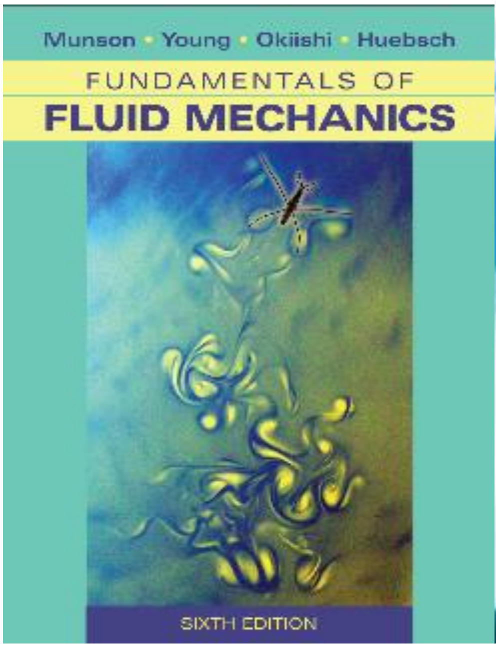
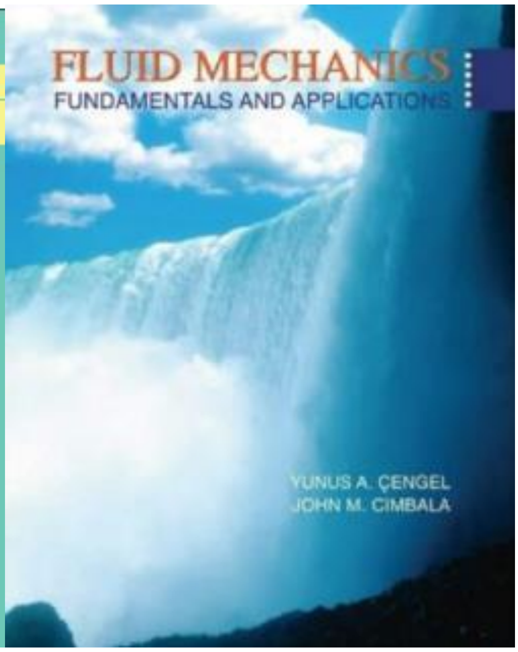
FLUID MECHANICS





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5. FLOW IN PIPES

5.1.3. Pressure and Shear Stress

Fully developed steady flow in a constant diameter pipe may be driven by gravity and/or pressure forces. For horizontal pipe flow, gravity has no effect except for a hydrostatic pressure variation across the pipe, $\gamma D,$ that is usually negligible. It is the pressure difference, $\Delta P = P_1 - P_2$ between one section of the horizontal pipe and another which forces the fluid through the pipe. Viscous effects provide the restraining force that exactly balances the pressure force, thereby allowing the fluid to flow through the pipe with no acceleration. If viscous effects were absent in such flows, the pressure would be constant throughout the pipe, except for the hydrostatic variation.

In non-fully developed flow regions, such as the entrance region of a pipe, the fluid accelerates or decelerates as it flows (the velocity profile changes from a uniform profile at the entrance of the pipe to its fully developed profile at the end of the entrance region). Thus, in the entrance region there is a balance between pressure, viscous, and inertia (acceleration) forces. The result is a pressure distribution along the horizontal pipe as shown in Fig.5.7. The magnitude of the pressure gradient, $\frac{\partial P}{\partial x}$, is larger in the entrance region than in the fully developed region, where it is a constant, $\frac{\partial P}{\partial x} = -\frac{\Delta P}{L} < 0.$

The fact that there is a nonzero pressure gradient along the horizontal pipe is a result of viscous effects. If the viscosity were zero, the pressure would not vary with *x*. The need for the pressure drop can be viewed from two different standpoints. In terms of a force balance, the pressure force is needed to overcome the viscous forces generated. In terms of an energy balance, the work done by the pressure force is needed to overcome the viscous dissipation of energy throughout the fluid. If the pipe is not horizontal, the pressure gradient along it is due in part to the component of weight in that direction. This contribution due to the weight either enhances or retards the flow, depending on whether the flow is downhill or uphill.

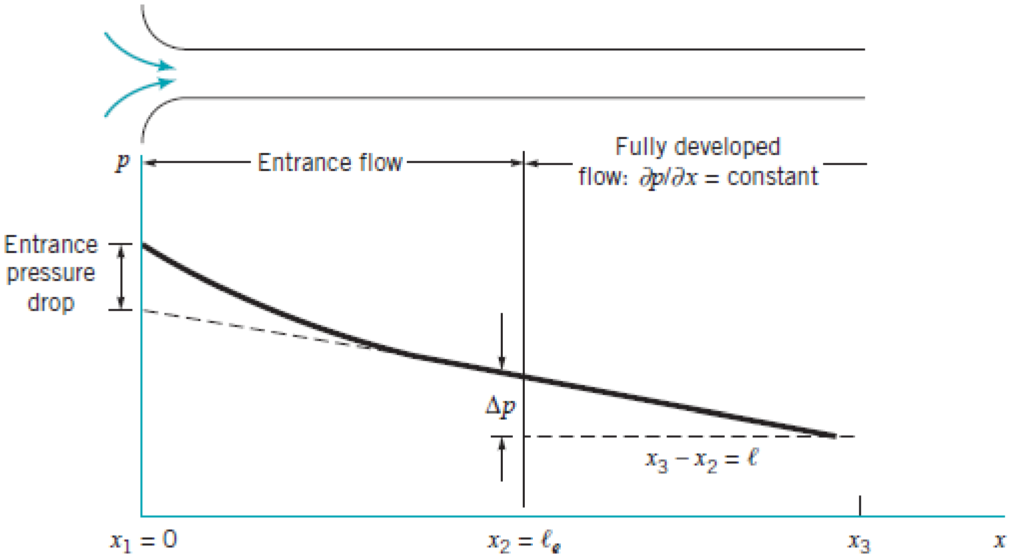


Figure 5.7. Pressure distribution along a horizontal pipe.

The nature of the pipe flow is strongly dependent on whether the flow is laminar or turbulent. This is a direct consequence of the differences in the nature of the shear stress in laminar and turbulent flows. The shear stress in laminar flow is a direct result of momentum transfer among the randomly moving molecules (a microscopic phenomenon). The shear stress in turbulent flow is largely a result of momentum transfer among the randomly moving, finite-sized fluid particles (a macroscopic phenomenon). The net result is that the physical properties of the shear stress are quite different for laminar flow than for turbulent flow.

5.2. Fully Developed Laminar Flow

We mentioned that flow in pipes is laminar for $Re \leq 2100$, and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length) so that the entrance effects are negligible. In this section we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe. We obtain the momentum equation by applying a momentum balance to a differential volume element, and obtain the velocity profile by solving it. Then we use it to obtain a relation for the friction factor. An important aspect of the analysis here is that it is one of the few available for viscous flow.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady and fully developed., denoted τ_w, the ***wall shear stress*** (Fig.5.8). Hence, the shear stress distribution throughout the pipe is a linear function of the radial coordinate. The following equations can be written for laminar flow. Although we are discussing laminar flow, a closer consideration of the assumptions involved in the derivation of below Eqs. *reveals that these equations are valid for both laminar and turbulent flow.*

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r} \qquad \tau = \frac{2\tau_w r}{D} \qquad \Delta p = \frac{4\ell\tau_w}{D}$$

Where: ΔP is the pressure difference (Pa), τ is the shear stress N/m^2, l is the length of pipe, r is the radial coordinate, τ_w is the *wall shear stress* (Pa) **(**At r = D/2 (the pipe wall) the shear stress is a maximum), D is the pipe diameter (m)

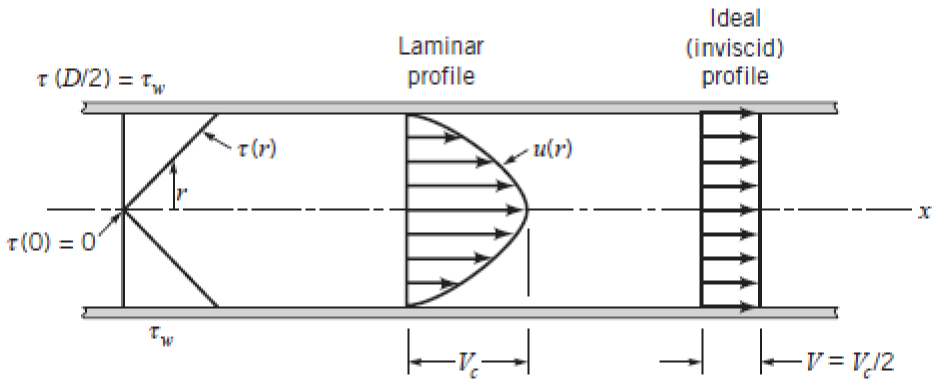


Figure 5.8. Shear stress distribution within the fluid in a pipe (laminar or turbulent flow) and typical velocity profiles

For laminar flow of a Newtonian fluid, the shear stress is simply proportional to the velocity gradient. In the notation associated with our pipe flow, this becomes

$$\tau = -\mu \frac{du}{dr}$$

The negative sign is included to give $\tau > 0$ with du/dr<0 (the velocity decreases from the pipe centerline to the pipe wall).

The velocity profile can be written as

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

$$u(r) = \mathbf{V_c}\left(1 - \frac{r^2}{R^2}\right)$$

Where; $V_c = \Delta P D^2/(16\mu l)$ is the centerline velocity. An alternative expression can be written by using the relationship between the wall shear stress and the pressure gradient to give

$$u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Where R=D/2 is the pipe radius.

The flowrate is can be written as follow.

$$V = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32\mu\ell}$$

$$\boxed{Q = \frac{\pi D^4 \, \Delta p}{128\mu\ell}}$$

The *maximum velocity* in fully developed laminar flow in a circular pipe are $V_c = 2V$

The equations for nonhorizontal pipes (Fig.5.9)

$$\frac{\Delta p - \gamma\ell \sin\theta}{\ell} = \frac{2\tau}{r}$$

Thus, all of the results for the horizontal pipe are valid provided the pressure gradient is adjusted for the elevation term, that is, $\Delta P\ is$ replaced by $\Delta P - \gamma l sin\theta$ so that

$$V = \frac{(\Delta p - \gamma\ell \sin\theta)D^2}{32\mu\ell}$$

$$Q = \frac{\pi(\Delta p - \gamma\ell \sin\theta)D^4}{128\mu\ell}$$

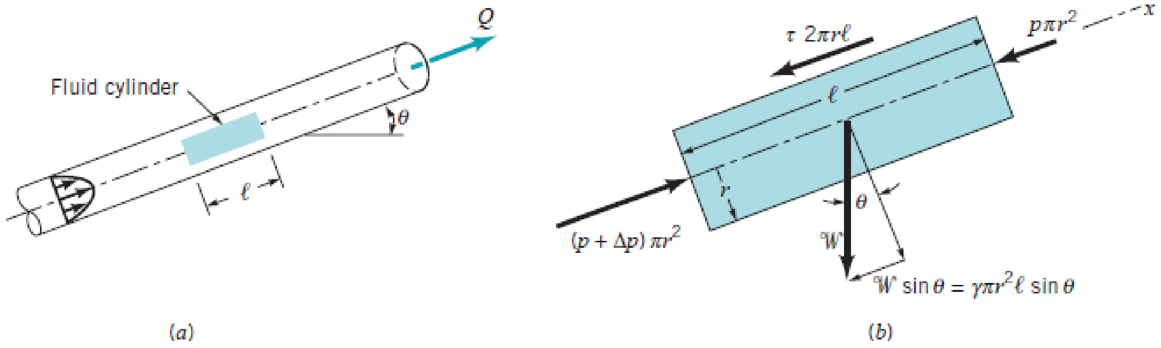


Figure 5.9. Free-body diagram of a fluid cylinder for flow in a nonhorizontal pipe.

It is seen that the driving force for pipe flow can be either a pressure drop in the flow direction, $\Delta P,$ or the component of weight in the flow direction, $-\gamma lsin\theta$. If the flow is downhill, gravity helps the flow (a smaller pressure drop is required; $sin\theta < 0$). If the flow is uphill, gravity works against the flow (a larger pressure drop is required; $sin\theta > 0$) . Note that $\gamma lsin\theta = \gamma \Delta z$ (where Δz *is* the change in elevation) is a hydrostatic type pressure term. If there is no flow V=0 and $\Delta P = \gamma lsin\theta = \gamma \Delta z,$ as expected for fluid statics (Fig 5.9).

It is usually advantageous to describe a process in terms of dimensionless quantities. To this end we rewrite the pressure drop equation for laminar horizontal pipe flow, as $\Delta P = 32\mu lV/D^2$ and divide both sides by the dynamic pressure, $\rho V^2/2$ obtain the dimensionless form as

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2} \qquad f = \Delta p(D/\ell)/(\rho V^2/2)$$

is termed the ***friction factor,*** or sometimes the *Darcy friction factor (*This parameter should not be confused with the less-used Fanning friction. which is defined to be f/4. In this text we will use only the Darcy fiction factor.) Thus the friction factor for laminar fully developed pipe flow is simply f= 64/Re. By substituting the pressure drop in terms of the wall shear stress, we obtain an alternate expression for the friction factor as a dimensionless wall shear stress

$$f = \frac{8\tau_w}{\rho V^2}$$

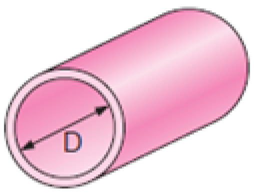
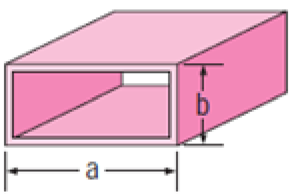
Knowledge of the friction factor will allow us to obtain a variety of information regarding pipe flow. For turbulent flow the dependence of the friction factor on the Reynolds number is much more complex than that given by f=64/Re for laminar flow.

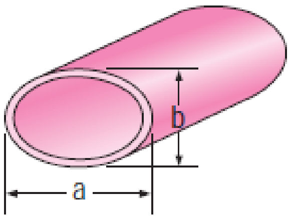
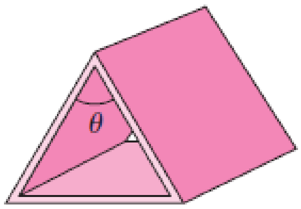
5.2.1. Laminar Flow in Noncircular Pipes

The friction factor f relations are given in the below Table 5.1. for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter D_h=4Ac /p, where Ac is the cross-sectional area of the pipe and p is its wetted perimeter.

Table 5.1. Friction factor for fully developed laminar flow

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $Re = V_{avg} D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re

Ellipse 	*a*/*b*	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle 	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re