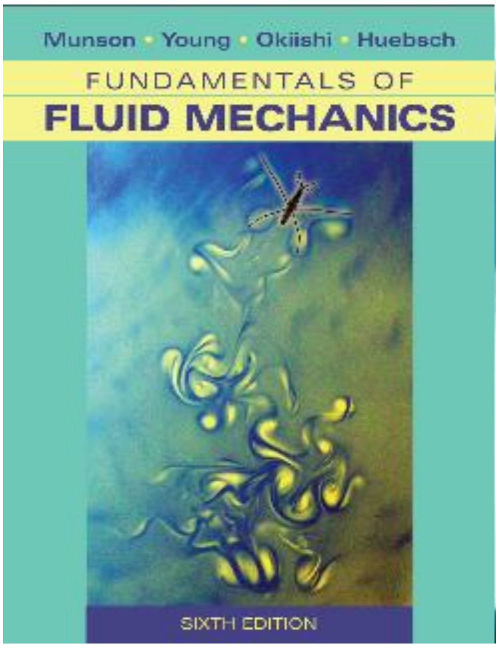
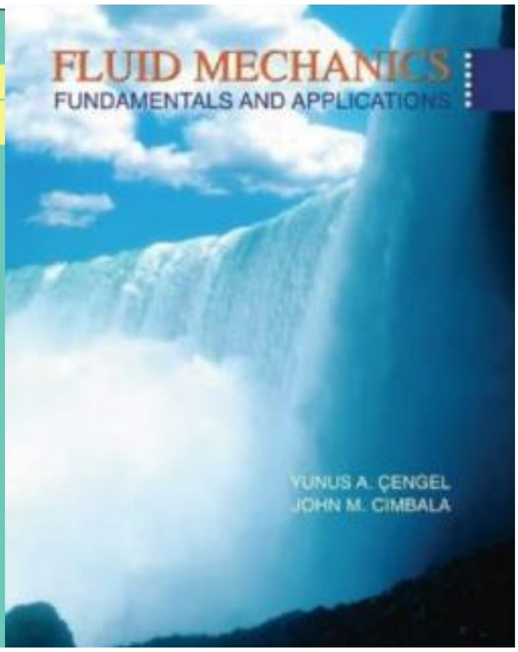
FLUID MECHANICS





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3-FLUID STATICS

3.8. Hydrostatic Force on a Curved Surface

For a submerged curved surface, the determination of the resultant hydrostatic force is more involved since it typically requires the integration of the pressure forces that change direction along the curved surface. The concept of the pressure prism in this case is not much help either because of the complicated shapes involved.

Consider a curved portion of the swimming pool shown in Fig.3.30*a*. We wish to find the resultant fluid force acting on section *BC* (which has a unit length perpendicular to the plane of the paper) shown in Fig.3.30*b*. We first isolate a volume of fluid that is bounded by the surface of interest, in this instance section *BC*, the horizontal plane surface *AB*, and the vertical plane surface *AC*. The free-body diagram for this volume is shown in Fig. 3.30*c*. The magnitude and location of forces F_1 and F_2 can be determined from the relationships for planar surfaces. The weight, W, is simply the specific weight of the fluid times the enclosed volume and acts through the center of gravity (CG) of the mass of fluid contained within the volume.

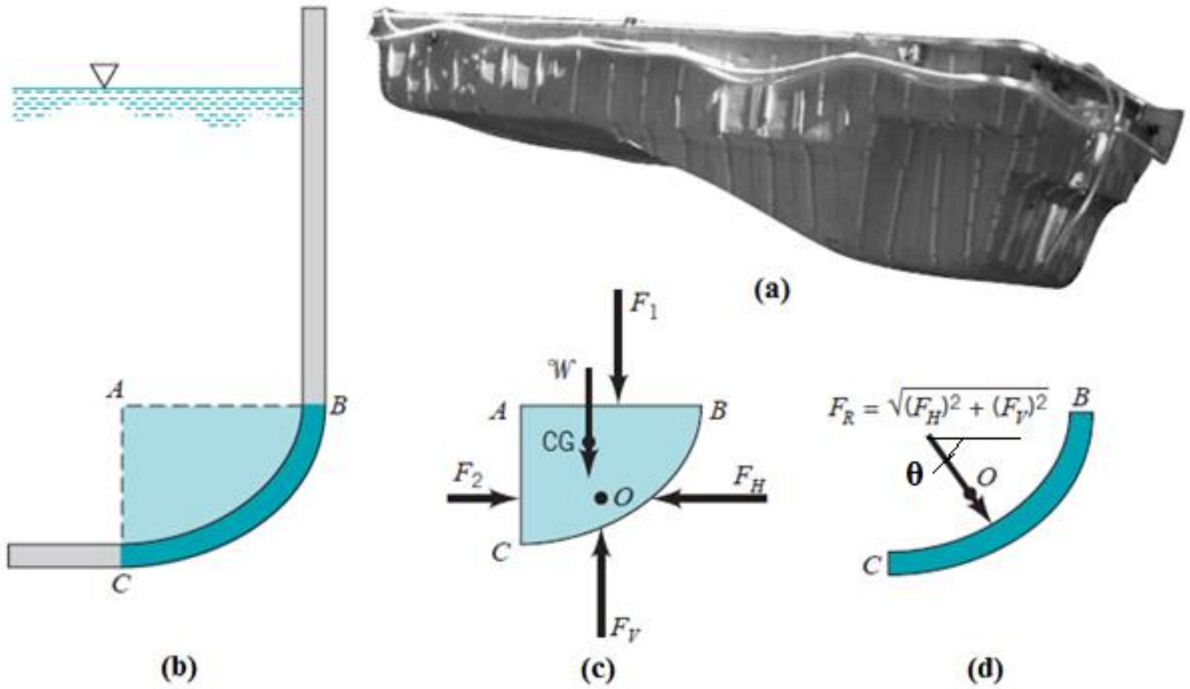


Figure 3.30. Hydrostatic force on a curved surface.

The forces F_H and F_V represent the components of the force that the tank *exerts on the fluid*. In order for this force system to be in equilibrium, the horizontal component must be equal in magnitude and collinear with F_2, and the vertical component F_V equal in magnitude and collinear with the resultant of the vertical forces F_1 and W. This follows since the three forces acting on the fluid mass (F_2, the resultant of F_1 and W, the resultant force that the tank exerts on the mass) must form a *concurrent* force system. That is, from the principles of statics, it is known that when a body is held in equilibrium by three nonparallel forces, they must be concurrent (their lines of action intersect at a common point), and coplanar. Thus,

$$F_H = F_2 \qquad\qquad F_V = F_1 + W$$

and the magnitude of the resultant is obtained from the equation

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

The tangent of the angle it makes with the horizontal is

$$\theta = \arctan\left(\frac{F_H}{F_V}\right)$$

The resultant force F_R passes through the point *O*, which can be located by summing moments about an appropriate axis. The resultant force of the fluid acting *on the curved surface BC* is equal and opposite in direction to that obtained from the free-body diagram of Fig.3.30*c*. The desired fluid force is shown in Fig.3.30*d*

We conclude that

1. The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.

2. The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block. These discussions are valid for all curved surfaces regardless of whether they are above or below the liquid. Note that in the case of a *curved surface above a liquid*, the weight of the liquid is *subtracted* from the vertical component of the hydrostatic force since they act in opposite directions (Fig.3.31)

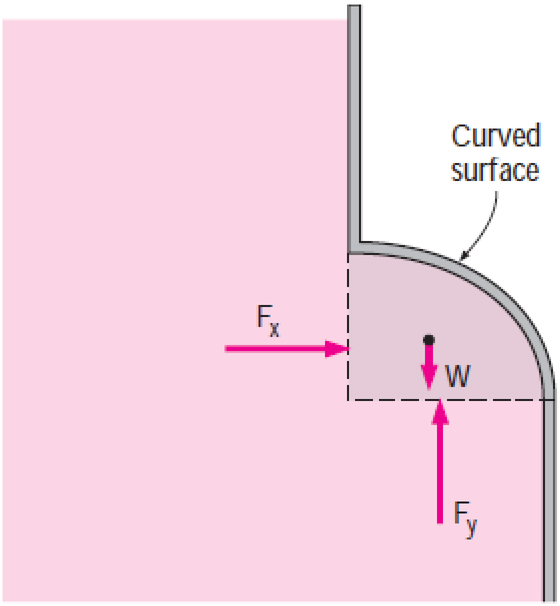
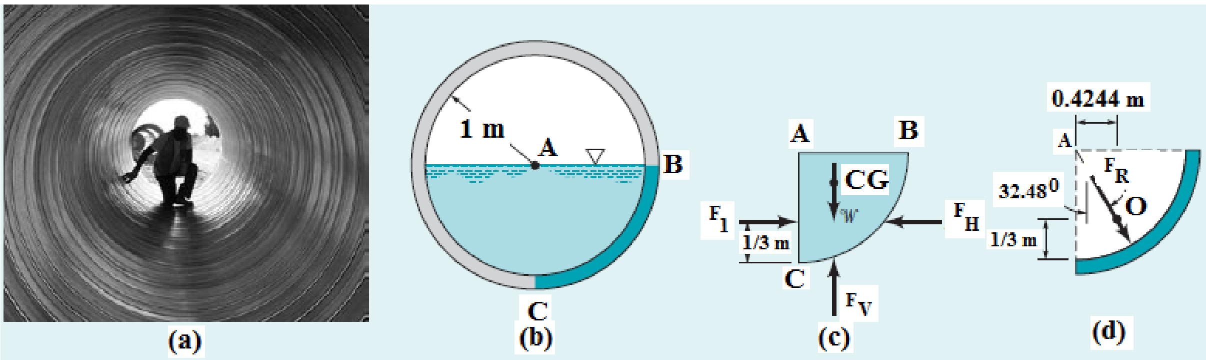


Figure 3.31. When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions

Example: A 2 m diameter drainage conduit of the type shown in Fig *a* is half full of water at rest, as shown in Fig. *b*. Determine the magnitude and line of action of the resultant force that the water exerts on a 1-m length of the curved section *BC* of the conduit wall.



Solution: We first isolate a volume of fluid bounded by the curved section *BC*, the horizontal surface *AB*, and the vertical surface *AC*, as shown in Fig.*c*. The volume has a length of 1 m. The forces acting on the volume are the horizontal force, F_1, which acts on the vertical surface *AC*, the weight,W, of the fluid contained within the volume, and the horizontal and vertical components of the force of the conduit wall on the fluid, F_H and F_V, respectively. The magnitude of F_1 is found from the equation.

$$F_1 = \gamma h_c A = 9810 \times \frac{1}{2} \times 1 \times 1 = 4905\ N$$

and this force acts 1/3 m above *C* as shown. The weight , $W = \gamma \forall$ where $\forall$ is the fluid volume, is

$$W = \gamma \forall = 9810 \times \frac{\pi 2^2}{4 \times 4} \times 1 = 7705\ N$$

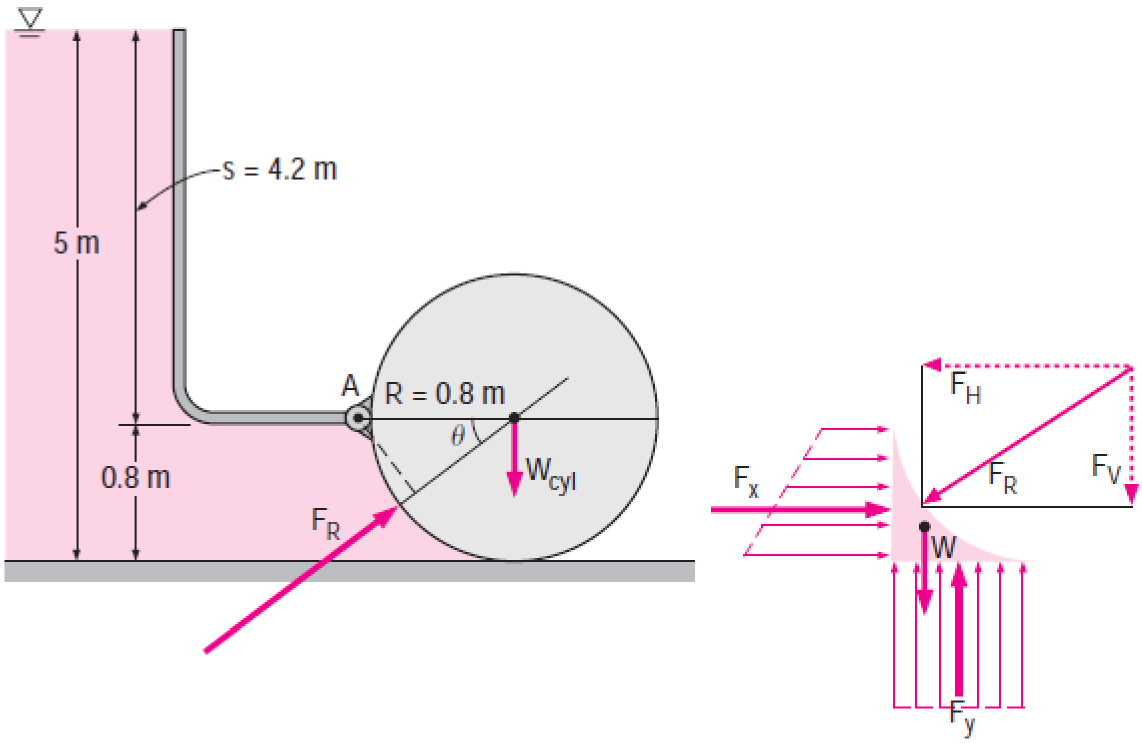
and acts through the center of gravity of the mass of fluid, which according to $\frac{4R}{3\pi} = \frac{4 \times 1}{3\pi} = 0.4244\ m$ to the right of *AC* as shown. Therefore, to satisfy equilibrium

$F_H = F_1 = 4905\ N$ $\qquad\qquad$ $F_V = W = 7705\ N$

$F_R = \sqrt{(4905)^2 + (7705)^2}$=9134 N $\qquad$ $\theta = \arctan\left(\frac{F_H}{F_V}\right) = \arctan\left(\frac{4905}{9134}\right) =$ 32.48°

The force the water exerts *on* the conduit wall is equal, but *opposite in direction*, to the forces F_H and F_V shown in Fig. *c*. Thus, the resultant force *on the conduit wall* is shown in Fig. *d*. This force acts through the point *O* at the angle shown.

Example: A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate, as shown in figure of the free-body diagram of the fluid underneath the cylinder When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per m length of the cylinder.



Solution: (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = 9810\times\left(4.2+\frac{0.8}{2}\right)(0.8\times1) = 36101\ N$$

Vertical force on horizontal surface (upward):

$$F_y = P_{ave}A = 9810\times5\times(0.8\times1) = 39240\ N$$

Weight of fluid block per m length (downward):

$$W = mg = \rho g\forall = \rho g\left(R^2-\frac{\pi R^2}{4}\right)(1)$$

$$W = 9810\left(0.8^2-0.8^2\times\frac{\pi}{4}\right)(1) = 1347\ N$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39240 - 1347 = 37893\ N$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

$$F_R = \sqrt{(36101)^2 + (37893)^2} = 52337 \text{ N}$$

The tangent of the angle it makes with the horizontal is

$$\theta = \arctan\left(\frac{F_V}{F_H}\right) = \arctan\left(\frac{37893}{36101}\right) = 46.4°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52 337 N per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.61° with the horizontal.

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R sin\theta = 52337 \times sin 46.4 = 37900\ N$$

3.9. Buoyancy and Floatation

It is a common experience that an object feels lighter and weighs less in a liquid than it does in air. Also, objects made of wood or other light materials float on water. These and other observations suggest that a fluid exerts an upward force on a body immersed in it. This force that tends to lift the body is called the ***buoyant force*** and is denoted by F_B.

The buoyant force is caused by the increase of pressure in a fluid with depth. Consider, for example, a flat plate of thickness *h* submerged in a liquid of density ρ_f parallel to the free surface, as shown in Fig.3.32.

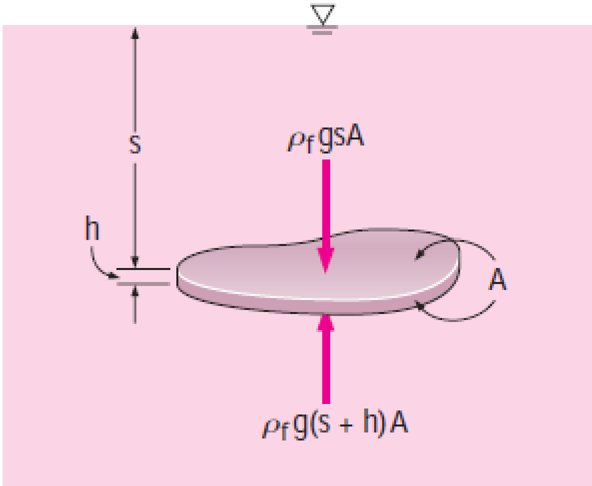


Figure 3.32. A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface

The area of the top (and also bottom) surface of the plate is *A*, and its distance to the free surface is *s*. The pressures at the top and bottom surfaces of the plate are $\rho_f\, g\, s$ and $\rho_f\, g\, (s+ h)$, respectively. Then the hydrostatic force $F_{top}= \rho_f gsA$ acts downward on the top surface, and the larger force F bottom: ρf g (s+h) A acts upward on the bottom surface of the plate. The difference between these two forces is a net upward force, which is the *buoyant force*,

$$F_B=F_{bottom}-F_{top}=\rho_f\, g\, (s+h)\, A-\, \rho_f\, g\, h\, A=\, \rho_f\, g\, \forall$$

$$F_B = \gamma \forall$$

Where; F_B is the buoyant force (N), γ is the specific weight of fluid (N/m^3), and ∀ is the volume of the body (m^3)

The direction of the buoyant force, which is the force of the fluid *on the body*, is opposite to that shown on the freebody diagram. Therefore, the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward.

The weight of the liquid whose volume is equal to the volume of the plate. We conclude that *the buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.* Note that the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body.
The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body. That is,

$$F_B = W \rightarrow \rho_f g \forall_{sub} = \rho_{ave,body} g \forall_{total} \rightarrow \frac{\forall_{sub}}{\forall_{total}} = \frac{\rho_{ave,body}}{\rho_f}$$

Therefore, the submerged volume fraction of a floating body is equal to the ratio of the average density of the body to the density of the fluid. Note that when the density ratio is equal to or greater than one, the floating body becomes completely submerged.

It follows from these discussions that a body immersed in a fluid

1) Remains at rest at any point in the fluid when its density is equal to the density of the fluid,
2) Sinks to the bottom when its density is greater than the density of the fluid, and
3) Rises to the surface of the fluid and floats when the density of the body is less than the density of the fluid (Fig.3.33).

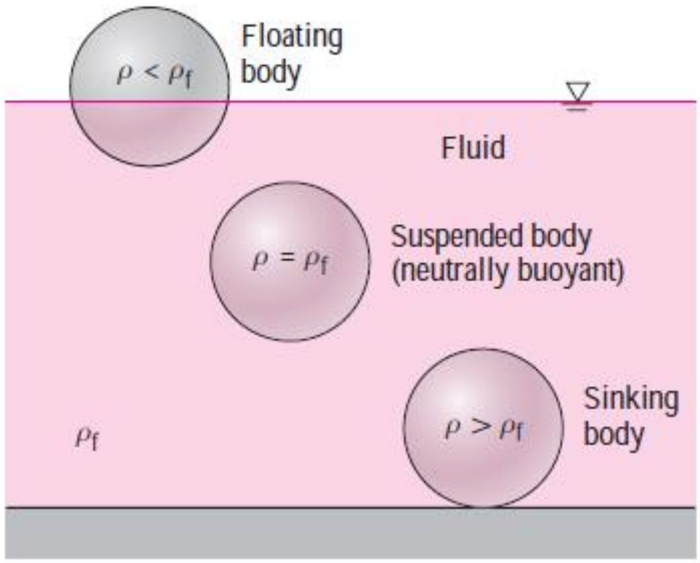


Figure 3.33. A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its density relative to the density of the fluid.

The buoyant force is proportional to the density of the fluid, and thus we might think that the buoyant force exerted by gases such as air is negligible. This is certainly the case in general, but there are significant exceptions. For example, the volume of a person is about 0.1 m^3, and taking the density of air to be 1.2 kg/m^3, the buoyant force exerted by air on the person is

$$F_B = \rho g \forall = 1.2 \times 9.81 \times 0.1 = 1.2\ N$$

The weight of an 80-kg person is 80 × 9.81= 788 N. Therefore, ignoring the buoyancy in this case results in an error in weight of just 0.15 percent, which is negligible. But the buoyancy effects in gases dominate some important natural phenomena such as the rise of warm air in a cooler environment and thus the onset of natural convection currents, the rise of hot-air or helium balloons, and air movements in the atmosphere. A helium balloon, for example, rises as a result of the buoyancy effect until it reaches an altitude where the density of air (which decreases with altitude) equals the density of helium in the balloon—assuming the balloon does not burst by then, and ignoring the weight of the balloon's skin. Archimedes' principle is also used in modern geology by considering the continents to be floating on a sea of magma.

Example: A crane is used to lower weights into the sea (density = 1025 kg/m^3) for an underwater construction project (Fig.). Determine the tension in the rope of the crane due to a rectangular 0.4×0.4×3 m concrete block (density= 2300 kg/m^3) when it is (*a*) suspended in the air and (*b*) completely immersed in water. The buoyancy of air is negligible. The weight of the ropes is negligible.

Solution: A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is in water. (*a*) Consider the free-body diagram of the concrete block. The forces acting on the concrete block in air are its weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$\forall = 0.4 \times 0.4 \times 3 = 0.48\ m^3$$

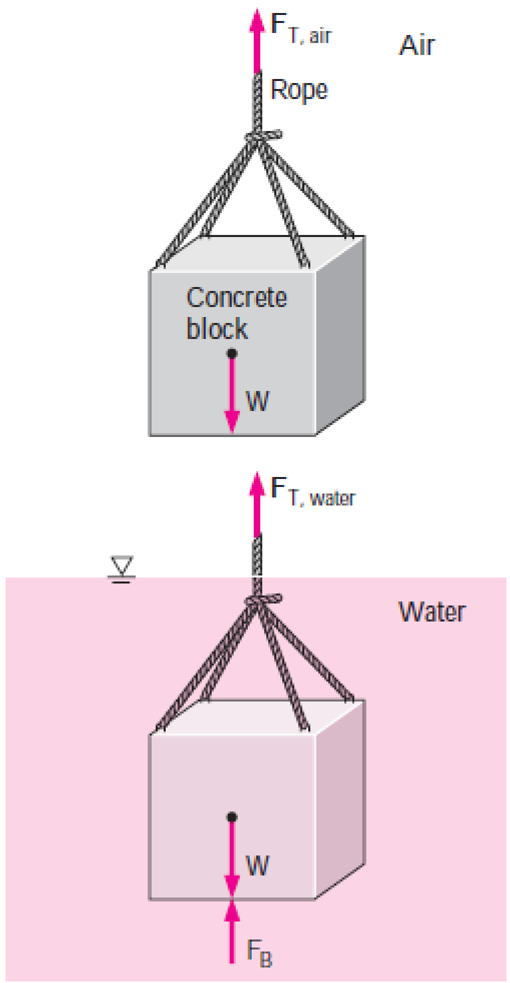
$$F_{T,air} = W = \rho_{concrete} g \forall = 2300 \times 9.81 \times 0.48 = 10830\ N$$

(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upward. The force balance in this case gives

$$F_B = \gamma \forall = 1025 \times 9.81 \times 0.48 = 4827\ N$$

$$F_{T,water} = W - F_B = 10830 - 4827 = 6003\ \text{N}$$

Note that the weight of the concrete block, and thus the tension of the rope, decreases by (10.8 - 6.0)/10.8= 55 percent in water.



3.10. Pressure Variation in a Fluid with Rigid-Body Motion

We knew that the pressure at a given point has the same magnitude in all directions, and thus it is a *scalar* function. In this section we obtain relations for the variation of pressure in fluids moving like a solid body with or without acceleration in the absence of any shear stresses (i.e., no motion between fluid layers relative to each other).

Many fluids such as milk and gasoline are transported in tankers. In an accelerating tanker, the fluid rushes to the back, and some initial splashing occurs. But then a new free surface (usually nonhorizontal) is formed, each fluid particle assumes the same acceleration, and the entire fluid moves like a rigid body. No shear stresses develop within the fluid body since there is no deformation and thus no change in shape. Rigid-body motion of a fluid also occurs when the fluid is contained in a tank that rotates about an axis.

if a container of fluid accelerates along a straight path, the fluid will move as a rigid mass (after the initial sloshing motion has died out) with each particle having the same acceleration. Since there is no deformation, there will be no shearing stresses. Similarly, if a fluid is contained in a tank that rotates about a fixed axis, the fluid will simply rotate with the tank as a rigid body.

3.10.1. Linear Motion

We first consider an open container of a liquid that is translating along a straight path with a constant acceleration **a** as illustrated in Fig.3.34. Since $a_x= 0$, it follows from the of equation of $-\frac{\partial P}{\partial x} = \rho a_x = 0$ that the pressure gradient in the *x* direction is zero In the *y* and *z* direction

$$\frac{\partial P}{\partial y} = -\rho a_y \qquad \frac{\partial P}{\partial z} = -\rho(g + a_z)$$

The change in pressure between two closely spaced points located at *y*, *z*,and y+dy, z+dz can be expressed as

$$dp = \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz$$

Or in terms of the the results from the above equations we can write

$$dP = -\rho a_y dy - \rho(g + a_z)dz$$

Along a line of constant pressure, dP=0 and therefore from the last equation it follows that the slope of this line is given by the relationship

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

This relationship is illustrated by the figure in the Fig.3.34d. Along a free surface the pressure is constant, so that for the accelerating mass shown in Fig. 3.34b the free surface will be inclined if $a_y \neq 0$. In addition, all lines of constant pressure will be parallel to the free surface as illustrated.

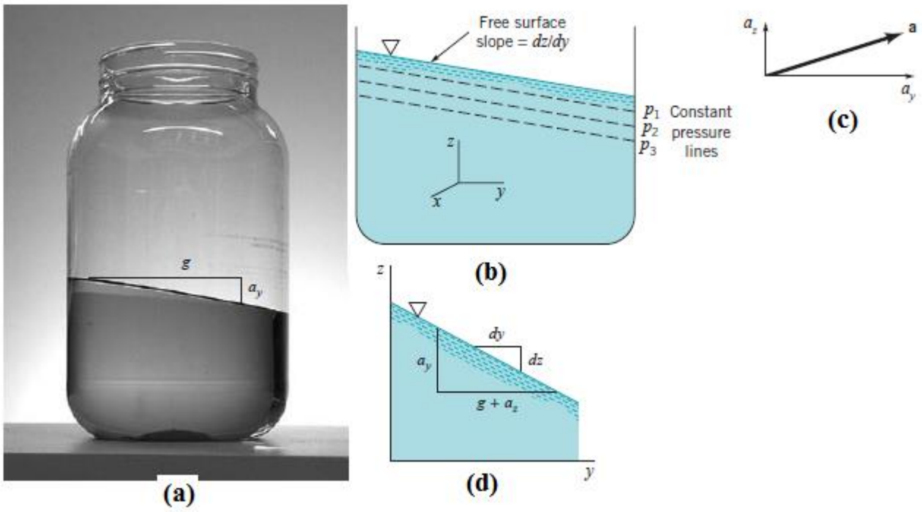


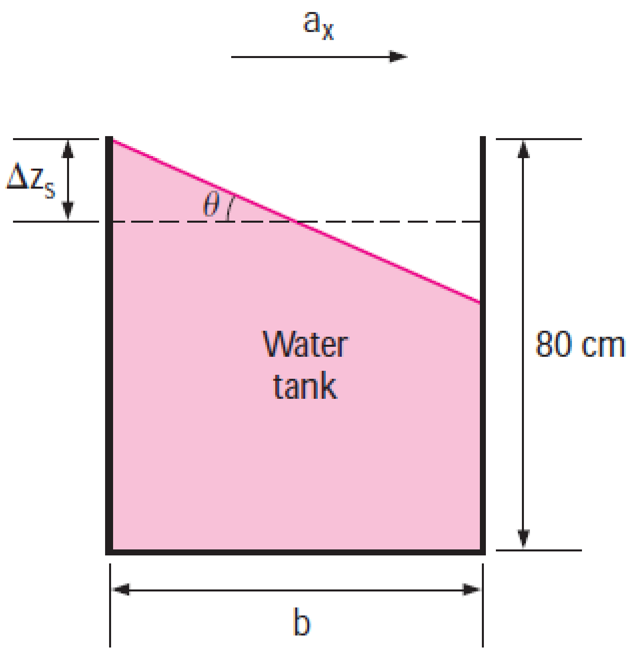
Figure 3.34.Linear motion

For the special circumstance in which a_y=0, a_z=0, which corresponds to the mass of fluid accelerating in the vertical direction, $\frac{dz}{dy} = -\frac{a_y}{g+a_z}$ indicates that the fluid surface will be horizontal. However, we see that the pressure distribution is not hydrostatic, but is given by the equation

$$\frac{dP}{dz} = -\rho(g + a_z)$$

For fluids of constant density this equation shows that the pressure will vary linearly with depth, but the variation is due to the combined effects of gravity and the externally induced acceleration, $\rho(g+a_z)$, rather than simply the specific weight ρg. Thus, for example, the pressure along the bottom of a liquid-filled tank which is resting on the floor of an elevator that is accelerating upward will be increased over that which exists when the tank is at rest (or moving with a constant velocity). It is to be noted that for a ***freely falling*** fluid mass (a_z=-g), the pressure gradients in all three coordinate directions are zero, which means that if the pressure surrounding the mass is zero, the pressure throughout will be zero. The pressure throughout a "blob" of orange juice floating in an orbiting space shuttle (a form of free fall) is zero. The only force holding the liquid together is surface tension.

Example: An 80-cm-high fish tank of cross section 2 m (0.6 m that is initially filled with water is to be transported on the back of a truck (Fig.). The truck accelerates from 0 to 90 km/h in 10 s. If it is desired that no water spills during acceleration, determine the allowable initial water height in the tank. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion? The road is horizontal during acceleration so that acceleration has no vertical component (az=*0*). Effects of splashing, braking, driving over bumps, and climbing hills are assumed to be secondary and are not considered. The cceleration remains constant.



Solution: We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction, and the origin to be the lower left corner of the tank. Noting that the truck goes from 0 to 90 km/h in 10 s, the acceleration of the truck is

$$a_x = \frac{\Delta x}{\Delta t} = \frac{(90-0)/3.6}{10} = 2.5\ m/s^2$$

The tangent of the angle the free surface makes with the horizontal is

$$tan\theta = \frac{a_x}{g + a_z} = \frac{2.5}{9.81+0} = 0.255 \rightarrow and\ thus\ \theta = 14.3°$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midplane experiences no rise or drop during acceleration since it is a plane of symmetry. Then the vertical rise at the back of the tank relative to the midplane for the two possible orientations becomes

Case 1: The long side is parallel to the direction of motion:

$$\Delta z_{s1} = \left(\frac{b_1}{2}\right) tan\theta = [2/2]\times 0.255 = 0.255\ m = 25.5\ cm$$

Case 2: The short side is parallel to the direction of motion:

$$\Delta z_{s2} = \left(\frac{b_2}{2}\right) tan\theta = [0.6/2]\times 0.255 = 0.076\, m = 7.6\, cm$$

Therefore, assuming tipping is not a problem, the tank should definitely be oriented such that its short side is parallel to the direction of motion. Emptying the tank such that its free surface level drops just 7.6 cm in this case will be adequate to avoid spilling during acceleration. Note that the orientation of the tank is important in controlling the vertical rise. Also, the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

3.10.2. Rigid-Body Rotation

We know from experience that when a glass filled with water is rotated about its axis, the fluid is forced outward as a result of the so-called centrifugal force, and the free surface of the liquid becomes concave. This is known as the *forced vortex motion*.

Consider a vertical cylindrical container partially filled with a liquid. The container is now rotated about its axis at a constant angular velocity of ω (Fig.3.35)After initial transients, the liquid will move as a rigid body together with the container. There is no deformation, and thus there can be no shear stress, and every fluid particle in the container moves with the same angular velocity.

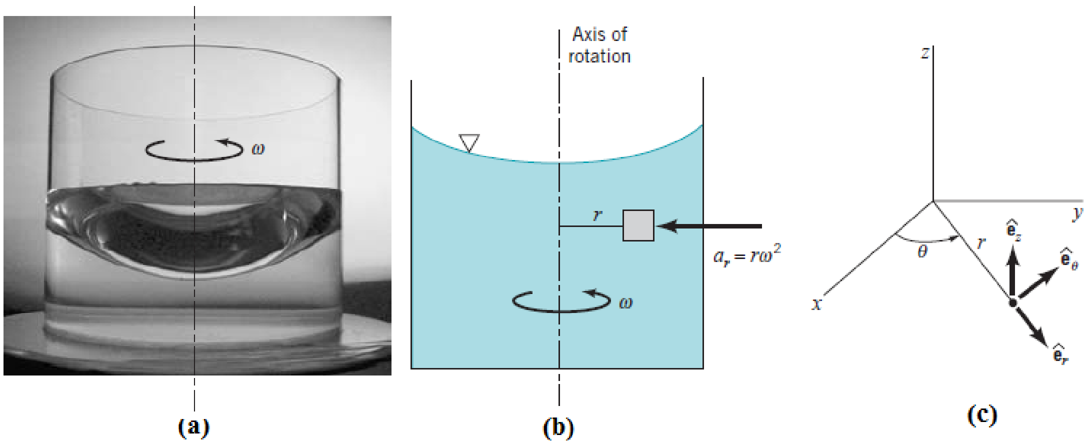


Figure 3.35. Rigid-body rotation of a liquid in a tank.

It is known from elementary particle dynamics that the acceleration of a fluid particle located at a distance *r* from the axis of rotation is equal in magnitude to $r\omega^2$ and the direction of the acceleration is toward the axis of rotation, as is illustrated in the figure. Since the paths of the fluid particles are circular, it is convenient to use cylindrical polar coordinates *r*, θ, and *z*, defined in the insert in Fig.3.35.

The equation for surfaces of constant pressure of free surface for rigid body rotation can be written as the following.

$$z = \frac{r^2\omega^2}{2g} + constant$$

Where; z is the distance of the free surface from the bottom of the container at radius r, ω is the angular velocity that calculated as $\pi(2r)^2 n/4$.

This equation reveals that these surfaces of constant pressure are parabolic, as illustrated in below Fig.3.36.

$$P = \frac{\rho r^2\omega^2}{2} - \gamma z + constant$$

The pressure varies with the distance from the axis of rotation, but at a fixed radius, the pressure varies hydrostatically in the vertical direction as shown inFig.

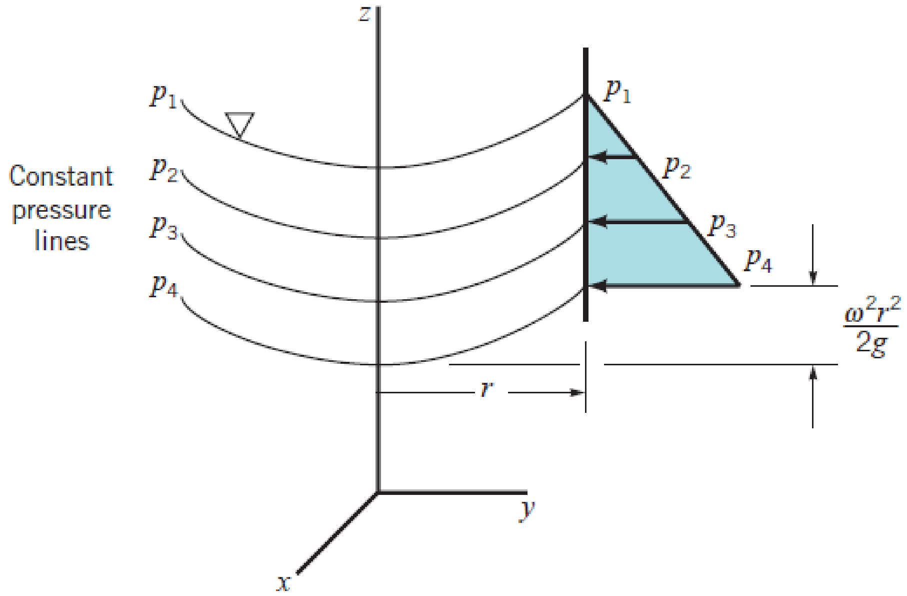
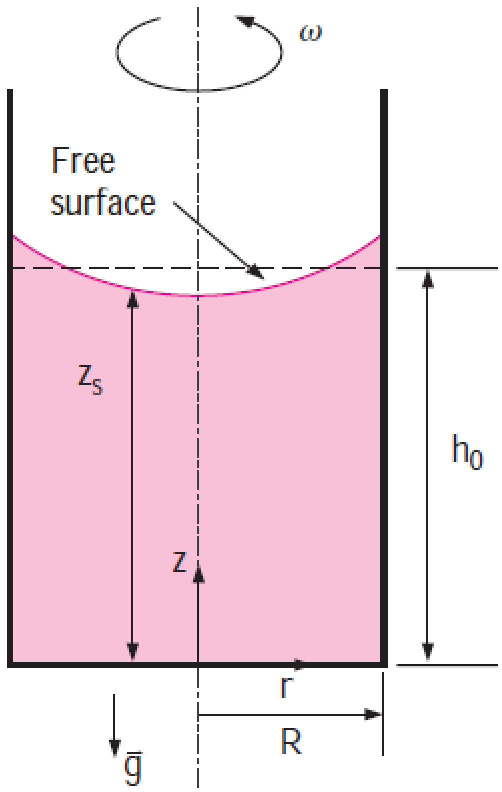


Figure 3.36. Pressure distribution in a rotating liquid.

Example: A 20-cm-diameter, 60-cm-high vertical cylindrical container, shown in Fig., is partially filled with 50-cm-high liquid whose density is 850 kg/m^3. Now the cylinder is rotated at a constant speed. Determine the rotational speed at which the liquid will start spilling from the edges of the container. The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. The bottom surface of the container remains covered with liquid during rotation (no dry spots).



$$z = \frac{r^2\omega^2}{2g} + constant \quad 0.20 = \frac{0.1^2\left(\frac{2\pi n}{60}\right)}{2\times 9.81} + 0 \rightarrow n = 189\, rpm$$

SUMMARY

Some of the important equations in this chapter are:

Pressure gradient in a stationary fluid: $\frac{dP}{dz} = -\gamma$

Pressure variation in a stationary incompressible fluid: $P_1 = \gamma h + P_2$

Hydrostatic force on a plane surface: $F_R = \gamma h_c A$

Location of hydrostatic force on a plane surface: $x_R = \frac{I_{xyc}}{y_c A} + x_c$; $y_R = \frac{I_{xc}}{y_c A} + y_c$

Boyant Force: $F_B = \gamma \forall$

Pressure gradient in rigid-body motion: $\frac{\partial P}{\partial x} = -\rho a_x$; $\frac{\partial P}{\partial y} = -\rho a_{y;}$ $\frac{\partial P}{\partial z} = \gamma + \rho a_z$

Pressure gradient in rigid –body rotation: $\frac{\partial P}{\partial r} = \rho r \omega^2$; $\frac{\partial P}{\partial \theta} = 0$; $\frac{\partial P}{\partial z} = -\gamma$

Pascal's Law: The pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present. $P_z=P_s=P_y$. *Pascal's law* states that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is a consequence of the pressure in a fluid remaining constant in the horizontal direction. An example of Pascal's principle is the operation of the hydraulic car jack.

Incompressible Fluid: a fluid with constant density is called an *incompressible fluid.*

Pressure Head: the *pressure head denoted h* is interpreted as the height of a column of fluid of specific weight γ required to give a pressure difference P_1-P_2. $h = (P_1 - P_{2)}/\gamma$.

Compressible Fluid: We normally think of gases such as air, oxygen, and nitrogen as being **compressible fluids** since the density of the gas can change significantly with changes in pressure and temperature.

Absolute Pressure: The pressure in the ideal gas law must be expressed as an *absolute pressure***,** denoted (abs), which means that it is measured relative to absolute zero pressure (a pressure that would only occur in a perfect vacuum).

Gage Pressure: In engineering it is common practice to measure pressure relative to the local atmospheric pressure, and when measured in this fashion it is called *gage pressure***.** Thus, the absolute pressure can be obtained from the gage pressure by adding the value of the atmospheric pressure.

Vacuum Pressure: A negative gage pressure is also referred to as a *suction* or *vacuum pressure.*

Barometer: The measurement of atmospheric pressure is usually accomplished with a ercury *barometer***,**which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury

Manometer: A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called *manometers*

Center of Pressure: The point through which the resultant force acts is called the *center of pressure*

Buoyant Force: When a stationary body is completely submerged in a fluid (such as the hot air balloon shown in the figure), or floating so that it is only partially submerged, the resultant fluid force acting on the body is called the *buoyant force*

Archimedes' Principle: the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward. This result is commonly referred to as *Archimedes' principle* in honor of Archimedes

Center of Buoyancy: The point through which the buoyant force acts is called the *center of buoyancy*.

EXAMPLES

Example: What is the difference between gage pressure and absolute pressure?

Solution: The pressure relative to the atmospheric pressure is called the *gage pressure*, and the pressure relative to an absolute vacuum is called *absolute pressure*. Most pressure gages (like your bicycle tire gage) read relative to atmospheric pressure, and therefore read the gage pressure.

Example: Explain why some people experience nose bleeding and some others experience shortness of breath at high elevations.

Solution: Atmospheric air pressure which is the external pressure exerted on the skin decreases with increasing elevation. Therefore, the pressure is lower at higher elevations. As a result, the difference between the blood pressure in the veins and the air pressure outside increases. This pressure imbalance may cause some thin-walled veins such as the ones in the nose to burst, causing bleeding. The shortness of breath is caused by the lower air density at higher elevations, and thus lower amount of oxygen per unit volume.

Example: Someone claims that the absolute pressure in a liquid of constant density doubles when the depth is doubled. Do you agree? Explain.

Solution: No, the absolute pressure in a liquid of constant density does not double when the depth is doubled. It is the *gage pressure* that doubles when the depth is doubled. This is analogous to temperature scales – when performing analysis using something like the ideal gas law, you *must* use absolute temperature (K), not relative temperature (°C), or you will run into the same kind of problem.

Example: Express Pascal's law, and give a real-world example of it.

Solution: *Pascal's law* states that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is a consequence of the pressure in a fluid remaining constant in the horizontal direction. An example of Pascal's principle is the operation of the hydraulic car jack. The above discussion applies to fluids at rest (hydrostatics). When fluids are in motion, Pascal's principle does not necessarily apply. However, as we shall see in later chapters, the differential equations of incompressible fluid flow contain only pressure *gradients*, and thus an increase in pressure *in the whole system* does *not* affect fluid motion.

Example: Consider two identical fans, one at sea level and the other on top of a high mountain, running at identical speeds. How would you compare (*a*) the volume flow rates and (*b*) the mass flow rates of these two fans?

Solution: The density of air at sea level is higher than the density of air on top of a high mountain. Therefore, the volume flow rates of the two fans running at identical speeds will be the same, but the mass flow rate of the fan at sea level will be higher.

Example: A vacuum gage connected to a chamber reads 24 kPa at a location where the atmospheric pressure is 92 kPa. Determine the absolute pressure in the chamber.

Solution: The absolute pressure in the chamber is determined from

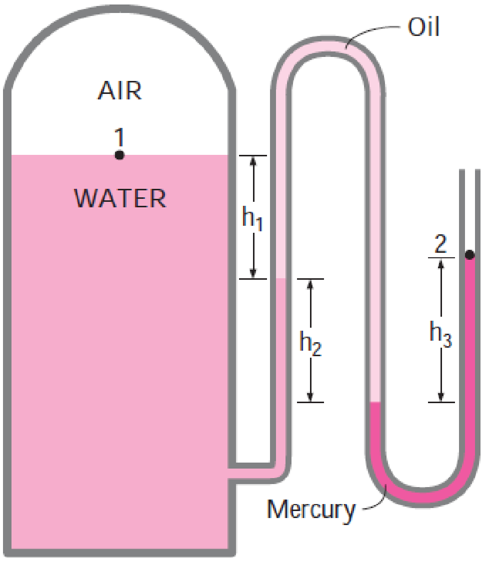
$$\boldsymbol{P_{abs} = P_{atm} - P_{vac} = 92 - 24 = 68\ kPa}$$

Example: Determine the atmospheric pressure at a location where the barometric reading is 750 mmHg. Take the density of mercury to be 13.600 kg/m^3.

Solution: The atmospheric pressure is determined directly from

$$P_{atm} = \rho g h = 13600 \times 9.81 \times 0.75 = 100062\ Pa$$

Example: The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig.. Determine the gage pressure of air in the tank if h_1= 0.2 m, h_2= 0.3 m, and h_3= 0.46 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13.600 kg/m^3, respectively.



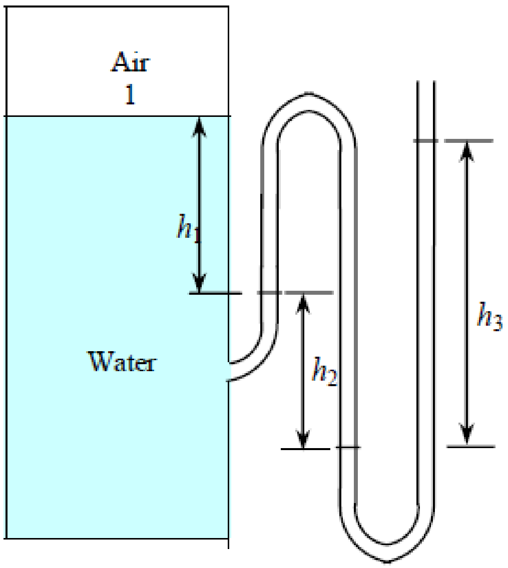
Solution: Starting with the pressure at point 1 at the air-water interface, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach point 2, and setting the result equal to *P*atm since the tube is open to the atmosphere gives

$$P_1 + \rho_w g h_1 + \rho_o g h_2 - \rho_m g h_3 = P_{atm}$$

$$P_1 = P_{atm} - \rho_w g h_1 - \rho_o g h_2 + \rho_m g h_3$$

$$P_1 = 0 - 1000\times9.81\times0.2 - 850\times9.81\times0.3 + 13600\times9.81\times0.46$$

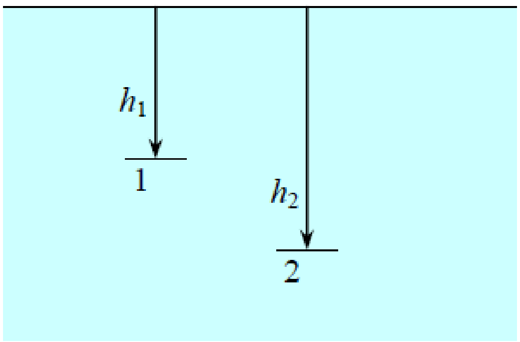
$$P_1 = 56907.81\ Pa$$



Example: The gage pressure in a liquid at a depth of 3 m is read to be 28 kPa. Determine the gage pressure in the same liquid at a depth of 12 m.

Solution: The gage pressure at two different depths of a liquid can be expressed as $P1 = \rho gh1$ and $P2 = \rho gh2$. Taking their ratio,

$$\frac{P_2}{P_1} = \frac{\rho\, g\, h_2}{\rho\, g\, h_1} = \frac{h_2}{h_1} \rightarrow P_2 = P_1 \frac{h_2}{h_1} = 28 \frac{12}{3} = 112\ kPa$$



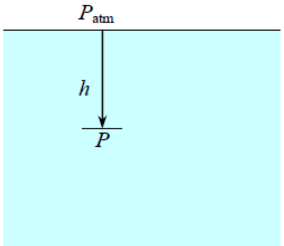
Example: The absolute pressure in water at a depth of 5 m is read to be 145 kPa. Determine (*a*) the local atmospheric pressure, and (*b*) the absolute pressure at a depth of 5 m in a liquid whose specific gravity is 0.85 at the same location. The liquid and water are incompressible.

Solution: The specific gravity of the fluid is given to be SG = 0.85. We take the density of water to be 1000 kg/m^3. Then density of the liquid is obtained by multiplying its specific gravity by the density of water,

(*a*) Knowing the absolute pressure, the atmospheric pressure can be determined from $P = P_{atm} + \rho gh$

$$P_{atm} = P - \rho gh = 145000 - 1000{\times}9.81{\times}5 = 95950\ Pa$$

(*b*) The absolute pressure at a depth of 5 m in the other liquid is
$P = P_{atm} + \rho gh = 95950 - 1000{\times}9.81{\times}5 = 46900\ Pa$



Example: Consider a 70-kg woman who has a total foot imprint area of 400 cm^2. She wishes to walk on the snow, but the snow cannot withstand pressures greater than 0.5 kPa. Determine the minimum size of the snowshoes needed (imprint area per shoe) to enable her to walk on the snow without sinking.

Solution: The weight of the person is distributed uniformly on the imprint area of the shoes. One foot carries the entire weight of a person during walking, and the shoe is sized for walking conditions (rather than standing). The weight of the shoes is negligible. The mass of the woman is given to be 70 kg. For a pressure of 0.5 kPa on the snow, the imprint area of one shoe must be

$$A = \frac{W}{P} = \frac{70\ kg \times 9.81\ m/s^2}{500} = 1.3734\ m^2$$

Example: A vacuum gage connected to a tank reads 30 kPa at a location where the barometric reading is 755 mmHg. Determine the absolute pressure in the tank. Take ρ_{Hg}= 13.590 kg/m^3

Solution: The atmospheric (or barometric) pressure can be expressed as

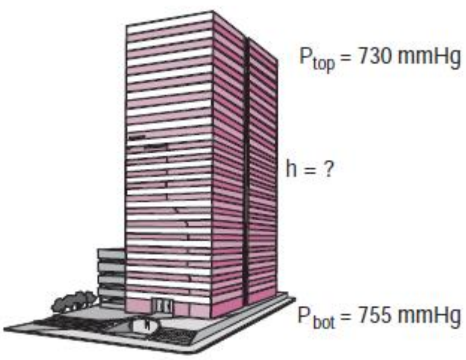
$$P_{atm} = \rho g h = 13590 \times 9.81 \times 0.755 = 100655\ Pa$$

Example: A pressure gage connected to a tank reads 500 kPa at a location where the atmospheric pressure is 94 kPa. Determine the absolute pressure in the tank.

Solution: The absolute pressure in the tank is determined from

$$P_{abs} = P_{gage} + P_{atm} = 500 + 94 = 594\ kPa$$

Example: The basic barometer can be used to measure the height of a building. If the barometric readings at the top and at the bottom of a building are 730 and 755 mmHg, respectively, determine the height of the building. Assume an average air density of 1.18 kg/m^3. The variation of air density with altitude is negligible.



Solution: Atmospheric pressures at the top and at the bottom of the building are

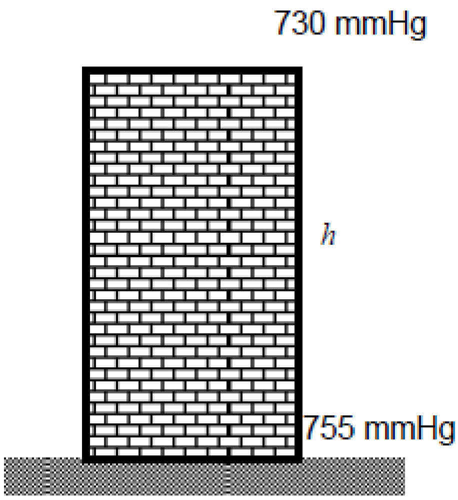
$$P_{top} = (\rho g h)_{top} = 13600 \times 9.81 \times 0.73 = 97394\ Pa$$

$$P_{bottom} = (\rho g h)_{bottom} = 13600 \times 9.81 \times 0.755 = 100729\ Pa$$

Taking an air column between the top and the bottom of the building, we write a force per unit base area.

$$\frac{W_{air}}{A} = P_{bottom} - P_{top} = (\rho g h)_{air} = 100729 - 97394 = 3335\ Pa$$

$$1.18 \times 9.81 \times h = 3335\ Pa \rightarrow h = 288.1\ m$$

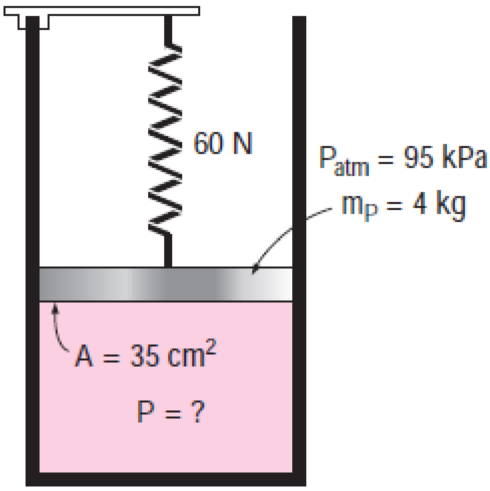


Example: Determine the absolute pressure exerted on a diver at 30 m below the free surface of the sea. Assume a barometric pressure of 101 kPa and a specific gravity of 1.03 for seawater.

Solution: The pressure exerted on a diver at 30 m below the free surface of the sea is the absolute pressure at that location:

$$P_{abs} = \rho g h + P_{atm} = 1.03 \times 9810 \times 30 + 101000 = 404129\ Pa$$

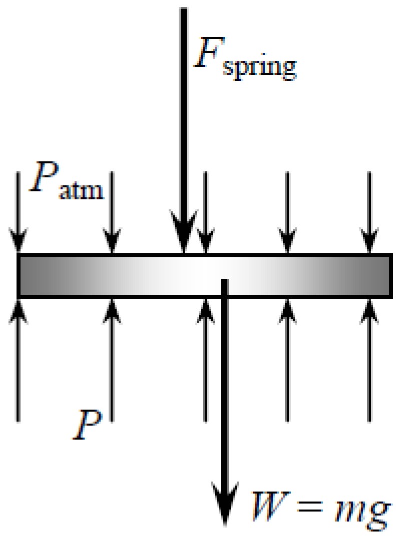
Example: A gas is contained in a vertical, frictionless piston–cylinder device. The piston has a mass of 4 kg and a crosssectional area of 35 cm^2. A compressed spring above the piston exerts a force of 60 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.



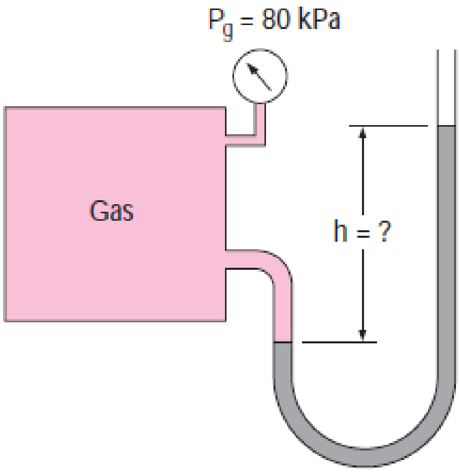
Solution: Drawing the free body diagram of the piston and balancing the vertical forces yields

$$P \times A = P_{atm}A + W + F_{spring}$$

$$P = P_{atm} + \frac{mg + F_{spring}}{A} = 95000 + \frac{4 \times 9.81 + 60}{35 \times 10^{-4}} = 123354\ Pa$$



Example: Both a gage and a manometer are attached to a gas tank to measure its pressure. If the reading on the pressure gage is 80 kPa, determine the distance between the two fluid levels of the manometer if the fluid is (*a*) mercury (ρ= 13.600 kg/m^3) or (*b*) water (ρ=1000 kg/m^3).

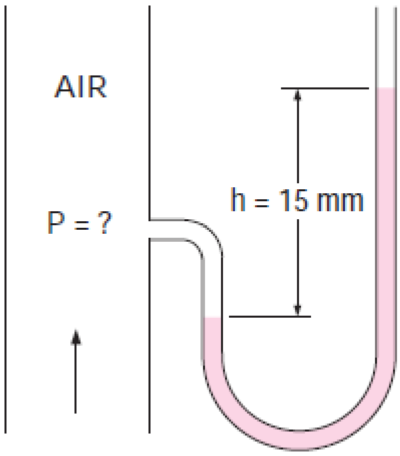


Solution: The gage pressure is related to the vertical distance *h* between the two fluid levels by

(*a*) For mercury, $P_{gage} = \rho g h \rightarrow h = \frac{P_{gage}}{\rho g} = \frac{80000}{13600 \times 9.81} = 0.60\ m$

(*b*) For water, $P_{gage} = \rho g h \rightarrow h = \frac{P_{gage}}{\rho g} = \frac{80000}{1000 \times 9.81} = 8.16\ \ m$

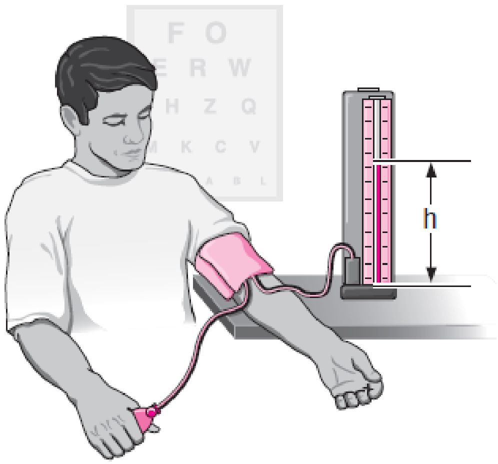
Example: A mercury manometer (ρ=13.600 kg/m^3) is connected to an air duct to measure the pressure inside. The difference in the manometer levels is 15 mm, and the atmospheric pressure is 100 kPa. (*a*) Judging from Fig., determine if the pressure in the duct is above or below the atmospheric pressure. (*b*) Determine the absolute pressure in the duct.



Solution: (*a*) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level. (*b*) The absolute pressure in the duct is determined from

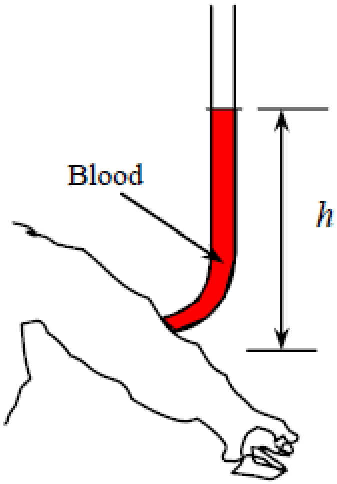
$$P = \rho g h + P_{atm} = 13600 \times 9.81 \times 0.015 + 100000 = 102001\ Pa$$

Example: The maximum blood pressure in the upper arm of a healthy person is about 120 mmHg. If a vertical tube open to the atmosphere is connected to the vein in the arm of the person, determine how high the blood will rise in the tube. Take the density of the blood to be 1050 kg/m^3. The density of blood is constant. The gage pressure of blood is 120 mmHg.



Solution: For a given gage pressure, the relation $P = \rho gh$ can be expressed for mercury and blood as $P = \rho_{blood}\, g\, h_{blood}$ and $P = \rho_{mercury}\, g\, h_{mercury}$. Setting these two relations equal to each other we get $P = \rho_{blood}\, g\, h_{blood} = \rho_{mercury}\, g\, h_{mercury}$. Solving for blood height and substituting gives

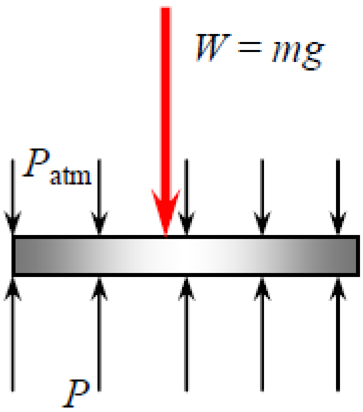
$$h_{blood} = \frac{\rho_{mercury}}{\rho_{blood}} h_{mercury} = \frac{13600}{1050} 0.12 = 1.554\, m$$



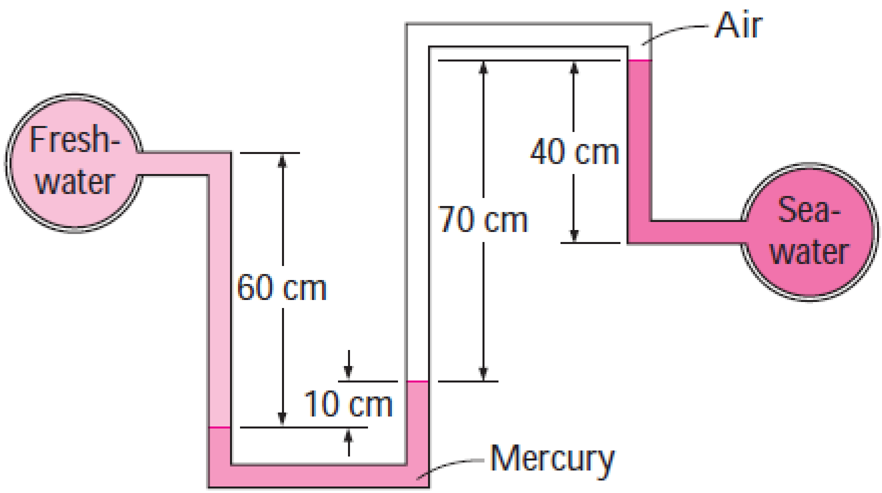
Example: The hydraulic lift in a car repair shop has an output diameter of 30 cm and is to lift cars up to 2000 kg. Determine the fluid gage pressure that must be maintained in the reservoir.

Solution: Pressure is force per unit area, and thus the gage pressure required is simply the ratio of the weight of the car to the area of the lift,

$$P_{gage} = \frac{W}{A} = \frac{2000\times 9.81}{\pi 0.3^2/4} = 277566\ Pa$$



Example: Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in Fig.. Determine the pressure difference between the two pipelines. Can the air column be ignored in the analysis? The densities of seawater and mercury are given to be ρsea = 1035 kg/m^3 and ρ_{Hg} = 13,600 kg/m^3. We take the density of water to be ρ_w =1000 kg/m^3.



Solution: Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρ*gh* terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{air} g h_{air} + \rho_{sea} g h_{sea} = P_2$$

Rearranging and neglecting the effect of air column on pressure

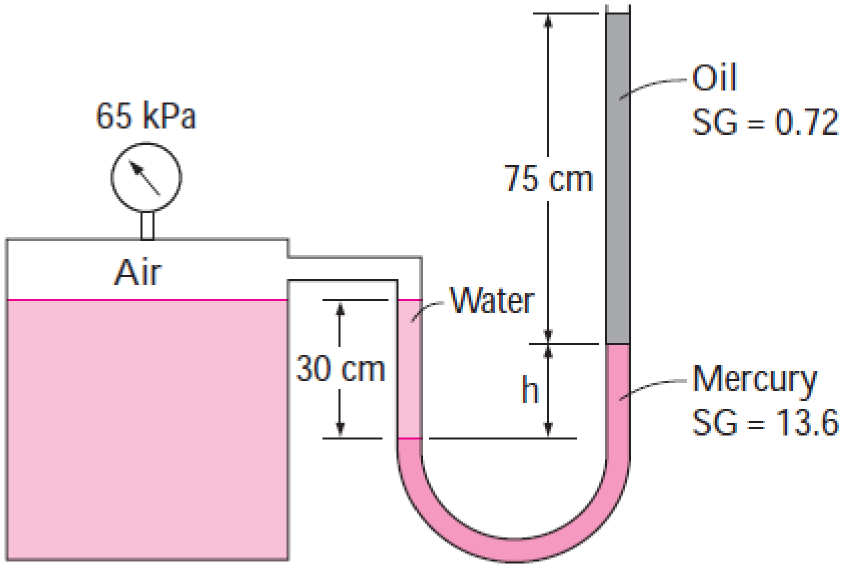
$$P_1 - P_2 = -\rho_w g h_w + \rho_{Hg} g h_{Hg} + \rho_{air} g h_{air} - \rho_{sea} g h_{sea}$$

$$P_1 - P_2 = g(\rho_{Hg} h_{Hg} \; - \rho_w h_w - \rho_{sea} h_{sea})$$

$$P_1 - P_2 = 9.81(13600 \times 0.10 - 1000 \times 0.60 - 1035 \times 0.40)$$

$$P_1 - P_2 = 3394 \; Pa$$

Example: The gage pressure of the air in the tank shown in Fig. is measured to be 65 kPa. Determine the differential height *h* of the mercury column. We take the density of water to be ρ_w=1000 kg/m^3. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.



Solution: Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρ*gh* terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to *P*atm gives,

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$ Rearranging,

$$P_1 - P_{atm} = -\rho_w g h_w + \rho_{Hg} g h_{Hg} + \rho_{oil} g h_{oil}$$

$$\frac{P_{1\,gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{Hg} h_{Hg} - h_w$$

$$\frac{65000}{9810} = 0.72 \times 0.75 + 13.6 \times h_{Hg} - 0.3 \rightarrow h_{Hg} = 0.47 \; m$$

Example: Consider a 4-m-long, 4-m-wide, and 1.5-m-high aboveground swimming pool that is filled with water to the rim. (*a*) Determine the hydrostatic force on each wall and the distance of the line of action of this force from the ground. (*b*) If the height of the walls of the pool is doubled and the pool is filled, will the hydrostatic force on each wall double or quadruple? Why? We take the density of water to be 1000 kg/m^3 throughout.

Solution: Atmospheric pressure acts on both sides of the wall of the pool, and thus it can be ignored in calculations for convenience. The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{ave} = P_c = \rho g h_c = \rho g \left(\frac{h}{2}\right) = 1000 \times 9.81 \times \frac{1.5}{2} = 7357.5\ Pa$$

Then the resultant hydrostatic force on each Wall becomes

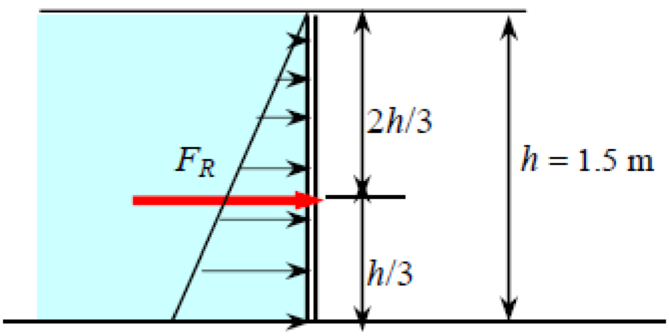
$$F_R = P_{ave} A = 7357.5 \times 4 \times 1.5 = 44145\ N$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface and $h/3$ from the bottom of the pool. Therefore, the distance of the line of action from the ground is

$$y_p = \frac{h}{3} = \frac{1.5}{3} = 0.5\ m \text{ (from the bottom)}$$

If the height of the walls of the pool is doubled, the hydrostatic force **quadruples** since

$$F_R = \rho g h_c A = \rho g \left(\frac{h}{2}\right)(h \times w) = \rho g w h^2/2$$



Example: A room in the lower level of a cruise ship has a 30-cm-diameter circular window. If the midpoint of the window is 5 m below the water surface, determine the hydrostatic force acting on the window, and the pressure center. Take the specific gravity of seawater to be 1.025.

Solution: The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m^3. The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

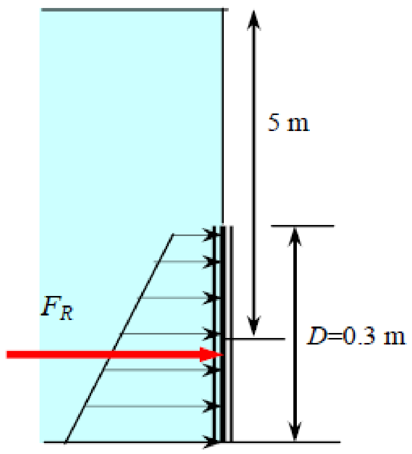
$$P_{ave} = P_c = \rho g h_c = 1025 \times 9.81 \times 5 = 50276\ Pa$$

Then the resultant hydrostatic force on each Wall becomes

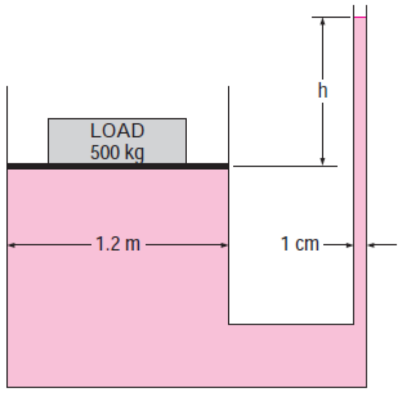
$$F_R = P_{ave}A = P_{ave}[\pi D^2/4] = 50276\ [\pi\ 0.3^2/4] = 3554\ N$$

The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

$$y_P = y_C + \frac{I_{xx,C}}{y_C A} = y_C + \frac{\pi R^2/4}{y_C \pi R^2} = y_C + \frac{R^2}{4 y_C} = 5 + \frac{0.15^2}{4 \times 5} = 5.0011\ m \cong 5\ m$$



Example: The 500-kg load on the hydraulic lift shown in Fig. is to be raised by pouring oil (ρ=780 kg/m^3) into a thin tube. Determine how high h should be in order to begin to raise the weight.



Solution: The cylinders of the lift are vertical. There are no leaks. Atmospheric pressure act on both sides, and thus it can be disregarded. Noting that pressure is force per unit area, the gage pressure in the fluid under the load is simply the ratio of the weight to the area of the lift,

$$P_{gage} = \frac{W}{A} = \frac{mg}{\pi D^2/4} = \frac{500\times9.81}{\pi\times1.2^2/4}) = 4337\ Pa$$

The required oil height that will cause 4337 Pa of pressure rise is

$$P_{gage} = \rho gh \rightarrow h = \frac{P_{gage}}{\rho g} = \frac{4337}{780\times9.81} = 0.567\ m$$

Example: Pressure is often given in terms of a liquid column and is expressed as "pressure head." Express the standard atmospheric pressure in terms of (*a*) mercury (SG=13.6), (*b*) water (SG=1.0), and (*c*) glycerin (SG=1.26) columns. Explain why we usually use mercury in manometers.

Solution: The atmospheric pressure is expressed in terms of a fluid column height as

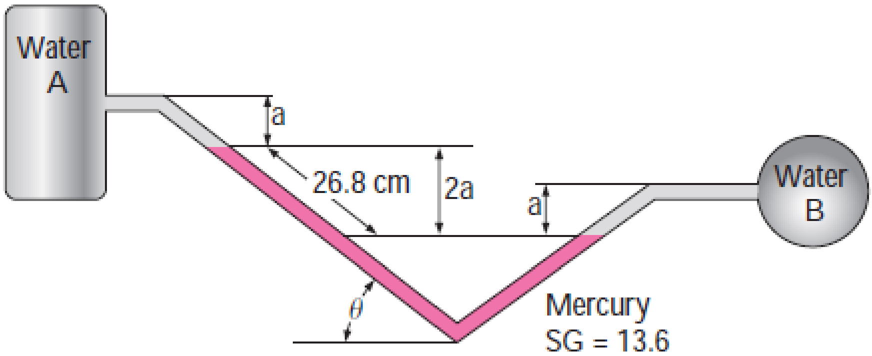
$$P_{atm} = \rho gh = SG\rho gh \rightarrow h = \frac{P_{atm}}{SG\ \rho h}$$

a) Mercury: $h = \frac{P_{atm}}{SG\ \rho h} = \frac{101325}{13.6\times1000\times9.81} = 0.759\ m$

b) Water: : $h = \frac{P_{atm}}{SG\ \rho h} = \frac{101325}{1\times1000\times9.81} = 10.33\ m$

c) Glycerin: : $h = \frac{P_{atm}}{SG\ \rho h} = \frac{101325}{1.26\times1000\times9.81} = 8.20\ m$

Example: Two water tanks are connected to each other through a mercury manometer with inclined tubes, as shown in Fig.. If the pressure difference between the two tanks is 20 kPa, calculate *a* and u. The specific gravity of mercury is given to be 13.6. We take the standard density of water to be ρ_w =1000 kg/m^3.



Solution: Starting with the pressure in the tank A and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach tank B, and setting the result equal to *PB* give

$$P_A + \rho_w g a + \rho_{Hg} g 2a - \rho_w g a = P_B \rightarrow P_A - P_B = 2\rho_{Hg} g a$$

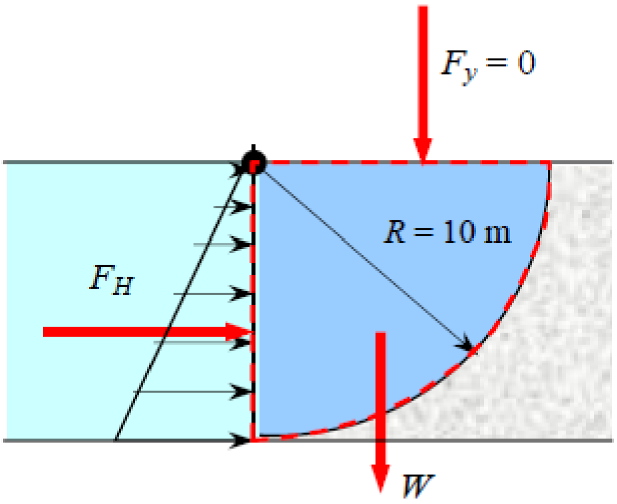
Rearranging and substituting the known values,

$$a = \frac{P_A - P_B}{2\rho_{Hg} g} = \frac{20000}{2\times 13.6\times 1000\times 9.81} = 0.075\ m = 7.50\ cm$$

From geometric considerations,

$$26.8\times sin\theta = 2a \rightarrow sin\theta = \frac{2a}{26.8} = \frac{2\times 7.5}{26.8} = 0.560 \rightarrow \theta = 34.0°$$

Example: The water side of the wall of a 100-m-long dam is a quarter circle with a radius of 10 m. Determine the hydrostatic force on the dam and its line of action when the dam is filled to the rim. We take the density of water to be 1000 kg/m^3 throughout.



Solution: We consider the free body diagram of the liquid block enclosed by the circular surface of the dam and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_c A = \rho g \left(\frac{R}{2}\right) A$$

$$F_H = \rho g \left(\frac{R}{2}\right) A = 1000 \times 9.81 \times \left(\frac{10}{2}\right) (10 \times 100) = 49050000\ N$$

Vertical force on horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per m length is

$$F_V = W = \rho g \forall = \rho g \left[w \times \frac{\pi R^2}{4} \right] = 1000 \times 9.81 \times \left[100 \times \pi \times \frac{10^2}{4} \right]$$
$$= 77047560\ N$$

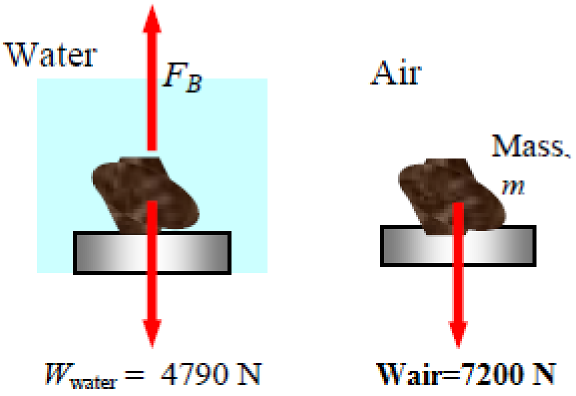
Then the magnitude and direction of the hydrostatic force acting on the surface of the dam become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{49050000^2 + 77047560^2} = 91335804\ N$$

$$tan\theta = \frac{F_V}{F_H} = \frac{77047560}{49050000} \rightarrow \theta = 57.5°$$

Therefore, the line of action of the hydrostatic force passes through the center of the curvature of the dam, making 57.5°downwards from the horizontal.

Example: The volume and the average density of an irregularly shaped body are to be determined by using a spring scale. The body weighs 7200 N in air and 4790 N in water. Determine the volume and the density of the body. State your assumptions. We take the density of water to be 1000 kg/m^3. The buoyancy force in air is negligible. The body is completely submerged in water.



Solution: The mass of the body is $m = \frac{W_{air}}{g} = \frac{7200}{9.81} = 733.9\ kg$

The difference between the weights in air and in water is due to the buoyancy force in water,

$F_B = W_{air} - W_{water} = 7200 - 4790 = 2410\ N$ Noting that $F_B = \rho_{water} g \forall$, the volume of the body is determined to be

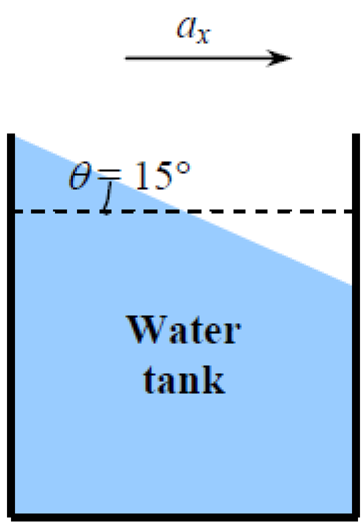
$\forall = \frac{F_B}{\rho_w g} = \frac{2410}{1000 \times 9.81} = 0.2457\ m^3$. Then the density of the body becomes

$$\rho = \frac{m}{\forall} = \frac{733.9}{0.2457} = 2987\ kg/m^3$$

Example: A water tank is being towed by a truck on a level road, and the angle the free surface makes with the horizontal is measured to be 15°. Determine the acceleration of the truck. The road is horizontal so that acceleration has no vertical component (*az* = 0). Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. The acceleration remains constant.

Solution: We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

$$tan\theta = \frac{a_x}{g + a_z} \rightarrow a_x = (g + z)tan\theta = (9.81 + 0)tan15° = 2.63\ m/s^2$$



Example: A 40-cm-diameter, 90-cm-high vertical cylindrical container is partially filled with 60-cm-high water. Now the cylinder is rotated at a constant angular speed of 120 rpm. Determine how much the liquid level at the center of the cylinder will drop as a result of this rotational motion. The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. The bottom surface of the container remains covered with liquid during rotation (no dry spots)

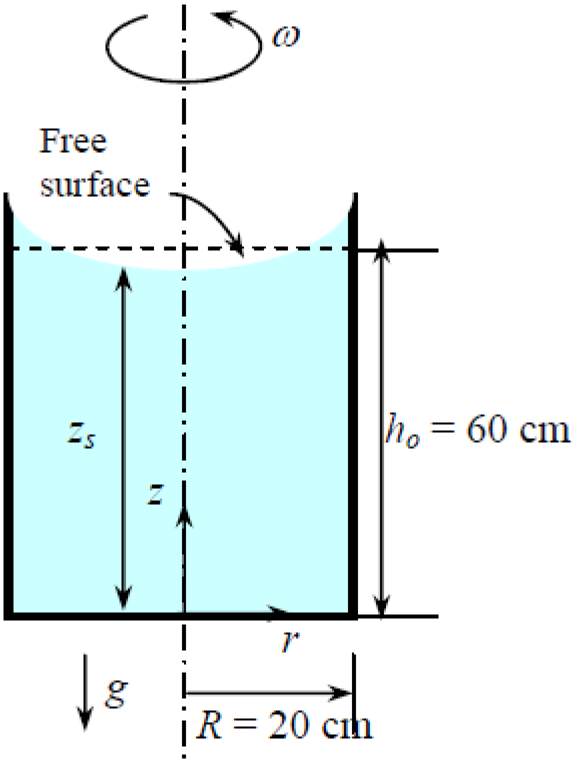
Solution:

$$z = \frac{r^2 w^2}{2g} = \frac{0.20^2 \times (2\pi n/60)^2}{2 \times 9.81} = 0.32\, m$$

$$z_s = h_0 - z = 0.60 - \frac{0.32}{2} = 0.44\, m$$

Therefore, the drop in the liquid level at the center of the cylinder is

$$\Delta h = h_0 - z_s = 0.60 - 0.44 = 0.16\, m \text{ or } \Delta h = \frac{z}{2} = \frac{0.322}{2} = 0.16$$



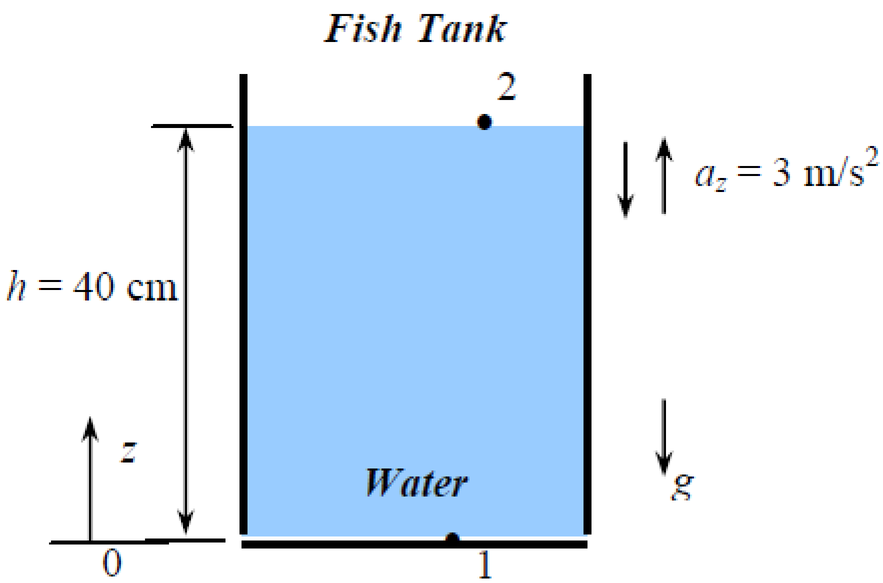
Example: A fish tank that contains 40-cm-high water is moved in the cabin of an elevator. Determine the pressure at the bottom of the tank when the elevator is (*a*) stationary, (*b*) moving up with an upward acceleration of 3 m/s^2, and (*c*) moving down with a downward acceleration of 3 m/s^2. We take the density of water to be 1000 kg/m^3.

Solution: The motion of a fish tank in the cabin of an elevator is considered. The pressure at the bottom of the tank when the elevator is stationary, moving up with a specified acceleration, and moving down with a specified acceleration is to be determined. The pressure difference between two points 1 and 2 in an incompressible fluid is given by

$$P_2 - P_2 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \rightarrow P_2 - P_2 = \rho(g + a_z)(z_2 - z_1)$$

Since a_x=0. Taking point 2 at the free surface and point 1 at the tank bottom. We have P2=Patm and z_2-z_1 =h and thus

$P_{1gage}=P_{bottom}=\rho(g+a_z)h$



(**a**) **Tank stationary**: We have *az* = 0, and thus the gage pressure at the tank bottom is

$$P_{bottom}=\rho gh=1000\times 9.81\times 0.4=3924\ Pa$$

(**b**) **Tank moving up**: We have *az* = +3 m/s2, and thus the gage pressure at the tank bottom is

$$P_{bottom}=\rho(g+a_z)h_B=1000\times(9.81+3)\times 0.4=5124\ Pa$$

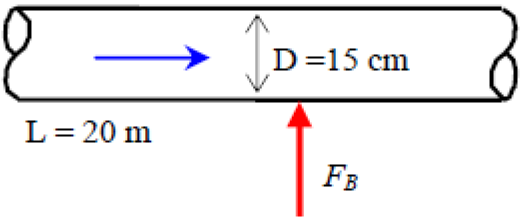
(**c**) **Tank moving down**: We have *az* = -3 m/s^2, and thus the gage pressure at the tank bottom is

$$P_{bottom}=\rho(g+a_z)h_B=1000\times(9.81-3)\times 0.4=2724\ Pa$$

Note that the pressure at the tank bottom while moving up in an elevator is almost twice that while moving down, and thus the tank is under much greater stress during upward acceleration.

Example: An air-conditioning system requires a 20-m-long section of 15-cm-diameter ductwork to be laid underwater. Determine the upward force the water will exert on the duct.
Take the densities of air and water to be 1.3 kg/m^3 and 1000 kg/m^3, respectively. The diameter given is the outer diameter of the duct (or, the thickness of the duct material is negligible). The weight of the duct and the air in is negligible.

Solution: The density of air is given to be ρ = 1.30 kg/m^3. We take the density of water to be 1000 kg/m^3. Noting that the weight of the duct and the air in it is negligible, the net upward force acting on the duct is the buoyancy force exerted by water. The volume of the underground section of the duct is



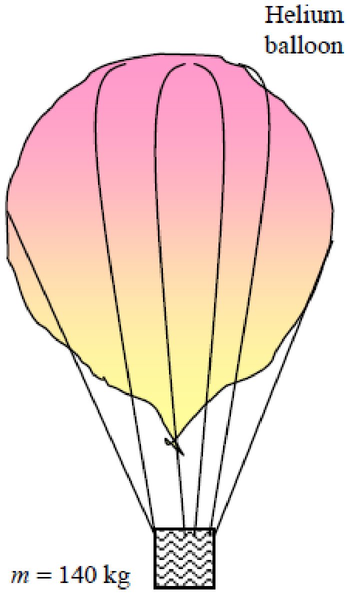
$$\forall = AL = \left(\frac{\pi D^2}{4L}\right) = \left[\pi \times \frac{0.15^2}{4}\right] \times 20 = 0.3534\ m^3$$

Then the buoyancy force becomes

$F_B = \rho g \forall = 1000 \times 9.81 \times 0.3534 = 3467$ Pa

Example: Balloons are often filled with helium gas because it weighs only about one-seventh of what air weighs under identical conditions. The buoyancy force, which can be expressed as $F_b = \rho_{air} g \forall_{balloon}$, will push the balloon upward. If the balloon has a diameter of 10 m and carries two people, 70 kg each. Assume the density of air is ρ=1.16 kg/m^3, and neglect the weight of the ropes and the cage. a) Determine the acceleration of the balloon when it is first released. b) Determine the maximum amount of load, in kg, the balloon.

Solution: A helium balloon tied to the ground carries 2 people. The acceleration of the balloon when it is first released is to be determined.



a) The density of air is given to be ρ = 1.16 kg/m^3. The density of helium gas is 1/7th of this. The buoyancy force acting on the balloon is

$$\forall_{balloon} = \frac{4r^3}{3} = \frac{4\pi 5^3}{3} = 523.6\ m^3$$

$$F_B = \rho_{air} g \forall_{balloon} = 1.16 \times 9.81 \times 523.6 = 5958.4\ N$$

The total mass is $m_{He} = \rho_{He} \forall = \frac{1.16}{7} 523.6 = 86.8\ kg$

$$m_{total} = m_{He} + m_{people} = 86.8 + 2 \times 70 = 226.8\ kg$$

The total weight is $W=m_{total}g=226.8\times 9.81=2224.9$ N

Thus the net force acting on the balloon is $F_{net}=F_B-W=5958.6-2224.5=3733.5$ N

Then the acceleration becomes $a = \frac{F_{net}}{m_{total}} = \frac{3733.5}{226.8} = 16.5\ m/s^2$

This is almost twice the acceleration of gravity – aerodynamic drag on the balloon acts quickly to slow down the acceleration.

b) In the limiting case, the net force acting on the balloon will be zero. That is, the buoyancy force and the weight will balance each other:

$W=mg=F_B$

$$m_{total} = \frac{F_B}{g} = \frac{5958.4}{9.81} = 607.4\ kg$$

Thus $m_{people}=m_{total}-m_{He}=607.4-86.8=520.6$ kg=521 kg

Example: The basic barometer can be used as an altitudemeasuring device in airplanes. The ground control reports a barometric reading of 753 mmHg while the pilot's reading is 690 mmHg. Estimate the altitude of the plane from ground level if the average air density is 1.20 kg/m^3. The density of mercury is given to be 13.600 kg/m^3.

Solution: Atmospheric pressures at the location of the plane and the ground level are

$P_{plane}=(\rho g h)_{plane}=13600\times 9.81\times 0.690=92060$ Pa

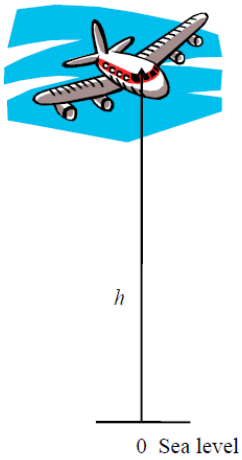
$P_{ground}=(\rho gh)_{ground}=13600\times9.81\times0.753=100460$ Pa

Taking an air column between the airplane and the ground and writing a force balance per unit base area, we obtain

$W_{air} / A=P_{ground} - P_{plane}$ $(\rho gh)air = P_{ground} - P_{plane}$

$1.20\times9.81\times h=100460-92060=8400$

h=714 m. Obviously, a mercury barometer is not practical on an airplane – an electronic barometer is used instead.

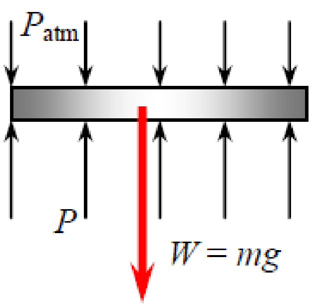


Example: A vertical, frictionless piston–cylinder device contains a gas at 500 kPa. The atmospheric pressure outside is 100 kPa, and the piston area is 30 cm2. Determine the mass of the piston.

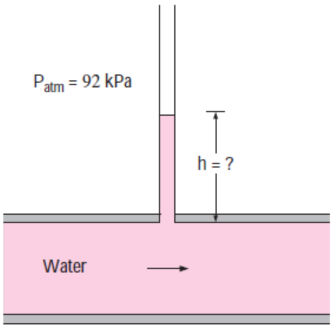
Solution: Drawing the free body diagram of the piston and balancing the vertical forces yield $W=PA-P_{atm}A$, $mg=(P-P_{atm})A$

$m\times9.81=(500000-100000)\times0.003$ ise m=122 kg

The gas cannot distinguish between pressure due to the piston weight and atmospheric pressure – both “feel” like a higher pressure acting on the top of the gas in the cylinder.



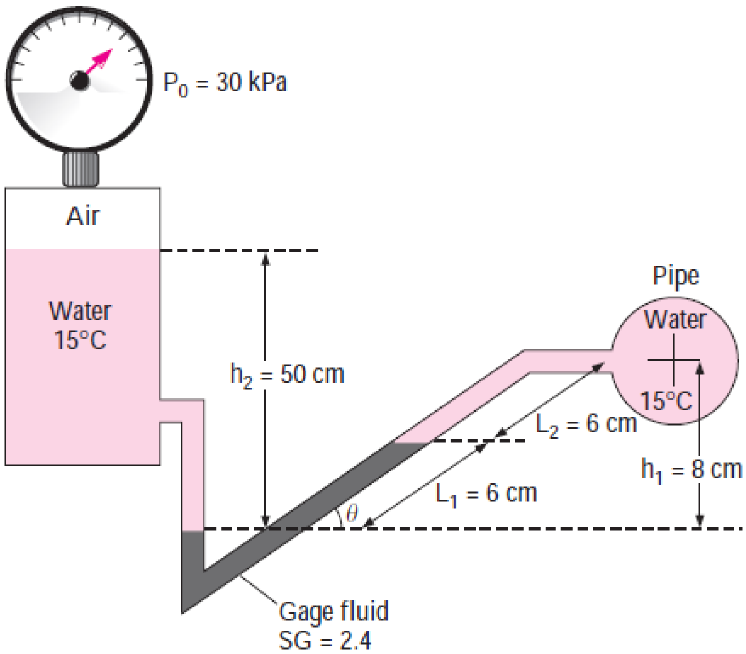
Example: A glass tube is attached to a water pipe, as shown in Fig.. If the water pressure at the bottom of the tube is 115 kPa and the local atmospheric pressure is 92 kPa, determine how high the water will rise in the tube, in m. Assume g= 9.8 m/s^2 at that location and take the density of water to be 1000 kg/m^3.



Solution: A glass tube open to the atmosphere is attached to a water pipe, and the pressure at the bottom of the tube is measured. It is to be determined how high the water will rise in the tube. The pressure at the bottom of the tube can be expressed as

$P=P_{atm}+(\rho gh)_{tube}$ Solving for h, $h=\frac{P-P_{atm}}{\rho g}=\frac{115000-92000}{1000\times 9.8}=2.35\ m$

Example: The pressure of water flowing through a pipe is measured by the arrangement shown in Fig. For the values given, calculate the pressure in the pipe.



Solution: The specific gravity of gage fluid is given to be 2.4. We take the standard density of water to be ρw= 1000 kg/m^3.

Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the water pipe, and setting the result equal to P_{water} give

$$P_{gage} + \rho_w g h_{w1} - \rho_{gage} g h_{gage} - \rho_w g h_{w2} = P_{water}$$

$$P_{water} = P_{gage} + \rho_w g(h_{w1} - SG_{gage} h_{gage} - h_{w2}) =$$

$$P_{water} = P_{gage} + \rho_w g(h_{w1} - SG_{gage} L_1 sin\theta - L_2 sin\theta)$$

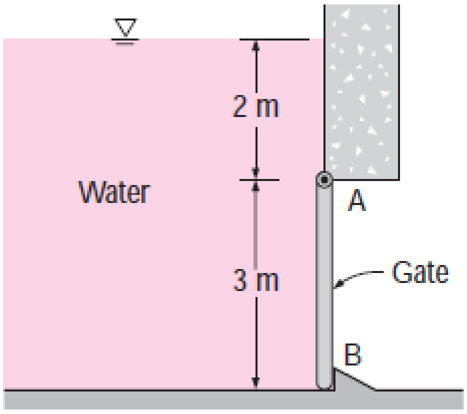
Noting that sinθ=8/12=0.6667 and substituting,

$$P_{water} = 30000 + 1000 \times 9.81[0.50 - 2.4 \times 0.06 \times 0.6667 - 0.06 \times 0.6667]$$
$$= 33600\ Pa$$

$$P_{water} = 33600\ Pa$$

Therefore, the pressure in the gasoline pipe is 3.6 kPa over the reading of the pressure gage.

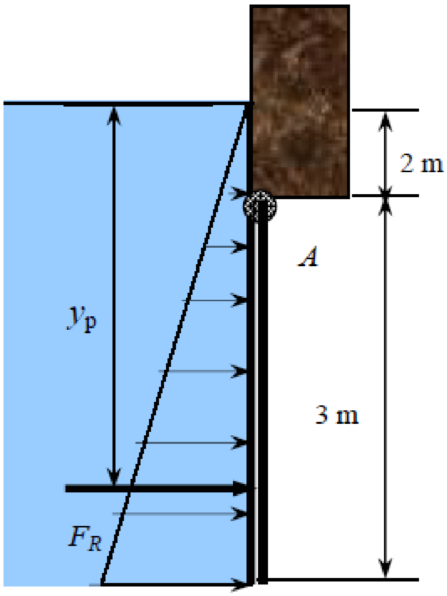
Example: A 3-m-high, 6-m-wide rectangular gate is hinged at the top edge at *A* and is restrained by a fixed ridge at *B*. Determine the hydrostatic force exerted on the gate by the 5-m-high water and the location of the pressure center. Determine the hydrostatic force exerted on the gate for a total water height of 2 m.



Solution: The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the gate,

$F_R = P_{ave}A = \rho g h_C A = 1000 \times 9.81 \times 3.5 \times 3 \times 6 = 206010$ N

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + \frac{b}{2})} = 2 + \frac{3}{2} + \frac{3^2}{12(2 + \frac{3}{2})} = 3.71\ m$$



b) The hydrostatic force exerted on the gate for a total water height of 2 m. The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the wetted plate area gives the resultant hydrostatic force on the gate,

$F_R = P_{ave}A = \rho g h_C A = 1000 \times 9.81 \times 1 \times (2 \times 6) = 117720$ N

The vertical distance of the pressure center from the free surface of water is

$$y_P = \frac{2h}{3} = \frac{2 \times 2}{3} = 1.33\ m$$