in the container.

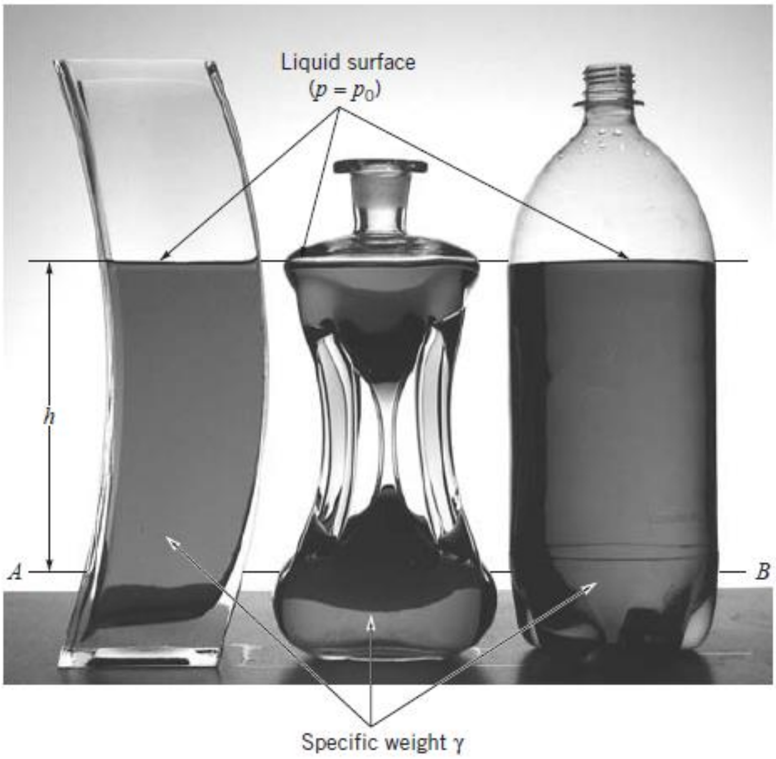
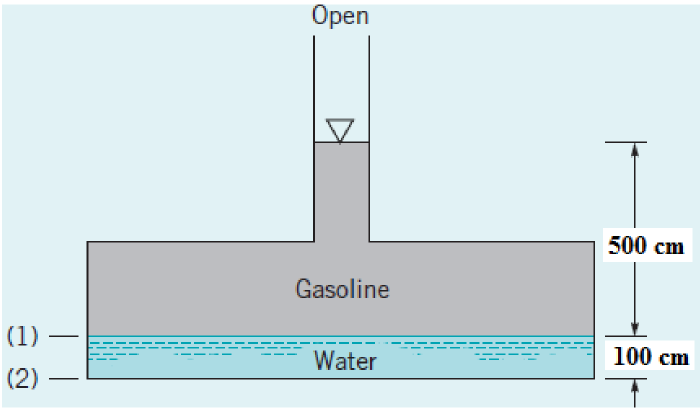


Figure 3.8. Fluid pressure in containers of arbitrary shape.

Example: Because of a leak in a buried gasoline storage tank, water has seeped in to the depth shown in the below figure. The specific gravity of the gasoline is *SG*= 0.68. Determine the pressure at the gasoline–water interface and at the bottom of the tank. Express the pressure in units of Pa and as a pressure head in meter of water.



Solution: Since we are dealing with liquids at rest, the pressure distribution will be hydrostatic, and therefore the pressure variation can be found from the equation:

$$P = \gamma h + P_0$$

With P_0 corresponding to the pressure at the free surface of the gasoline, then the pressure at the interface is

$$SG = \frac{\rho_{gasoline}}{\rho_{water}}$$

$$P_1 = \rho_g g h + P_0 = SG_g \, \rho_w g h + P_0 = 0.68 \times 1000 \times 9.81 \times 5 + P_0$$

$$P_1 = 33354 + P_0$$

If we measure the pressure relative to atmospheric pressure (gage pressure), it follows that $P_0 = 0$ and therefore

$$P_1 = 33354 \, Pa$$

$$h = \frac{33354 \, Pa}{9810 \, N/m^3} = 3.4 \, m$$

We can now apply the same relationship to determine the pressure at the tank bottom; that is,

$$P_2 = \gamma_w \, h_w + P_1 = 9810 \times 1 + 33354$$

$$P_2 = 43164 \, Pa$$

$$h = \frac{43164\ Pa}{9810\ N/m^3} = 4.4\ m$$

Observe that if we wish to express these pressures in terms of *absolute* pressure, we would have to add the local atmospheric pressure (in appropriate units) to the previous results.

For sea level, the pressures become;

$$P_{1b} = 33354 + 101325 = 134679\ Pa$$

$$P_{2a} = 43164 + 101325 = 144489\ Pa$$

Hydraulic Jacks

A consequence of the pressure in a fluid remaining constant in the horizontal direction is that *the pressure applied to a confined fluid increases the pressure throughout by the same amount*. This is called **Pascal's law,** after Blaise Pascal (1623–1662). Pascal also knew that the force applied by a fluid is proportional to the surface area. He realized that two hydraulic cylinders of different areas could be connected, and the larger could be used to exert a proportionally greater force than that applied to the smaller. "***Pascal's machine***" has been the source of many inventions that are a part of our daily lives such as hydraulic brakes and lifts. This is what enables us to lift a car easily by one arm, as shown in Fig.3.9. Noting that $P_1 = P_2$ since both pistons are at the same level (the effect of small height differences is negligible, especially at high pressures), the ratio of output force to input force is determined to be

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2}$$

The area ratio A_2/A_1 is called the ***ideal mechanical advantage*** of the hydraulic lift. Using a hydraulic car jack with a piston area ratio of $A_2/A_1 = 10$, for example, a person can lift a 1000-kg car by applying a force of just 100 kgf (= 908 N).

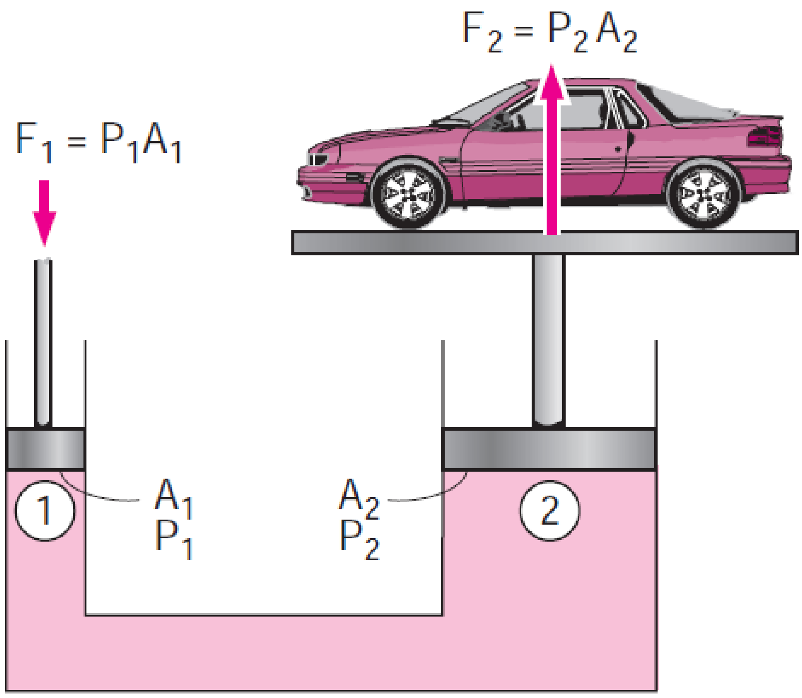


Figure 3.9. Lifting of a large weight by a small force by the application of Pascal's law.

The required equality of pressures at equal elevations throughout a system is important for the operation of hydraulic jacks (Fig.3.10*a*), lifts, and presses, as well as hydraulic controls on aircraft and other types of heavy machinery. The fundamental idea behind such devices and systems is demonstrated in Fig. 3.10*b*. A piston located at one end of a closed system filled with a liquid, such as oil, can be used to change the pressure throughout the system, and thus transmit an applied force F_1 to a second piston where the resulting force is F_2. Since the pressure p acting on the faces of both pistons is the same (the effect of elevation changes is usually negligible for this type of hydraulic device), it follows that

$$F_2 = F_1 \frac{A_2}{A_1}$$

The piston area A_2 can be made much larger than A_1 and therefore a large mechanical advantage can be developed; that is, a small force applied at the smaller piston can be used to develop a large force at the larger piston. The applied force could be created manually through some type of mechanical device, such as a hydraulic jack, or through compressed air acting directly on the surface of the liquid, as is done in hydraulic lifts commonly found in service stations.

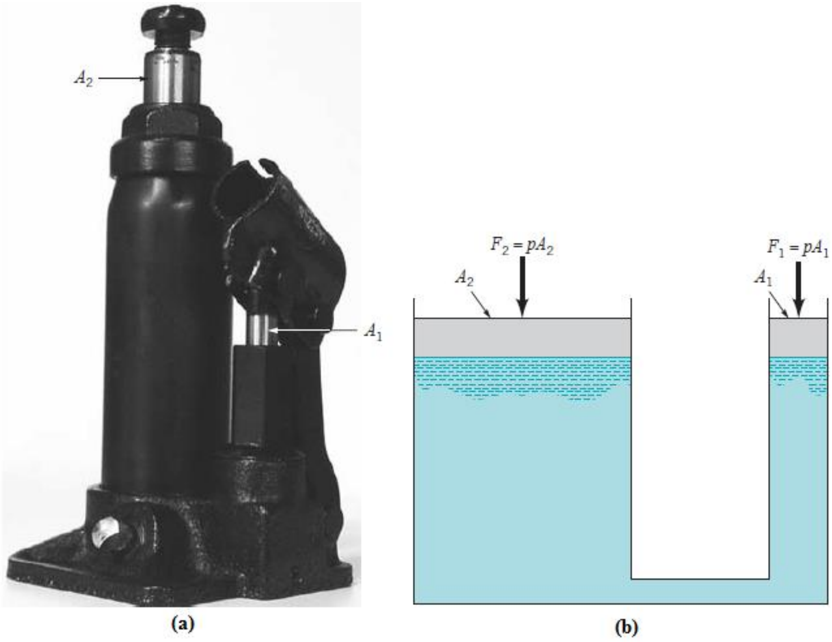


Figure 3.10. (*a*) Hydraulic jack, (*b*) Transmission of fluid pressure.

3.3.2. Compressible Fluid

We normally think of gases such as air, oxygen, and nitrogen as being ***compressible fluids*** since the density of the gas can change significantly with changes in pressure and temperature. Thus, it is necessary to consider the possible variation in before the equation can be integrated. The specific weights of common gases are small when compared with those of liquids. For example, the specific weight of air at sea level and is whereas the specific weight of water under the same conditions is Since the specific weights of gases are comparatively small, it follows from $\frac{dP}{dz} = -\gamma$ that the pressure gradient in the vertical direction is correspondingly small, and even over distances of several hundred feet the pressure will remain essentially constant for a gas. This means we can neglect the effect of elevation changes on the pressure in gases in tanks, pipes, and so forth in which the distances involved are small.

For those situations in which the variations in heights are large, on the order of thousands of feet, attention must be given to the variation in the specific weight. The equation of state for an ideal (or perfect) gas is

$$\rho = \frac{P}{RT}$$

Where; P is the absolute pressure, R is the gas constant, and T is the absolute temperature. This relationship can be combined with $\frac{dP}{dz} = -\gamma$ to give

$$\frac{dP}{dz} = -\frac{gP}{RT}$$

and by separating variables

$$\int_{P_1}^{P_2} \frac{dP}{P} = \ln\frac{P_2}{P_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

Where; g and R are assumed to be constant over the elevation change from z_1 to z_2. Although the acceleration of gravity, g, does vary with elevation, the variation is very small, and g is usually assumed constant at some average value for the range of elevation involved.

Before completing the integration, one must specify the nature of the variation of temperature with elevation. For example, if we assume that the temperature has a constant value T_0 over the range z_1 to z_2 (*isothermal conditions*), it then follows from the above equation that

$$P_2 = P_1 exp\left[-\frac{g(z_2 - z_1)}{R\,T_0}\right]$$

This equation provides the desired pressure–elevation relationship for an isothermal layer. As shown in the Fig. 3.11., even for a 3048 m altitude change the difference between the constant temperature (isothermal) and the constant density (incompressible) results are relatively minor. For nonisothermal conditions a similar procedure can be followed if the temperature–elevation relationship is known, as is discussed in the following section.

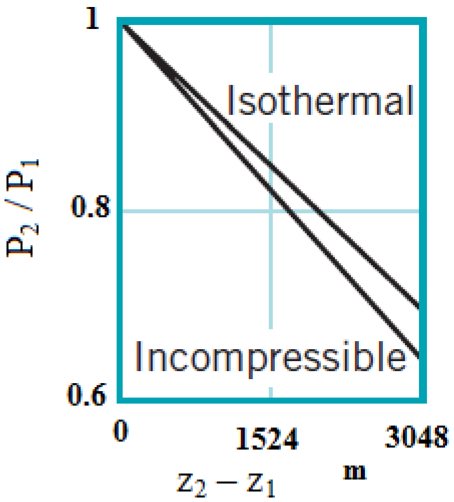


Figure 3.11. The relationship between elevation and pressure ratio

Example: **(a)** Estimate the ratio of the pressure at 694 m top of the building to the pressure at its base, assuming the air to be at a common temperature of 15 ℃ **(b)** Compare the pressure calculated in part (a) with that obtained by assuming the air to be incompressible with $\gamma = 12\ N/m^3$ at 101 325 Pa (values for air at standard sea level conditions).

Solution: For the assumed isothermal conditions, and treating air as a compressible fluid, the following equations can be applied to yield.

$$P_2 = P_1 exp\left[-\frac{g(z_2 - z_1)}{R\ T_0}\right]$$

$$\frac{P_2}{P_1} = exp\left[-\frac{g(z_2 - z_1)}{R\ T_0}\right]$$

$$\frac{P_2}{P_1} = exp\left[-\frac{9.81 \times 694\ m}{286.9 \times 288}\right] = 0.921$$

If the air is treated as an incompressible fluid we can apply

$$P_2 = P_1 - \gamma(z_2 - z_1)$$

$$\frac{P_2}{P_1} = 1 - \frac{\gamma\ (z_2 - z_1)}{P_1} = 1 - \frac{12 \times 694}{101325} = 0.918$$

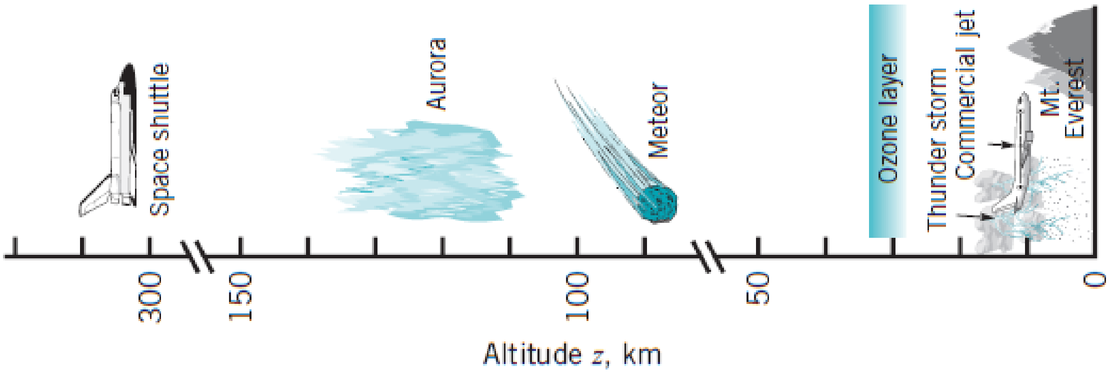
Note that there is little difference between the two results. Since the pressure difference between the bottom and top of the building is small, it follows that the variation in fluid density is small and, therefore, the compressible fluid and incompressible fluid analyses yield essentially the same result.

3.4. Standard Atmosphere

An important application of the following equation relates to the variation in pressure in the earth's atmosphere. Ideally, we would like to have measurements of pressure versus altitude over the specific range for the specific conditions (temperature, reference pressure) for which the pressure is to be determined. However, this type of information is usually not available. Thus, a "***standard atmosphere***" has been determined that can be used in the design of aircraft, missiles, and spacecraft, and in comparing their performance under standard conditions.

$$\int_{P_1}^{P_2} \frac{dP}{P} = \ln \frac{P_2}{P_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

The concept of a standard atmosphere was first developed in the 1920s. Several important properties for standard atmospheric conditions at ***sea level*** are listed in Fig. shows the temperature profile for the U.S. standard atmosphere. As is shown in this figure the temperature decreases with altitude in the region nearest the earth's surface (*troposphere*), then becomes essentially constant in the next layer (*stratosphere)*, and subsequently starts to increase in the next layer. Typical events that ocur in the atmosphere are shown in the Fig.3.12.



Property	SI Units	BG Units
Temperature, T	288.15 K (15 °C)	518.67 °R (59.00 °F)
Pressure, p	101.33 kPa (abs)	2116.2 lb/ft^2 (abs) [14.696 lb/in.^{2} (abs)]
Density, ρ	1.225 kg/m^3	0.002377 slugs/ft^3
Specific weight, γ	12.014 N/m^3	0.07647 lb/ft^3
Viscosity, μ	1.789×10^{-5} N · s/m^2	3.737×10^{-7} lb · s/ft^2

Acceleration of gravity at sea level = 9.807 m/s^2 = 32.174 ft/s^2.

Figure 3.12. Typical events that ocur in the atmosphere and properties of U.S. standar atmodphere at sea level

Since the temperature variation is represented by a series of linear segments, it is possible to integrate the above equation to obtain the corresponding pressure variation. For example, in the troposphere, which extends to an altitude of about 11 km the temperature variation is of the form

$$T = T_a - \beta\, z$$

Where; T_a is the temperature at sea level (z=0) and β is the lapse rate (the rate of change of temperature with elevation). For the standard atmosphere in the troposphere $\beta = 0.00650\ K/m$. From the following equations

$$\int_{P_1}^{P_2} \frac{dP}{P} = \ln\frac{P_2}{P_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T} \text{ and } T = T_a - \beta\, z$$

We can write the following equation.

$$P = P_a\left(1 - \frac{\beta z}{Ta}\right)^{\frac{g}{R\beta}}$$

Where P_a is the absolute pressure at z=0. With P_a, T_a and g obtained from the above table, and with the gas constant or the pressure variation throughout the troposphere can be determined from the above equation. This calculation shows that at the outer edge of the troposphere, where the temperature is -56.5 ℃ , the absolute pressure is about 23 kPa. It is to be noted that modern jetliners cruise at approximately this altitude. Pressures at other altitudes are shown in the following Fig.3.13.

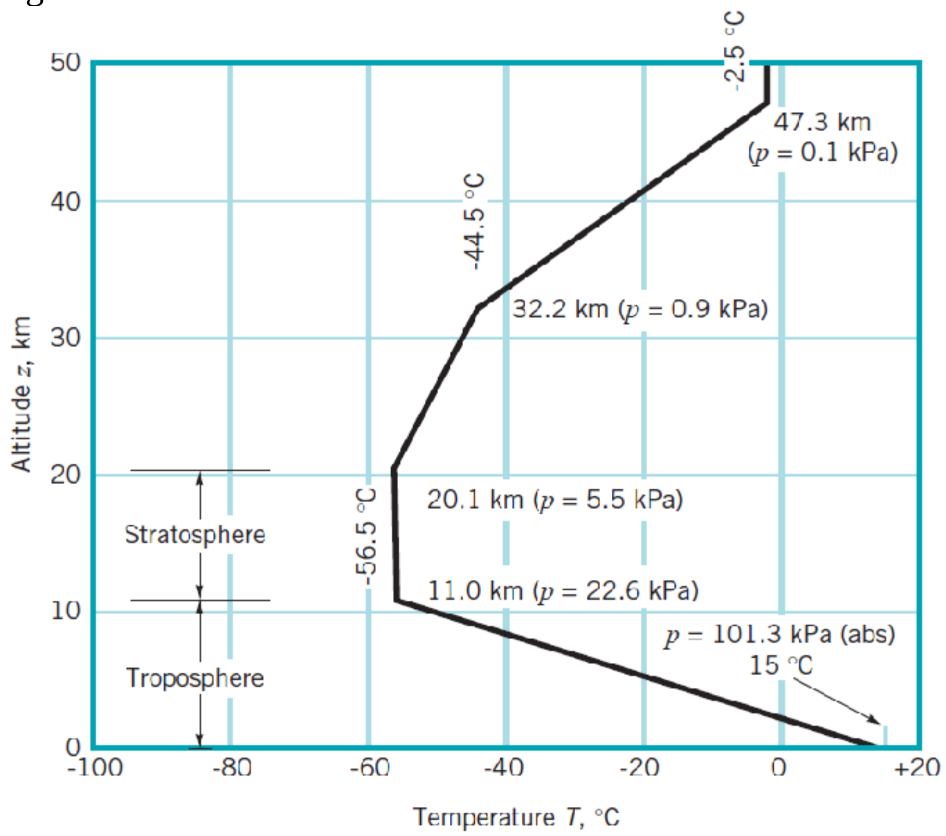


Figure 3.13. Variation of temperature with altitude in the U.S. standard atmosphere.