

FLUID MECHANICS



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3-FLUID STATICS

3.5. Measurement of Pressure

Since pressure is a very important characteristic of a fluid field, it is not surprising that numerous devices and techniques are used in its measurement. As is noted briefly before Chapters, the pressure at a point within a fluid mass will be designated as either an **absolute pressure** or a **gage pressure**. Absolute pressure is measured relative to a perfect vacuum (absolute zero pressure), whereas gage pressure is measured relative to the local atmospheric pressure. Thus, a gage pressure of zero corresponds to a pressure that is equal to the local atmospheric pressure. Absolute pressures are always positive, but gage pressures can be either positive or negative depending on whether the pressure is above atmospheric pressure (a positive value) or below atmospheric pressure (a negative value). A negative gage pressure is also referred to as a *suction* or **vacuum pressure**.

The measurement of atmospheric pressure is usually accomplished with a mercury **barometer**, which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in Fig.3.14.

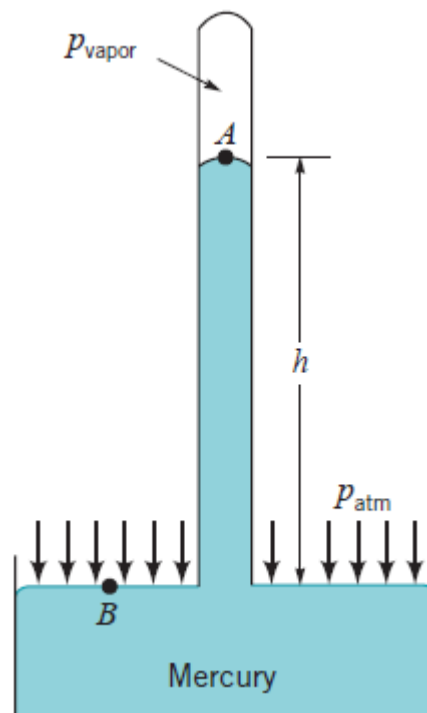


Figure 3.14. Mercury barometer

The tube is initially filled with mercury (inverted with its open end up) and then turned upside down (open end down), with the open end in the container of

mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor pressure (which develops in the space above the column) balances the force due to the atmospheric pressure. Thus,

$$P_{atm} = \gamma h + P_{vapor}$$

Where; γ is the specific weight of mercury.

For most practical purposes the contribution of the vapor pressure can be neglected since it is very small (for mercury $P_{vapor} = 0.16 \text{ Pa (abs)}$ at 20°C) so that $P_{atm} = \gamma h$. It is conventional to specify atmospheric pressure in terms of the height, h , in millimeters or inches of mercury. Note that if water were used instead of mercury, the height of the column would have to be approximately 11.33 m rather than 0.760 m of mercury for an atmospheric pressure of 1 atm (Fig.3.15). The concept of the mercury barometer is an old one, with the invention of this device attributed to Evangelista Torricelli in about 1644.

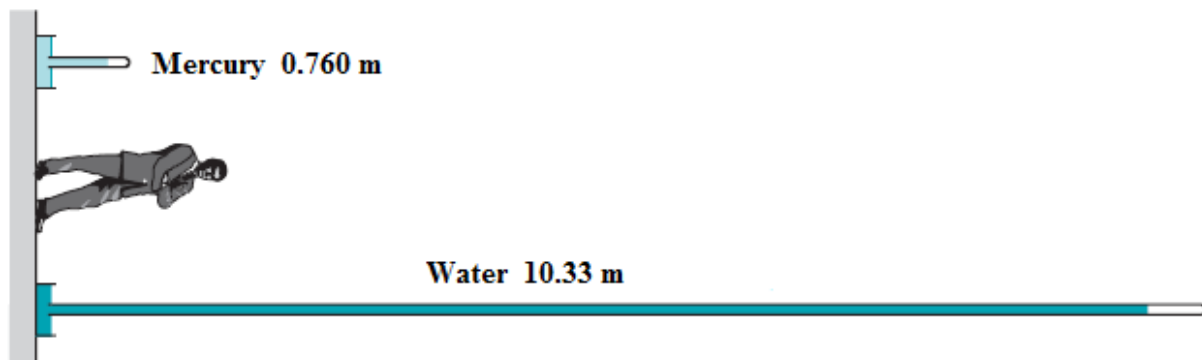


Figure 3.15. Atmospheric pressure in mHg and in mH₂O

Remember that the atmospheric pressure at a location is simply the weight of the air above that location per unit surface area. Therefore, it changes not only with elevation but also with weather conditions. The decline of atmospheric pressure with elevation has far-reaching ramifications in daily life. For example, cooking takes longer at high altitudes since water boils at a lower temperature at lower atmospheric pressures. Nose bleeding is a common experience at high altitudes since the difference between the blood pressure and the atmospheric pressure is larger in this case, and the delicate walls of veins in the nose are often unable to withstand this extra stress.

For a given temperature, the density of air is lower at high altitudes, and thus a given volume contains less air and less oxygen. So it is no surprise that we tire more easily and experience breathing problems at high altitudes. To compensate for this effect, people living at higher altitudes develop more efficient lungs. Similarly, a 2.0-L car engine will act like a 1.7-L car engine at 1500 m altitude

(unless it is turbocharged) because of the 15 percent drop in pressure and thus 15 percent drop in the density of air (Fig.3.16).

A fan or compressor will displace 15 percent less air at that altitude for the same volume displacement rate. Therefore, larger cooling fans may need to be selected for operation at high altitudes to ensure the specified mass flow rate. The lower pressure and thus lower density also affects lift and drag: airplanes need a longer runway at high altitudes to develop the required lift, and they climb to very high altitudes for cruising for reduced drag and thus better fuel efficiency.

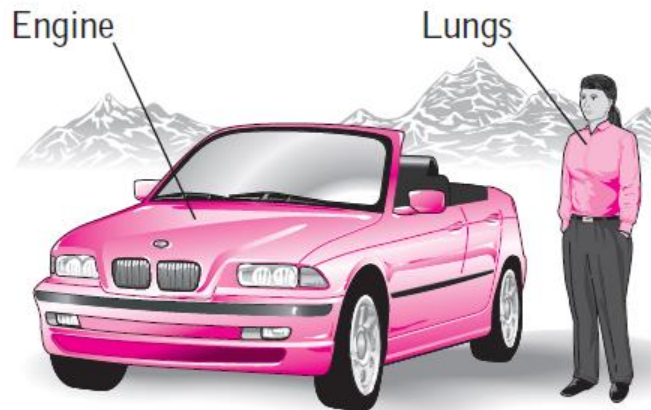


Figure 3.16. At high altitudes, a car engine generates less power and a person gets less oxygen because of the lower density of air

Example: Determine the atmospheric pressure at a location where the barometric reading is 730 mm Hg and the gravitational acceleration is $g = 9.81 \text{ m/s}^2$. Assume the temperature of mercury to be 10°C , at which its density is 13.570 kg/m^3 .

Solution: The barometric reading at a location in height of mercury column is given. The atmospheric pressure is to be determined. The temperature of mercury is assumed to be 10°C . The density of mercury is given to be 13.570 kg/m^3 .

$$P_{atm} = \rho gh = 13570 \times 9.81 \times 0.73 = 97178.84 \text{ Pa}$$

Note that density changes with temperature, and thus this effect should be considered in calculations.

Example: The piston of a vertical piston–cylinder device containing a gas has a mass of 50 kg and a cross-sectional area of 0.05 m^2 , as shown in Fig. The local atmospheric pressure is 96 000 Pa, and the gravitational acceleration is 9.81 m/s^2 . (a) Determine the pressure inside the cylinder. (b) If some heat is transferred to the gas and its volume is doubled, do you expect the pressure inside the cylinder to change?

Solution: A gas is contained in a vertical cylinder with a heavy piston. The pressure inside the cylinder and the effect of volume change on pressure are to be determined.

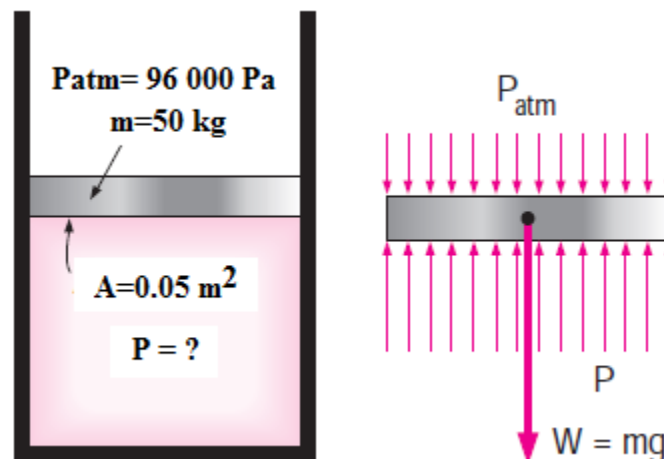
a) The gas pressure in the piston–cylinder device depends on the atmospheric pressure and the weight of the piston. Drawing the free-body diagram of the piston as shown in Fig. and balancing the vertical forces yield.

$$PA = P_{atm}A + W$$

Solving for P and substituting,

$$P = P_{atm} + \frac{mg}{A} = 96000 + \frac{50 \times 9.81}{0.05} = 105810 \text{ Pa}$$

b) The volume change will have no effect on the free-body diagram drawn in part (a), and therefore the pressure inside the cylinder will remain the same. If the gas behaves as an ideal gas, the absolute temperature doubles when the volume is doubled at constant pressure.



Example: A mountain lake has an average temperature of and a maximum depth of 50 m. The barometric pressure is 630 mm Hg. $\gamma_w = 9804 \frac{N}{m^3}$ at 10 °C. $\gamma_{Hg} = 133416 \frac{N}{m^3}$. Determine the absolute pressure (in pascals) at the deepest part of the lake.

Solution: The pressure in the lake at any depth, h , is given by the equation

$$P = \gamma h + P_0$$

where P_0 is the pressure at the surface. Since we want the absolute pressure, P_0 will be the local barometric pressure expressed in a consistent system of units; that is

$$\frac{P_{\text{barometric}}}{\gamma_{Hg}} = 630 \text{ mm} = 0.630 \text{ m}$$

$$P_0 = \gamma_{Hg} h = 133416 \times 0.630 = 84052 \text{ Pa}$$

$$P = \gamma_w h + P_0 = 9804 \times 50 + 84052 = 574252 \text{ Pa(abs)}$$

This simple example illustrates the need for close attention to the units used in the calculation of pressure; that is, be sure to use a *consistent* unit system, and be careful not to add a pressure head (m) to a pressure (Pa).

3.6. Manometry

A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called *manometers*. The mercury barometer is an example of one type of manometer, but there are many other configurations possible, depending on the particular application. Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

3.6.1. Piezometer Tube

The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Fig.3.17. The figure shows an important device whose operation is based upon this principle. It is a sphygmomanometer, the traditional instrument used to measure blood pressure.

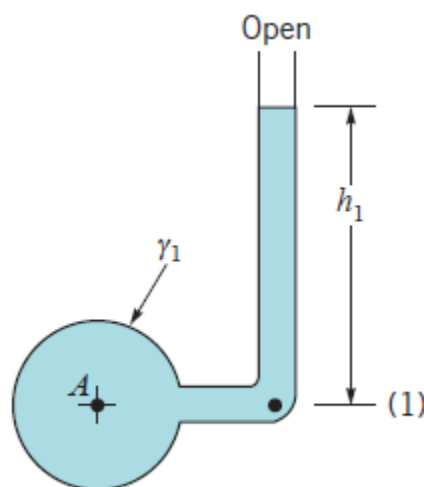


Figure 3.17. The basic manometry piezometer tube.

Since manometers involve columns of fluids at rest, the fundamental equation describing their use is

$$P = \gamma h + P_0.$$

which gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure and the vertical distance h between P and P_0 . Remember that in a fluid at rest pressure will *increase* as we move *downward* and will decrease as we move *upward*. Application of this equation to the piezometer tube of the above figure indicates that the pressure P_A can be determined by a measurement of h_1 through the relationship.

$$P_A = \gamma_1 h_1$$

Where; γ_1 is the specific weight of the liquid in the container. Note that since the tube is open at the top, the pressure P_0 can be set equal to zero (we are now using gage pressure), with the height h_1 measured from the meniscus at the upper surface to point (1). Since point (1) and point A within the container are at the same elevation, $P_A = P_1$.

Although the piezometer tube is a very simple and accurate pressure measuring device, it has several disadvantages. It is only suitable if the pressure in the container is greater than atmospheric pressure (otherwise air would be sucked into the system), and the pressure to be measured must be relatively small so the required height of the column is reasonable. Also, the fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.

3.6.2. U-Tube Manometer

To overcome the difficulties noted previously, another type of manometer which is widely used consists of a tube formed into the shape of a U, as is shown in Fig. The fluid in the manometer is called the *gage fluid*. To find the pressure P_A in terms of the various column heights, we start at one end of the system and work our way around to the other end, simply utilizing $P = \gamma h + P_0$. Thus, for the U-tube manometer shown in Fig.3.18, we will start at point A and work around to the open end. The pressure at points A and (1) are the same, and as we move from point (1) to (2) the pressure will increase by $\gamma_1 h_1$. The pressure at point (2) is equal to the pressure at point (3), since the pressures at equal elevations in a continuous mass of fluid at rest must be the same. Note that we could not simply “jump across” from point (1) to a point at the same elevation in the right-hand tube since these would not be points within the same continuous mass of fluid. With the pressure at point (3) specified, we now move to the open end where the pressure is zero.

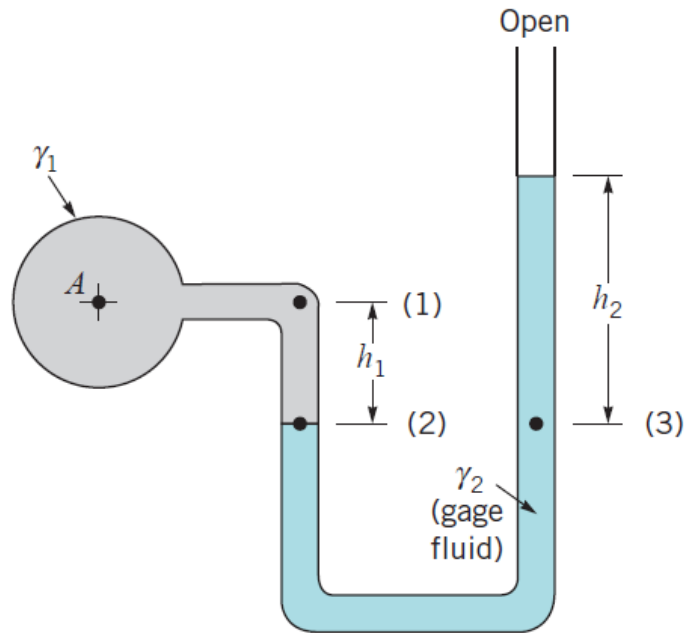


Figure 3.18. Simple U-tube manometer

As we move vertically upward the pressure decreases by an amount $\gamma_2 h_2$. In equation form these various steps can be expressed as

$$P_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

and, therefore, the pressure P_A can be written in terms of the column heights as

$$P_A = \gamma_2 h_2 - \gamma_1 h_1$$

A major advantage of the U-tube manometer lies in the fact that the gage fluid can be different from the fluid in the container in which the pressure is to be determined. For example, the fluid in A in Fig.3.18 can be either a liquid or a gas. If A does contain a gas, the contribution of the gas column, $\gamma_1 h_1$, is almost always negligible so that, and in this instance becomes

$$P_A = \gamma_2 h_2 - \gamma_1 h_1$$

$$P_A = \gamma_2 h_2$$

Thus, for a given pressure the height, h_2 , is governed by the specific weight, γ_2 , of the gage fluid used in the manometer. If the pressure P_A is large, then a heavy gage fluid, such as mercury, can be used and a reasonable column height (not too long) can still be maintained. Alternatively, if the pressure P_A is small, a lighter gage fluid, such as water, can be used so that a relatively large column height (which is easily read) can be achieved.

The U-tube manometer is also widely used to measure the *difference* in pressure between two containers or two points in a given system. Consider a manometer connected between containers *A* and *B* as is shown in differential U-tube manometer Figure 3.19.

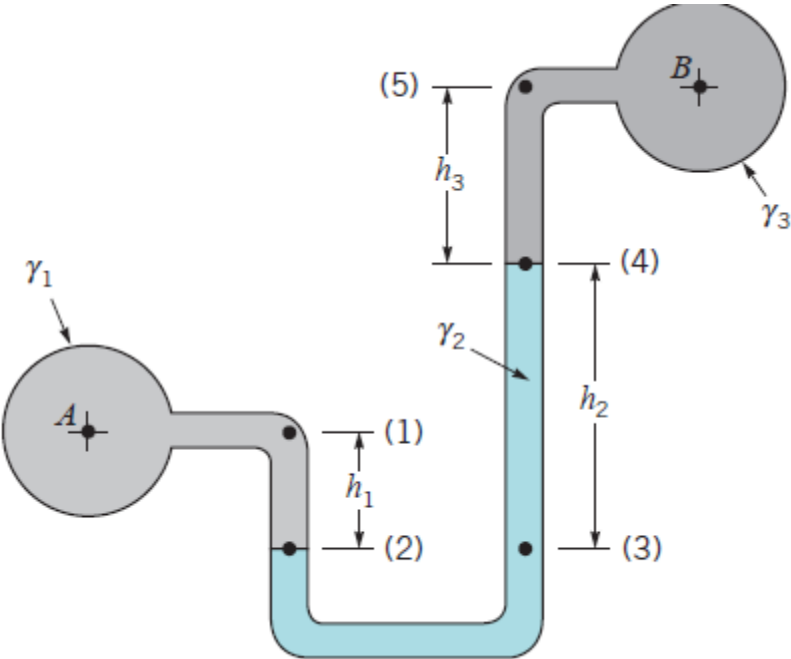


Figure 3.19. Differential U-tube manometer.

The difference in pressure between *A* and *B* can be found by again starting at one end of the system and working around to the other end. For example, at *A* the pressure P_A , is which is equal to P_1 , and as we move to point (2) the pressure increases by $\gamma_1 h_1$. The pressure at P_2 is equal to P_3 and as we move upward to point (4) the pressure decreases by $\gamma_2 h_2$. Similarly, as we continue to move upward from point (4) to (5) the pressure decreases by $\gamma_3 h_3$. Finally, since they are at equal elevations. Thus,

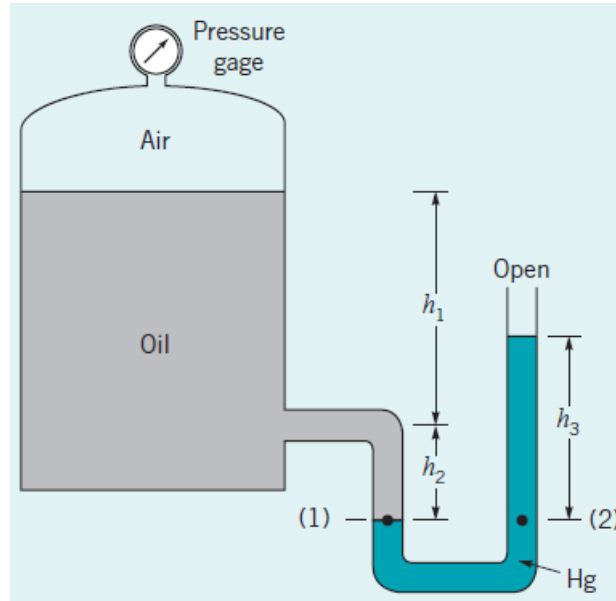
$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

Or, as indicated in the figure, we could start at *B* and work our way around to *A* to obtain the same result. In either case, the pressure difference is

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

When the time comes to substitute in numbers, be sure to use a consistent system of units.

Example: A closed tank contains compressed air and oil ($SG_{oil} = 0.92$) as is shown in Fig. U-tube manometer using Mercury ($SG_{Hg} = 13.6$) is connected to the tank as shown. The column heights are $h_1=90$ cm, $h_2= 15$ cm and $h_3=23$ cm. Determine the pressure reading (Pa) of the gage.



Solution: Following the general procedure of starting at one end of the manometer system and working around to the other, we will start at the air–oil interface in the tank and proceed to the open end where the pressure is zero. The pressure at level (1) is

$$P_1 = P_{air} + \gamma_{oil}(h_1 + h_2)$$

This pressure is equal to the pressure at level (2), since these two points are at the same elevation in a homogeneous fluid at rest. As we move from level (2) to the open end, the pressure must decrease by and at the open end the pressure is zero. Thus, the manometer equation can be expressed as

$$P_{air} + \gamma_{oil}(h_1 + h_2) - \gamma_{Hg}h_3 = 0 \quad \text{or}$$

$$P_{air} + (SG_{oil})(\gamma_{H_2O})(h_1 + h_2) - (SG_{Hg})(\gamma_{H_2O})h_3 = 0$$

For given values

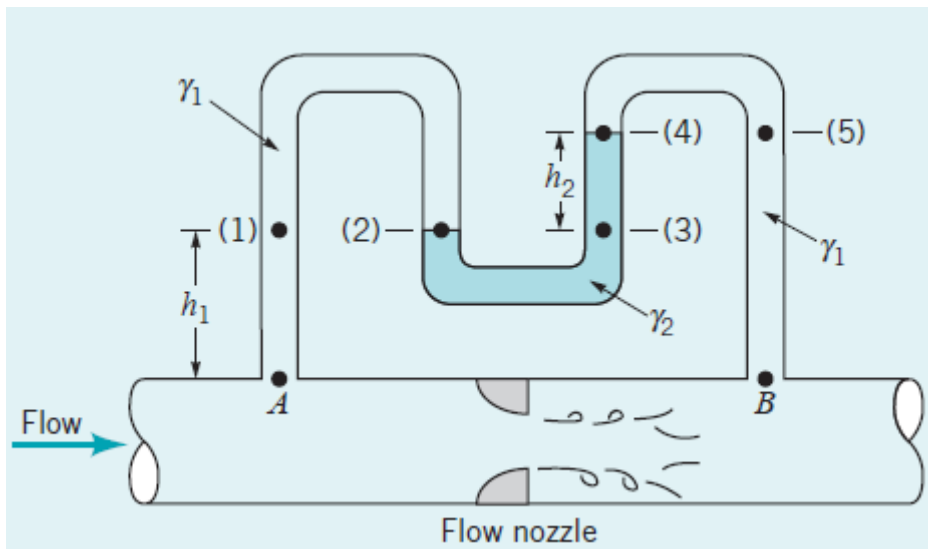
$$P_{air} = -(SG_{oil})(\gamma_{H_2O})(h_1 + h_2) + (SG_{Hg})(\gamma_{H_2O})h_3$$

$$P_{air} = -(0.92)(9810)(1.05) + (13.6)(9810) \times 0.23 = 21209 \text{ Pa}$$

So that $P_{air} = 21209 Pa$

Note that the air pressure is a function of the height of the mercury in the manometer and the depth of the oil (both in the tank and in the tube). It is not just the mercury in the manometer that is important.

Example: The volume rate of flow, Q , through a pipe can be determined by means of a flow nozzle located in the pipe as illustrated in Fig. a) Determine an equation for $P_A - P_B$ in terms of the specific weight of the following fluid, γ_1 , the specific weights of the gage fluid, γ_2 , and the various heights indicated. b) For $\gamma_1 = 9810 \frac{N}{m^3}$, $\gamma_2 = 15600 \frac{N}{m^3}$, $h_1 = 1.1 m$, $h_2 = 0.6 m$
What is the value of the pressure drop, $P_A - P_B$?



Solution: Although the fluid in the pipe is moving, the fluids in the columns of the manometer are at rest so that the pressure variation in the manometer tubes is hydrostatic. If we start at point A and move vertically upward to level (1), the pressure will decrease by $\gamma_1 h_1$ and will be equal to the pressure at (2) and at (3). We can now move from (3) to (4) where the pressure has been further reduced by $\gamma_2 h_2$. The pressures at levels (4) and (5) are equal, and as we move from (5) to B the pressure will increase by $\gamma(h_1 + h_2)$. Thus, in equation form

$$\text{a) } P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1 (h_1 + h_2) = P_B \quad \text{or} \quad P_A - P_B = h_2 (\gamma_2 - \gamma_1)$$

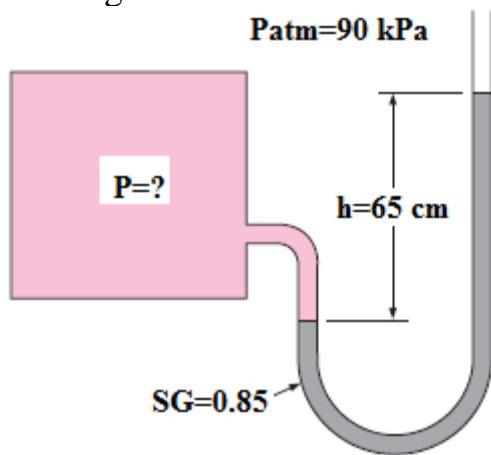
b) The specific value of the pressure drop for the data given is

$$P_A - P_B = 0.6 (15600 - 9810) = 3474 Pa$$

Example: A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 65 cm, as

shown in Fig.. If the local atmospheric pressure is 90 kPa, determine the absolute pressure within the tank.

Solution: The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined. The fluid in the tank is a gas whose density is much lower than the density of manometer fluid. The specific gravity of the manometer fluid is given to be 0.85. We take the standard density of water to be 1000 kg/m^3 . The density of the fluid is obtained by multiplying its specific gravity by the density of water, which is taken to be 1000 kg/m^3 :



$$P = \rho gh + P_{atm} = 0.85 \times 1000 \times 9.81 \times 0.65 + 90\,000 = 95420 \text{ Pa}$$

3.6.3. Inclined-Tube Manometer

To measure small pressure changes, a manometer of the type shown in Fig. 3.20 is frequently used. One leg of the manometer is inclined at an angle θ , and the differential reading l_2 is measured along the inclined tube. The difference in pressure $P_A - P_B$ can be expressed as

$$P_A + \gamma_1 h_1 - \gamma_2 l_2 \sin\theta - \gamma_3 h_3 = P_B \quad \text{or} \quad P_A - P_B = \gamma_2 l_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

Where; it is to be noted the pressure difference between points (1) and (2) is due to the *vertical* distance between the points, which can be expressed as $l_2 \sin\theta$. Thus, for relatively small angles the differential reading along the inclined tube can be made large even for small pressure differences. The inclined-tube manometer is often used to measure small differences in gas pressures so that if pipes *A* and *B* contain a gas then

$$P_A - P_B = \gamma_2 l_2 \sin\theta \quad l_2 = \frac{P_A - P_B}{\gamma_2 \sin\theta}$$

Where the contributions of the gas columns h_1 and h_3 have been neglected. The above equation 2.16 and the Fig. 3.20 show that the differential reading l_2 (for a given pressure difference) of the inclined-tube manometer can be increased over that obtained with a conventional U-tube manometer by the factor $1/\sin\theta$.

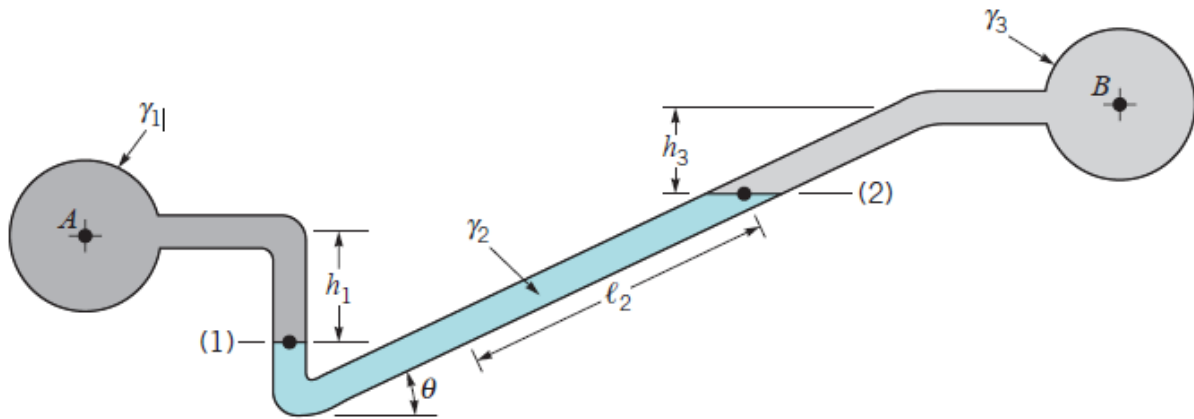


Figure 3.20. Inclined-tube manometer

3.7. Hydrostatic Force on a Plane Surface

The **fluid statics** deals with problems associated with fluids at rest. The fluid can be either gaseous or liquid. Fluid statics is generally referred to as *hydrostatics* when the fluid is a liquid and as *aerostatics* when the fluid is a gas. In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the *normal stress*, which is the pressure, and the variation of pressure is due only to the weight of the fluid. Therefore, the topic of fluid statics has significance only in gravity fields, and the force relations developed naturally involve the gravitational acceleration g . The force exerted on a surface by a fluid at rest is normal to the surface at the point of contact since there is no relative motion between the fluid and the solid surface, and thus no shear forces can act parallel to the surface.

Fluid statics is used to determine the forces acting on floating or submerged bodies and the forces developed by devices like hydraulic presses and car jacks. The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics. The complete description of the resultant hydrostatic force acting on a submerged surface requires the determination of the magnitude, the direction, and the line of action of the force. We consider the forces acting on both plane and curved surfaces of submerged bodies due to pressure.

When a surface is submerged in a fluid, forces develop on the surface due to the fluid. The determination of these forces is important in the design of storage tanks,

ships, dams, and other hydraulic structures. For fluids at rest we know that the force must be *perpendicular* to the surface since there are no shearing stresses present. We also know that the pressure will vary linearly with depth as shown in the below Fig.3.21 if the fluid is incompressible.

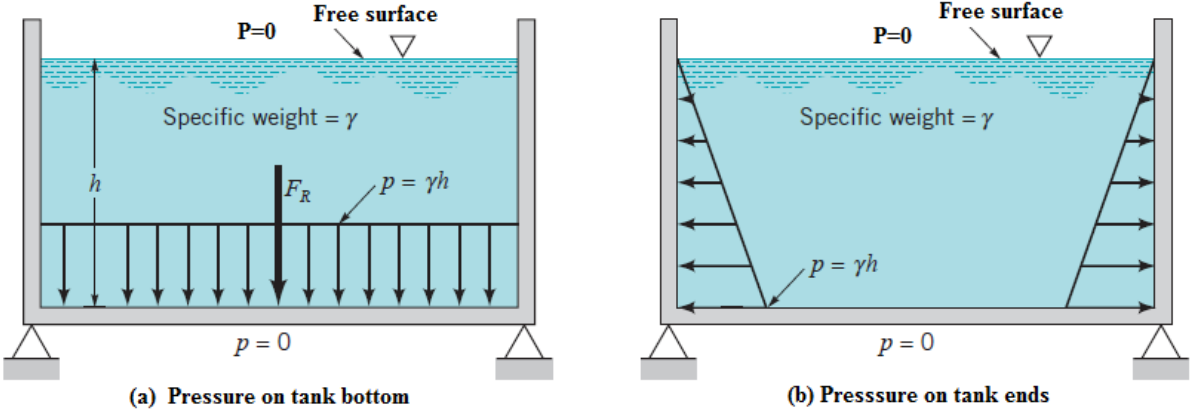


Figure 3.21. (a) Pressure distribution and resultant hydrostatic force on the bottom of an open tank. (b) Pressure distribution on the ends of an open tank.

For a horizontal surface, such as the bottom of a liquid-filled tank (Fig.3.21a), the magnitude of the resultant force is simply $F_R = PA$ where P is the uniform pressure on the bottom and A is the area of the bottom. For the open tank shown, $P = \gamma h$. Note that if atmospheric pressure acts on both sides of the bottom, as is illustrated, the *resultant* force on the bottom is simply due to the liquid in the tank. Since the pressure is constant and uniformly distributed over the bottom, the resultant force acts through the centroid of the area as shown in Fig.3.21a. As shown in Fig. 3.21b, the pressure on the ends of the tank is not uniformly distributed. Determination of the resultant force for situations such as this is presented below.

For the more general case in which a submerged plane surface is inclined, as is illustrated in the below figure, the determination of the resultant force acting on the surface is more involved. For the present we will assume that the fluid surface is open to the atmosphere. Let the plane in which the surface lies intersect the free surface at 0 and make an angle with this surface as in Fig.3.22. The x - y coordinate system is defined so that 0 is the origin and $y=0$ (i.e., the x -axis) is directed along the surface as shown. The area can have an arbitrary shape as shown. We wish to determine the direction, location, and magnitude of the resultant force acting on one side of this area due to the liquid in contact with the area. At any given depth, h , the force acting on dA (the differential area of Fig.3.22) is $dF = \gamma h dA$ and is perpendicular to the surface. Thus, the magnitude of the resultant force can be found by summing these differential forces over the entire surface.

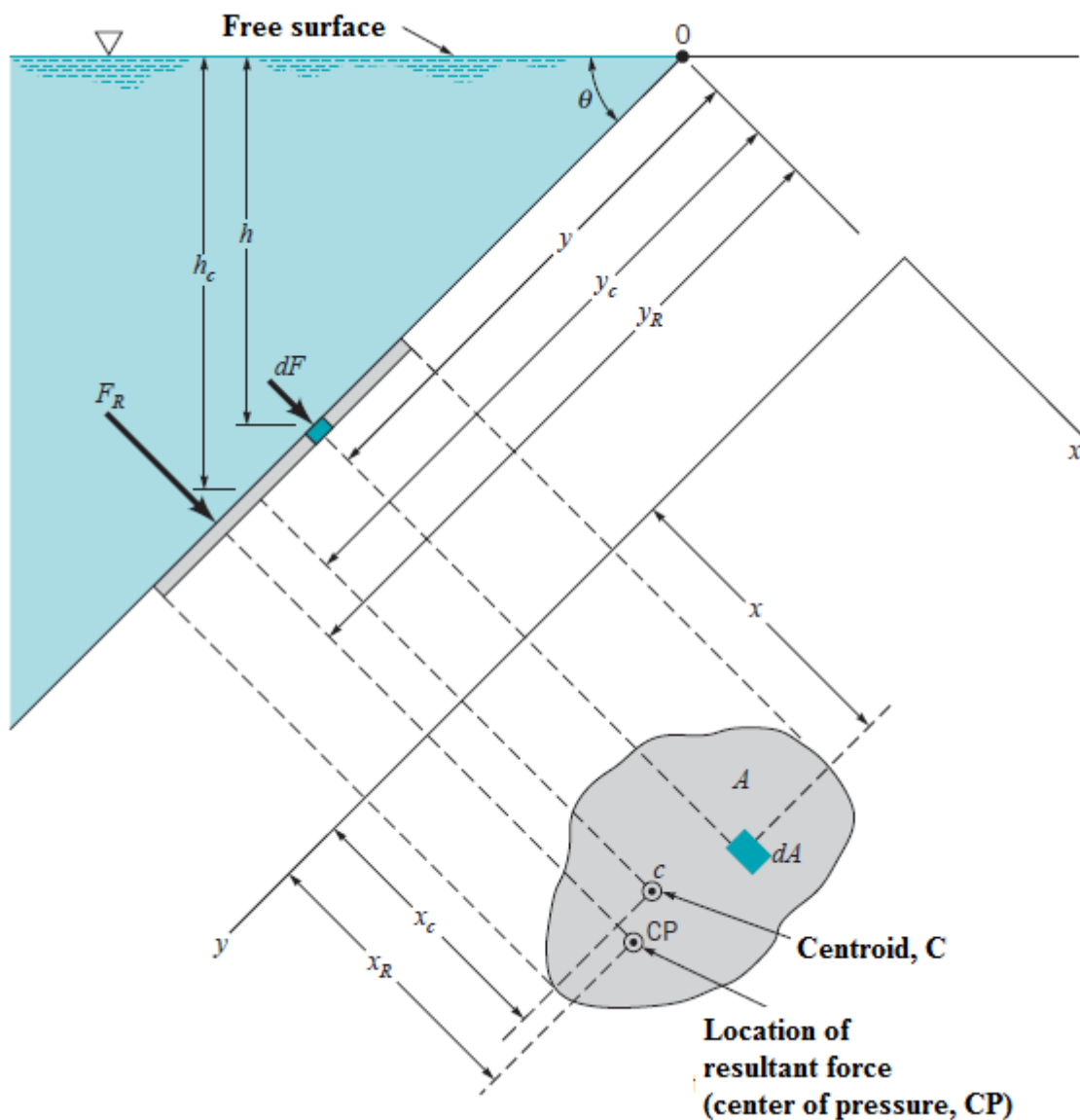


Figure 3.22. Notation for hydrostatic force on an inclined plane surface of arbitrary shape.

In equation form

$$F_R = \int_A \gamma h dA = \int_A \gamma y \sin \theta dA$$

Where; $h = y \sin \theta$ For constant γ and θ ;

$$F_R = \gamma \sin \theta \int_A y dA$$

The integral appearing in this equation is the *first moment of the area* with respect to the x axis, so we can write $\int_A y dA = y_c A$

where y_c is the y coordinate of the centroid of area A measured from the x axis which passes through 0. The last equation can thus be written as

$$F_R = \gamma A y_c \sin\theta \text{ or more simply as } F_R = \gamma h_c A$$

Where; h_c is the vertical distance from the fluid surface to the centroid of the area. Note that the magnitude of the force is independent of the angle θ . As indicated by the below Fig.3.23, it depends only on the specific weight of the fluid, the total area, and the depth of the centroid of the area below the surface. In effect, ***the last equation indicates that the magnitude of the resultant force is equal to the pressure at the centroid of the area multiplied by the total area.*** Since all the differential forces that were summed to obtain are perpendicular to the surface, the resultant must also be perpendicular to the surface.

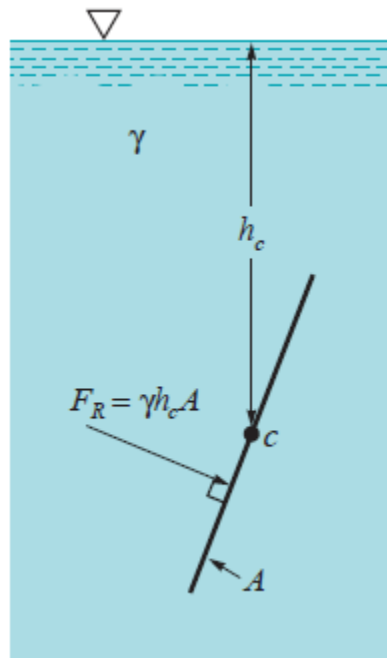


Figure 3.23. The magnitude of the resultant fluid force is equal to the pressure acting at the centroid of the area multiplied by the total area.

Although our intuition might suggest that the resultant force should pass through the centroid of the area, this is not actually the case. The y coordinate, y_R , of the resultant force can be determined by summation of moments around the x axis. That is, the moment of the resultant force must equal the moment of the distributed pressure force, or $F_R y_R = \int_A y dF = \int_A \gamma \sin\theta y^2 dA$ and,

$$\text{therefore, since } F_R = \gamma A y_c \sin\theta \quad y_R = \frac{\int_A y^2 dA}{y_c A}$$

The integral in the numerator is the *second moment of the area (moment of inertia)*, I_x , with respect to an axis formed by the intersection of the plane containing the surface and the free surface (x axis). Thus, we can write

$$y_R = \frac{I_x}{y_c A} \cdot \gamma \sin \theta$$

$$I_x = I_{xc} + Ay_c^2$$

Where I_{xc} is the second moment of the area with respect to an axis passing through its *centroid* and parallel to the x axis. Thus,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

As shown by this equation and the below figure, the resultant force does not pass through the centroid but for nonhorizontal surfaces is always *below* it, since $\frac{I_{xc}}{y_c A} > 0$.

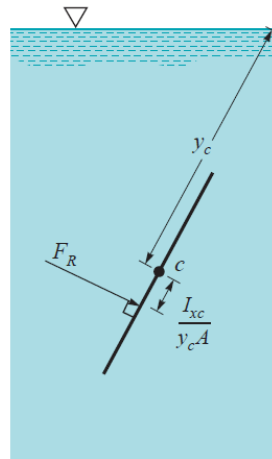


Figure 3.24. The resultant force does not pass through the centroid but for nonhorizontal surfaces is always *below* it

The x coordinate, x_R , for the resultant force can be determined in a similar manner by summing moments about the y axis,

$$F_R x_R = \int_A \gamma \sin \theta xy dA \quad \text{and, therefore} \quad x_R = \frac{\int_A xy dA}{y_c A} = \frac{I_{xy}}{y_c A}$$

where I_{xy} is the product of inertia with respect to the x and y axes. Again, using the parallel axis theorem, we can write

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

Where; I_{xyc} is the product of inertia with respect to an orthogonal coordinate system passing through the *centroid* of the area and formed by a translation of the x - y coordinate system. If the submerged area is symmetrical with respect to an axis passing through the centroid and parallel to either the x or y axes, the resultant

force must lie along the line $x = x_c$, since I_{xy_c} is identically zero in this case. The point through which the resultant force acts is called the **center of pressure**. It is to be noted from above equations that as y_c increases the center of pressure moves closer to the centroid of the area. Since $y_c = h_c / \sin\theta$, the distance y_c will increase if the depth of submergence, h_c increases, or, for a given depth, the area is rotated so that the angle, θ , decreases. Thus, the hydrostatic force on the right-hand side of the gate shown in the figure acts closer to the centroid of the gate than the force on the left-hand side. Centroidal coordinates and moments of inertia for some common areas are given in Fig.3.25.

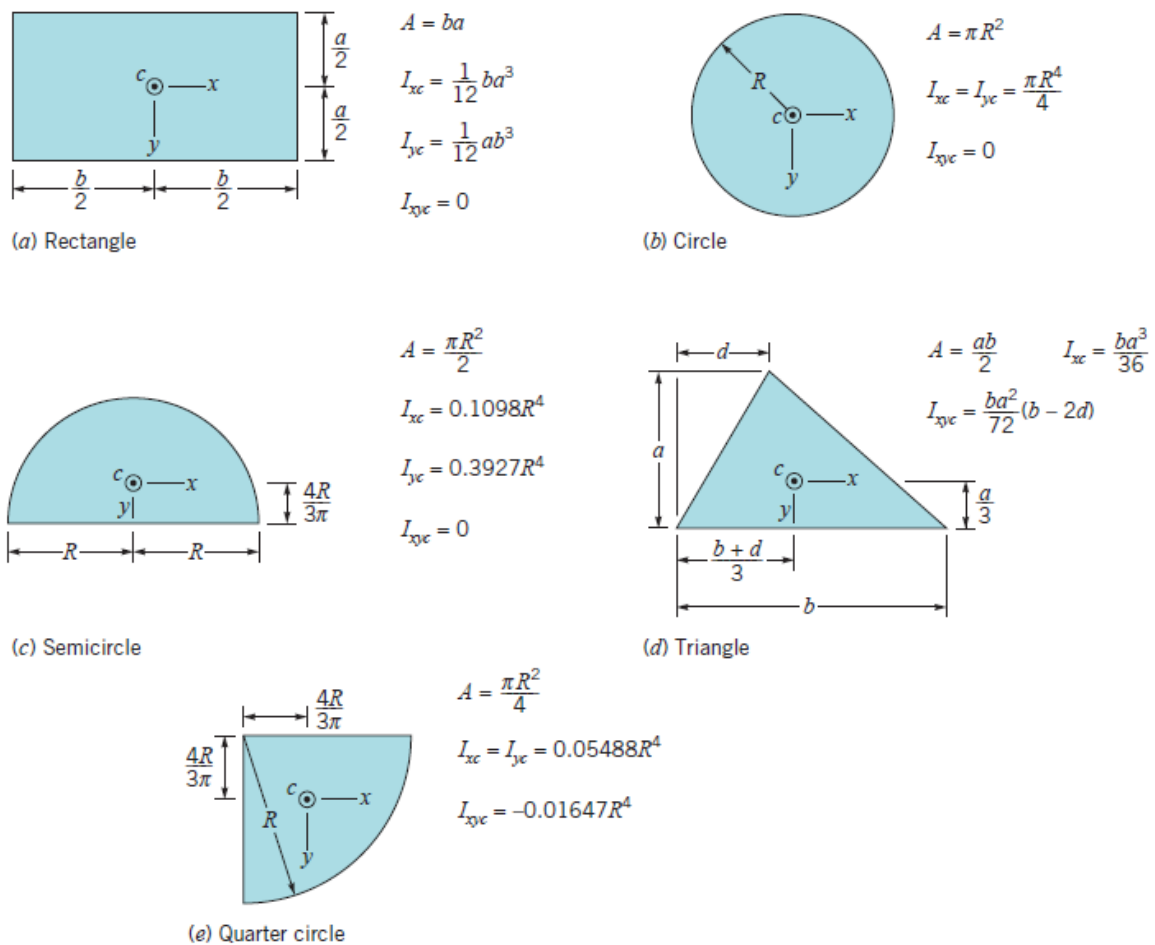
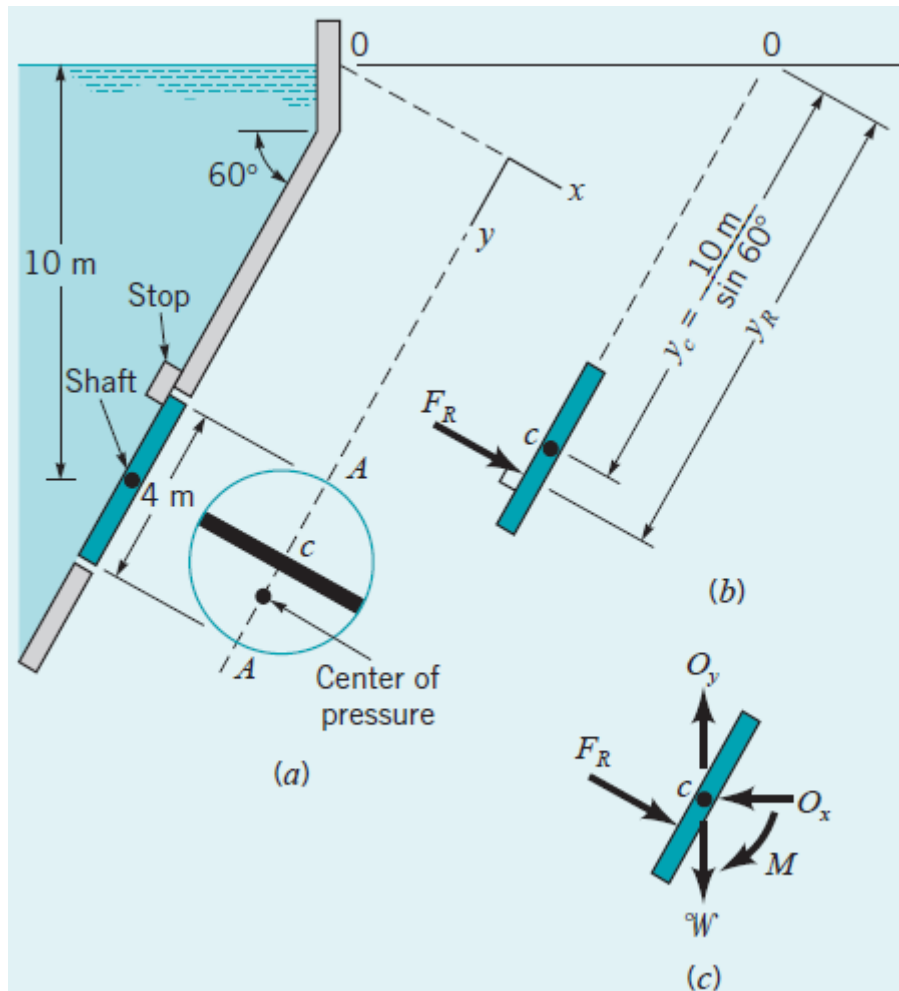


Figure 3.25. Geometric properties of some common shapes (Recall that the parallel axis theorem for the product of inertia of an area states that the product of inertia with respect to an orthogonal set of axes (x - y coordinate system) is equal to the product of inertia with respect to an orthogonal set of axes parallel to the original set and passing through the centroid of the area, plus the product of the area and the x and y coordinates of the centroid of the area. Thus, $I_{xy} = I_{xy_c} + Ax_c y_c$.

Example: The 4-m-diameter circular gate of Fig. *a* is located in the inclined wall of a large reservoir containing water $\gamma=9810 \text{ N/m}^3$. The gate is mounted on a shaft along its horizontal diameter, and the water depth is 10 m above the shaft. Determine (a) the magnitude and location of the resultant force exerted on the gate by the water and (b) the moment that would have to be applied to the shaft to open the gate.



Solution: To find the magnitude of the force of the water we can apply the following formula. $F_R = \gamma h_c A$ and since the vertical distance from the fluid surface to the centroid of the area is 10 m, it follows that

$$F_R = 9810 \times 10 \times \frac{\pi 4^2}{4} = 1232761 \text{ N}$$

To locate the point (center of pressure) through which F_R acts, we use the following equations.

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \quad y_R = \frac{I_{xc}}{y_c A} + y_c$$

For the coordinate system shown, $x_R = 0$ since the area is symmetrical, and the center of pressure must lie along the diameter A-A. To obtain y_R , $I_{xc} = \frac{\pi R^4}{4}$ and y_c is shown in Fig. b.

$$y_R = \frac{\left(\frac{\pi}{4}\right)2^4}{\left(\frac{10}{\sin 60}\right)(4\pi)} + \frac{10}{\sin 60} = 0.0866 + 11.547 = 11.634 \text{ m}$$

and the distance (along the gate) below the shaft to the center of pressure is $y_R - y_c = 0.0866 \text{ m}$

We can conclude from this analysis that the force on the gate due to the water has a magnitude of 1232761 N and acts through a point along its diameter A-A at a distance of 0.0866 m (along the gate) below the shaft. The force is perpendicular to the gate surface as shown in Fig. b.

Example: A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig.). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

Solution: A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed. The density of lake water to be 1000 kg/m^3 throughout. The average pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

$$P_{ave} = P_c = \rho g h_c = \rho g \left(s + \frac{b}{2} \right) = 1000 \times 9.81 \left(8 + \frac{1.1}{2} \right) = 83875.5 \text{ Pa}$$

Then the resultant hydrostatic force on the door becomes

$$F_R = P_{ave} A = 83875.5 \times (1 \times 1.2) = 100650.6 \text{ N}$$

The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from below equation

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(1)(1.2)^3}{8.6 \times (1 \times 1.2)} + 8.6 = 0.014 + 8.6 = 8.614 \text{ m}$$

3.7.1. Pressure Prism

An informative and useful graphical interpretation can be made for the force developed by a fluid acting on a plane rectangular area. Consider the pressure distribution along a vertical wall of a tank of constant width b , which contains a liquid having a specific weight γ . Since the pressure must vary linearly with depth, we can represent the variation as is shown in Fig.3.26 *a*, where the pressure is equal to zero at the upper surface and equal to γh at the bottom.

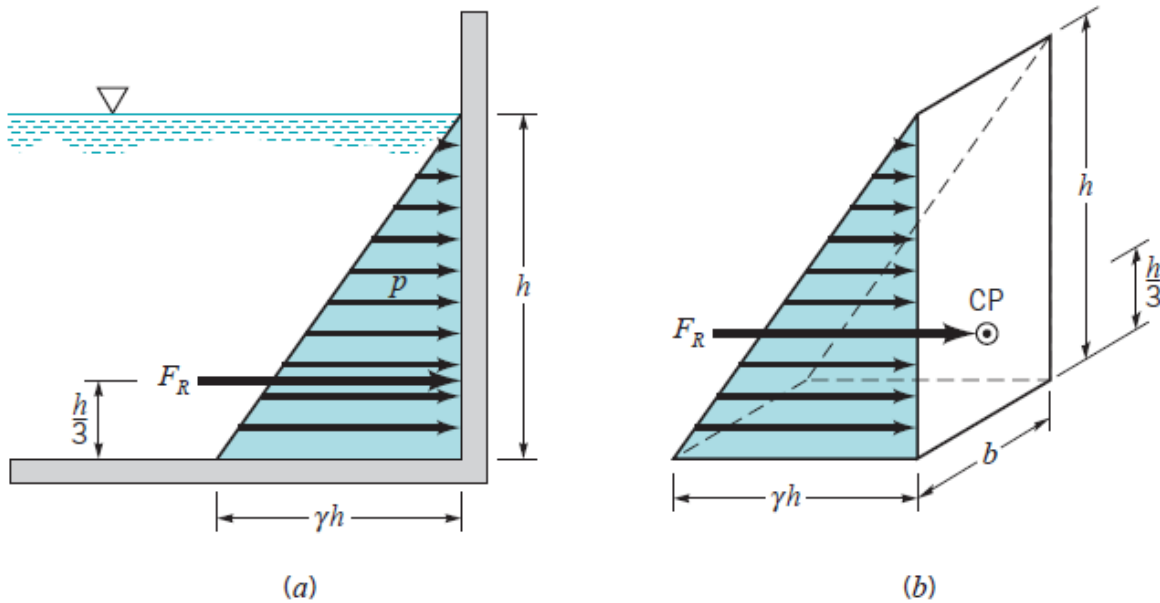


Figure 3.26. Pressure prism for vertical rectangular area.

It is apparent from this diagram that the average pressure occurs at the depth $h/2$ and therefore the resultant force acting on the rectangular area $A = bh$ is

$$F_R = P_{ave}A = \gamma \left(\frac{h}{2} \right) A$$

which is the same result as obtained from $F_R = \gamma h_c A$. The pressure distribution shown in Fig.3.26*a* applies across the vertical surface so we can draw the three-dimensional representation of the pressure distribution as shown in Fig. 3.26*b*. The base of this “volume” in pressure-area space is the plane surface of interest, and its altitude at each point is the pressure. This volume is called the **pressure prism**, and it is clear that the magnitude of the resultant force acting on the rectangular surface is equal to the volume of the pressure prism. Thus, for the prism of Fig. 3.26*b* the fluid force is

$$F_R = volume = \frac{1}{2}(\gamma h)(bh) = \gamma \left(\frac{h}{2} \right) A$$

Where bh is the area of the rectangular surface, A .

The resultant force must pass through the **centroid** of the pressure prism. For the volume under consideration the centroid is located along the vertical axis of

symmetry of the surface, and at a distance of $h/3$ above the base (since the centroid of a triangle is located at $h/3$ above its base). This result can readily be shown to be consistent with that obtained from Eqs. y_R and x_R .

This same graphical approach can be used for plane rectangular surfaces that do not extend up to the fluid surface, as illustrated in Fig.3.27a. In this instance, the cross section of the pressure prism is trapezoidal. However, the resultant force is still equal in magnitude to the volume of the pressure prism, and it passes through the centroid of the volume. Specific values can be obtained by decomposing the pressure prism into two parts, $ABDE$ and BCD , as shown in Fig.3.27b. Thus,

$$F_R = F_1 + F_2$$

Where the components can readily be determined by inspection for rectangular surfaces. The location of F_R can be determined by summing moments about some convenient axis, such as one passing through A . In this instance

$$F_R y_A = F_1 y_1 + F_2 y_2$$

and y_1 and y_2 can be determined by inspection.

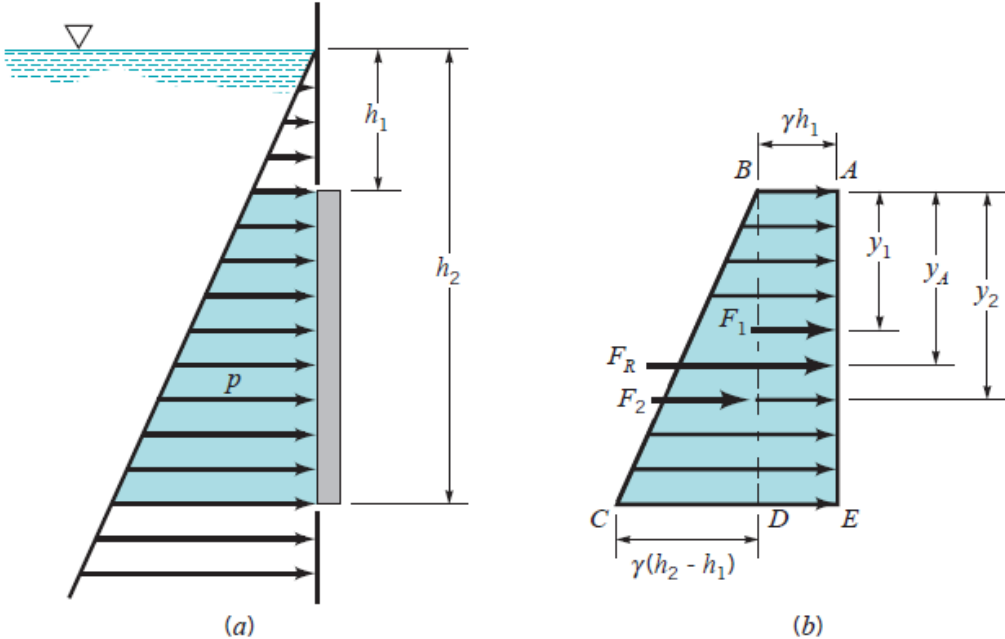


Figure 3.27. Graphical representation of hydrostatic forces on a vertical rectangular surface

For inclined plane rectangular surfaces the pressure prism can still be developed, and the cross section of the prism will generally be trapezoidal, as is shown in Fig.3.28. Although it is usually convenient to measure distances along the inclined surface, the pressures developed depend on the vertical distances as illustrated.

The use of pressure prisms for determining the force on submerged plane areas is convenient if the area is rectangular so the volume and centroid can be easily determined. However, for other nonrectangular shapes, integration would generally be needed to determine the volume and centroid. In these circumstances it is more convenient to use the equations developed in the previous section, in which the necessary integrations have been made and the results presented in a convenient and compact form that is applicable to submerged plane areas of any shape.

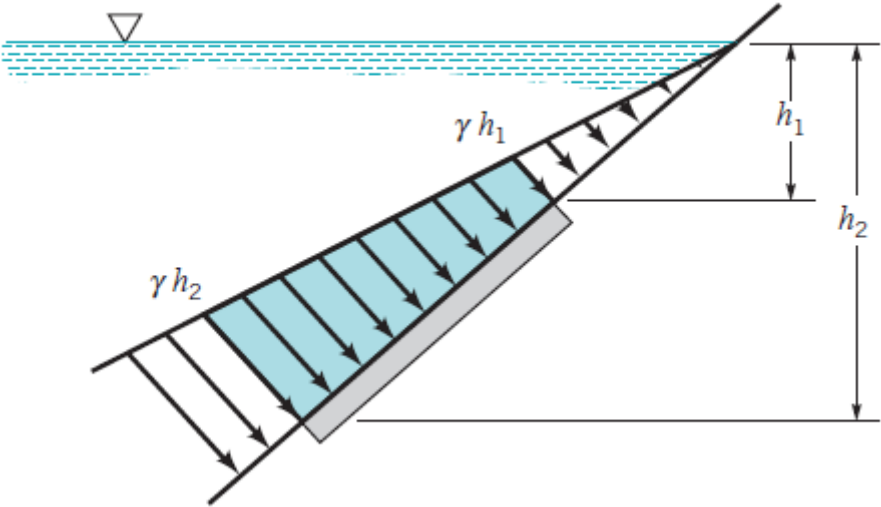


Figure 3.28. Pressure variation along an inclined plane area.

The effect of atmospheric pressure on a submerged area has not yet been considered, and we may ask how this pressure will influence the resultant force. If we again consider the pressure distribution on a plane vertical wall, as is shown in Fig.3.29a, the pressure varies from zero at the surface to gh at the bottom. Since we are setting the surface pressure equal to zero, we are using atmospheric pressure as our datum, and thus the pressure used in the determination of the fluid force is gage pressure. If we wish to include atmospheric pressure, the pressure distribution will be as is shown in Fig.3.29 b.

We note that in this case the force on one side of the wall now consists of F_R as a result of the hydrostatic pressure distribution, plus the contribution of the atmospheric pressure, $P_{atm} A$, where A is the area of the surface. However, if we are going to include the effect of atmospheric pressure on one side of the wall, we must realize that this same pressure acts on the outside surface (assuming it is exposed to the atmosphere), so that an equal and opposite force will be developed as illustrated in the figure.

Thus, we conclude that the *resultant* fluid force on the surface is that due only to the gage pressure contribution of the liquid in contact with the surface—the atmospheric pressure does not contribute to this resultant. Of course, if the surface pressure of the liquid is different from atmospheric pressure (such as might occur in a closed tank), the resultant force acting on a submerged area, A , will be changed in magnitude from that caused simply by hydrostatic pressure by an amount $P_s A$, where P_s is the gage pressure at the liquid surface (the outside surface is assumed to be exposed to atmospheric pressure).

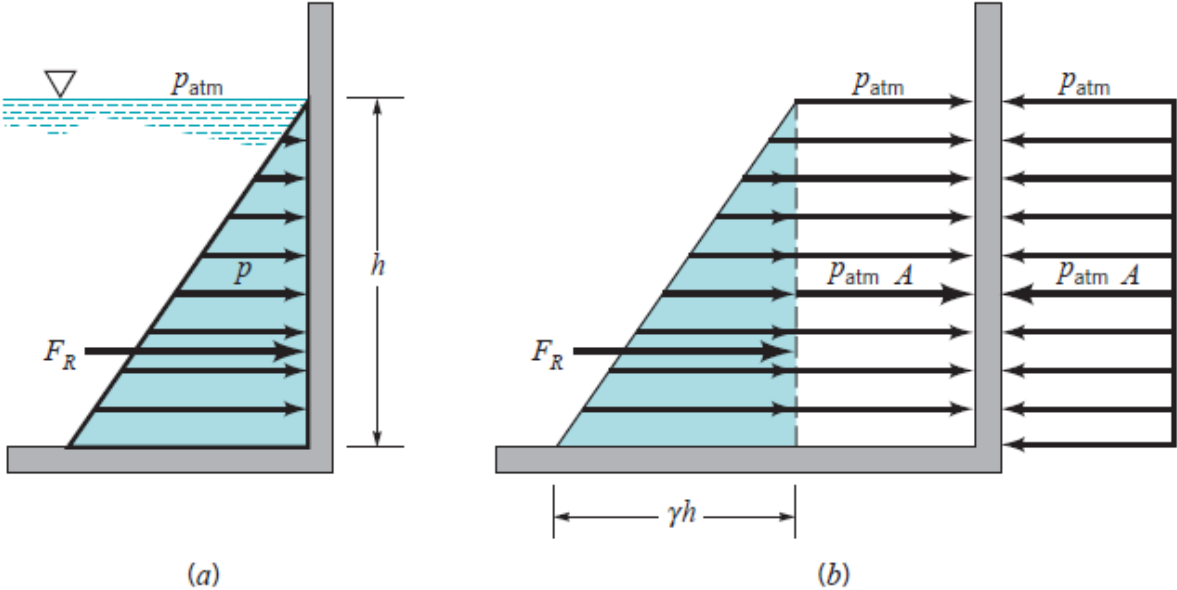


Figure 3.29. Effect of atmospheric pressure on the resultant force acting on a plane vertical wall.

Example: A pressurized tank contains oil ($SG=0.90$) and has a square, 0.6-m by 0.6-m plate bolted to its side, as is illustrated in Fig. *a*. The pressure gage on the top of the tank reads 50 kPa, and the outside of the tank is at atmospheric pressure. What is the magnitude and location of the resultant force on the attached plate?

Solution: The pressure distribution acting on the inside surface of the plate is shown in Fig. *b*. The pressure at a given point on the plate is due to the air pressure, P_s , at the oil surface, and the pressure due to the oil, which varies linearly with depth as is shown in the figure. The resultant force on the plate (having an area A) is due to the components, where F_1 and F_2 are due to the rectangular and triangular portions of the pressure distribution, respectively.

