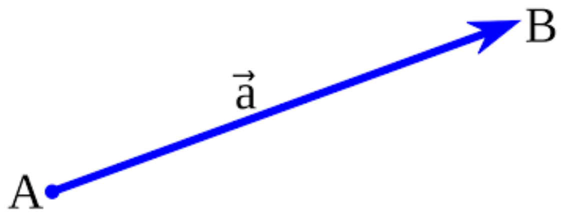
PHYSICS I

Assoc.Prof.Dr. Yeşim Moğulkoç



Vectors

Lecture 2

Vector quantities

- Physical quantities that have both numerical and directional properties

Mathematical operations of vectors in this chapter

- Addition
- Subtraction

What are vectors?

Examples

Vectors	Scalars
Position **r**	Time *t*
Velocity **v**	Mass *m*
Acceleration **a**	Volume *V*
Force **F**	Density *m/V*
Momentum **p**	Vector components

Coordinate Systems

Used to describe the position of a point in space

Common coordinate systems are:

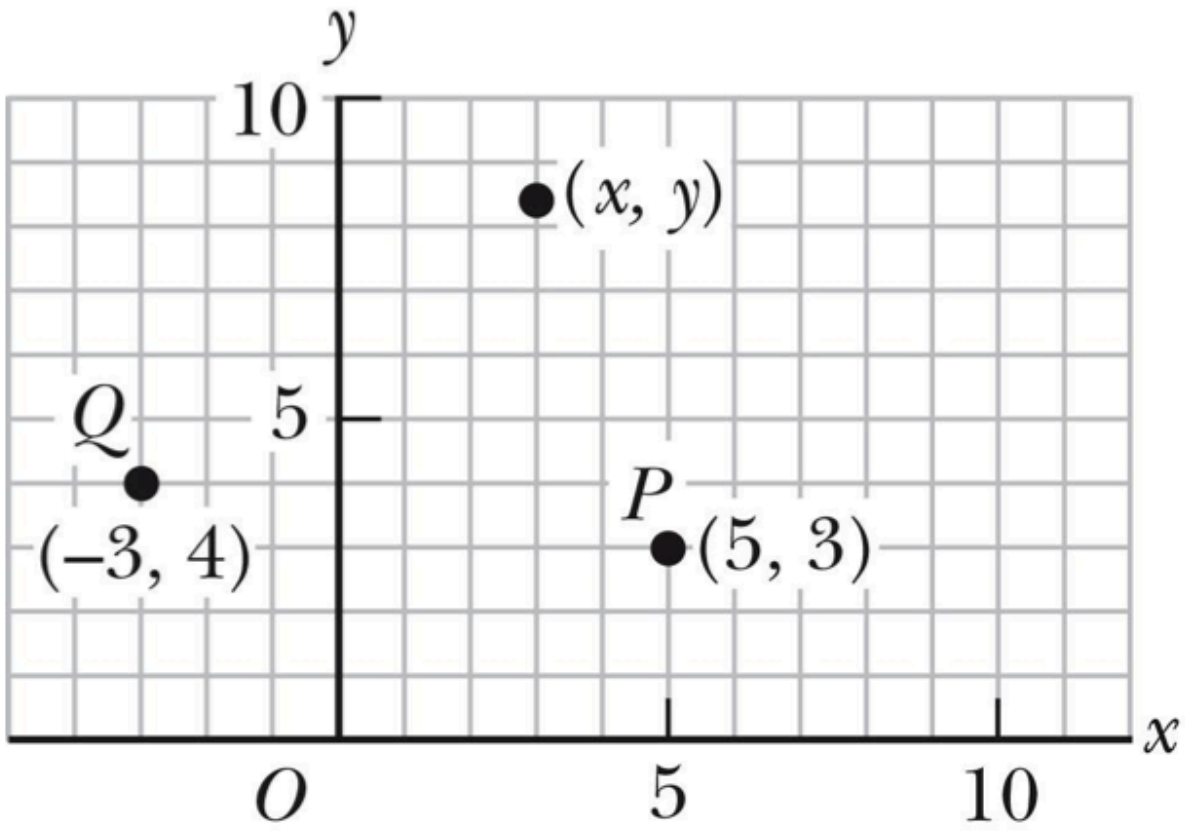
- Cartesian
- Polar

Cartesian Coordinate System

Also called rectangular coordinate system

x- and *y*- axes intersect at the origin

Points are labeled (*x*,*y*)



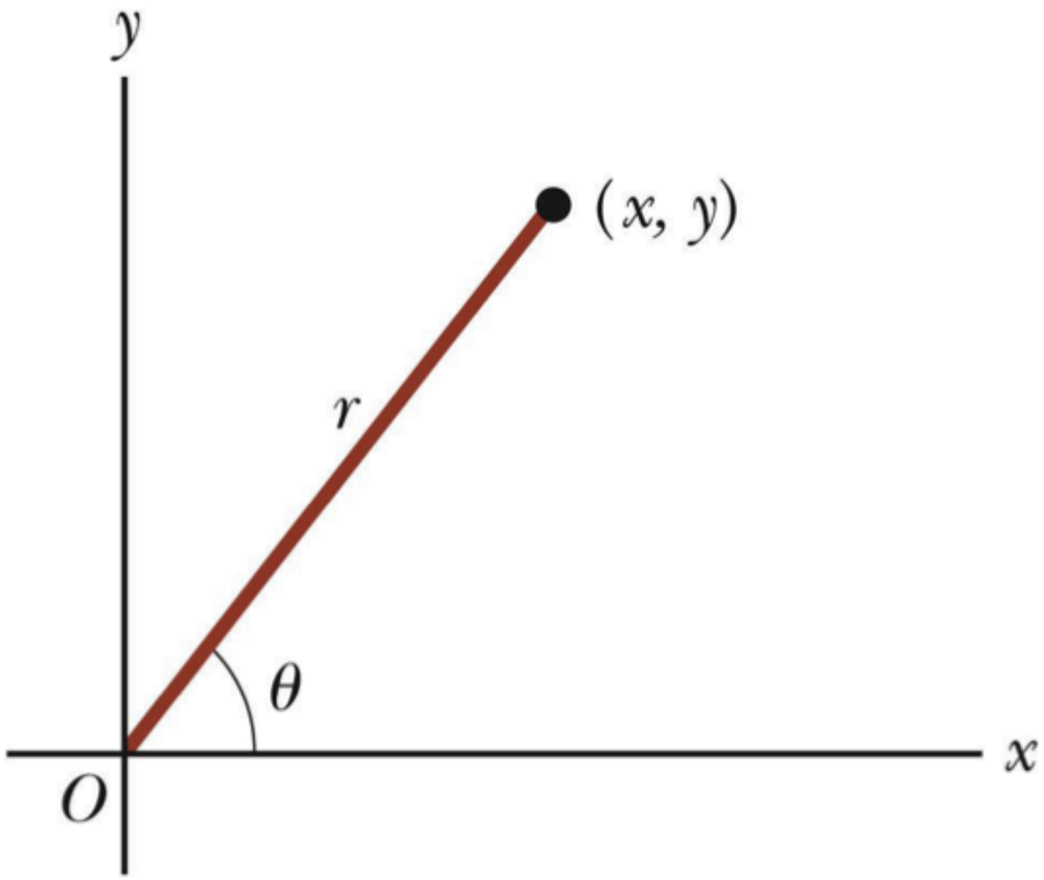
Polar Coordinate System

Origin and reference line are noted

Point is distance *r* from the origin in the direction of angle θ, ccw from reference line

- The reference line is often the x-axis.

Points are labeled (r, θ)



Polar to Cartesian Coordinates

Based on forming a right triangle from r and θ

$x = r\cos\theta$

$y = r\sin\theta$

If the Cartesian coordinates are known:

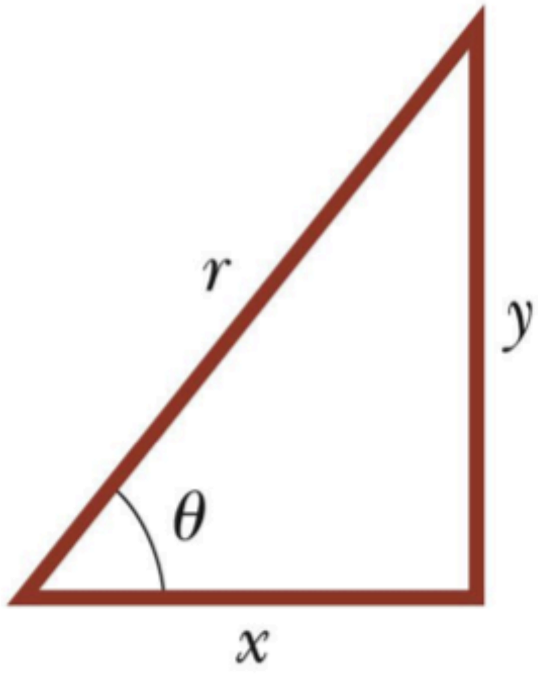
$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}$$

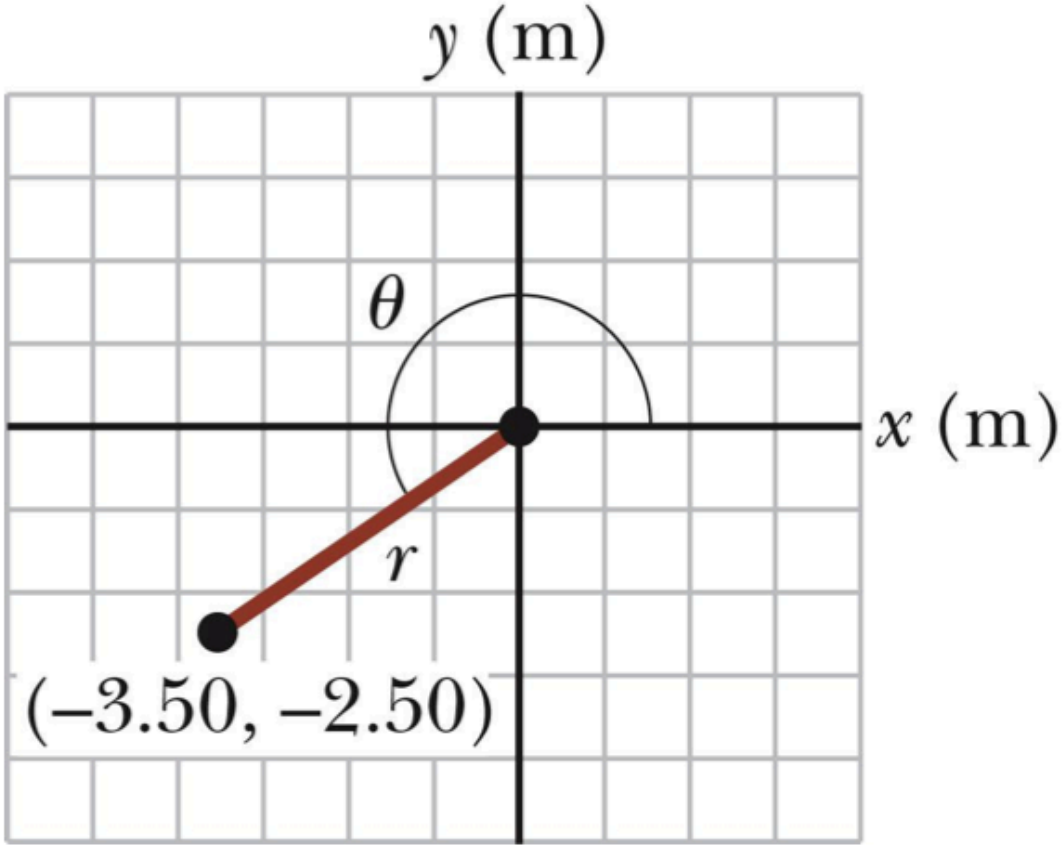
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



Example 3.1

The Cartesian coordinates of a point in the *xy* plane are $(x,y) = (-3.50, -2.50)$ m, as shown in the figure. Find the polar coordinates of this point.



Solution: From Equation 3.4,

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2}$$
$$= 4.30\text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$
$$\theta = 216^\circ \quad \text{(signs give quadrant)}$$

Vectors and Scalars

A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.

- Many are always positive
- Some may be positive or negative
- Rules for ordinary arithmetic are used to manipulate scalar quantities.

A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

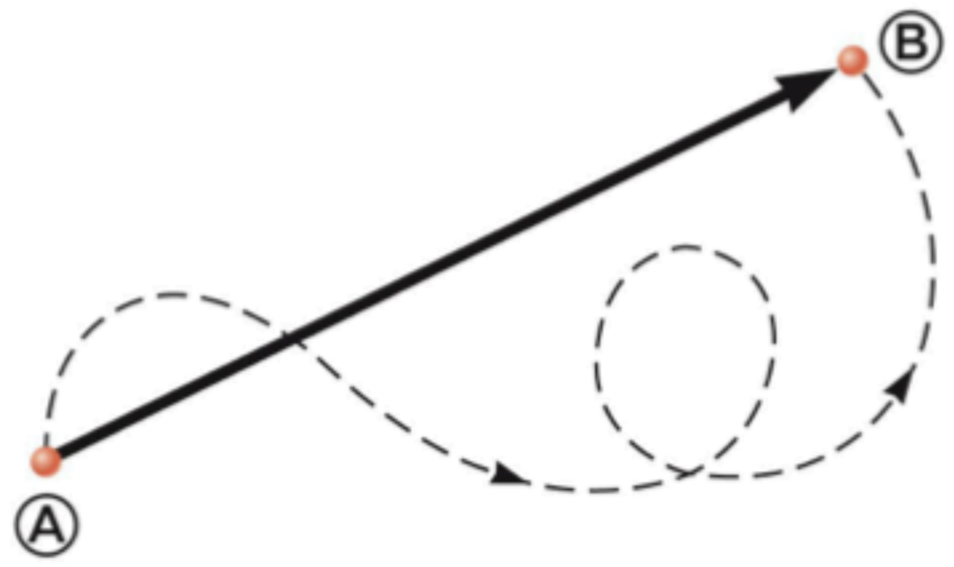
Vector Example

A particle travels from A to B along the path shown by the broken line.

- This is the ***distance*** traveled and is a scalar.

The ***displacement*** is the solid line from A to B

- The displacement is independent of the path taken between the two points.
- Displacement is a vector.



Vector Notation

Text uses bold with arrow to denote a vector: $\vec{\mathbf{A}}$

Also used for printing is simple bold print: **A**

When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|\vec{\mathbf{A}}|$

- The magnitude of the vector has physical units.
- The magnitude of a vector is always a positive number.

When handwritten, use an arrow: $\vec{A}$

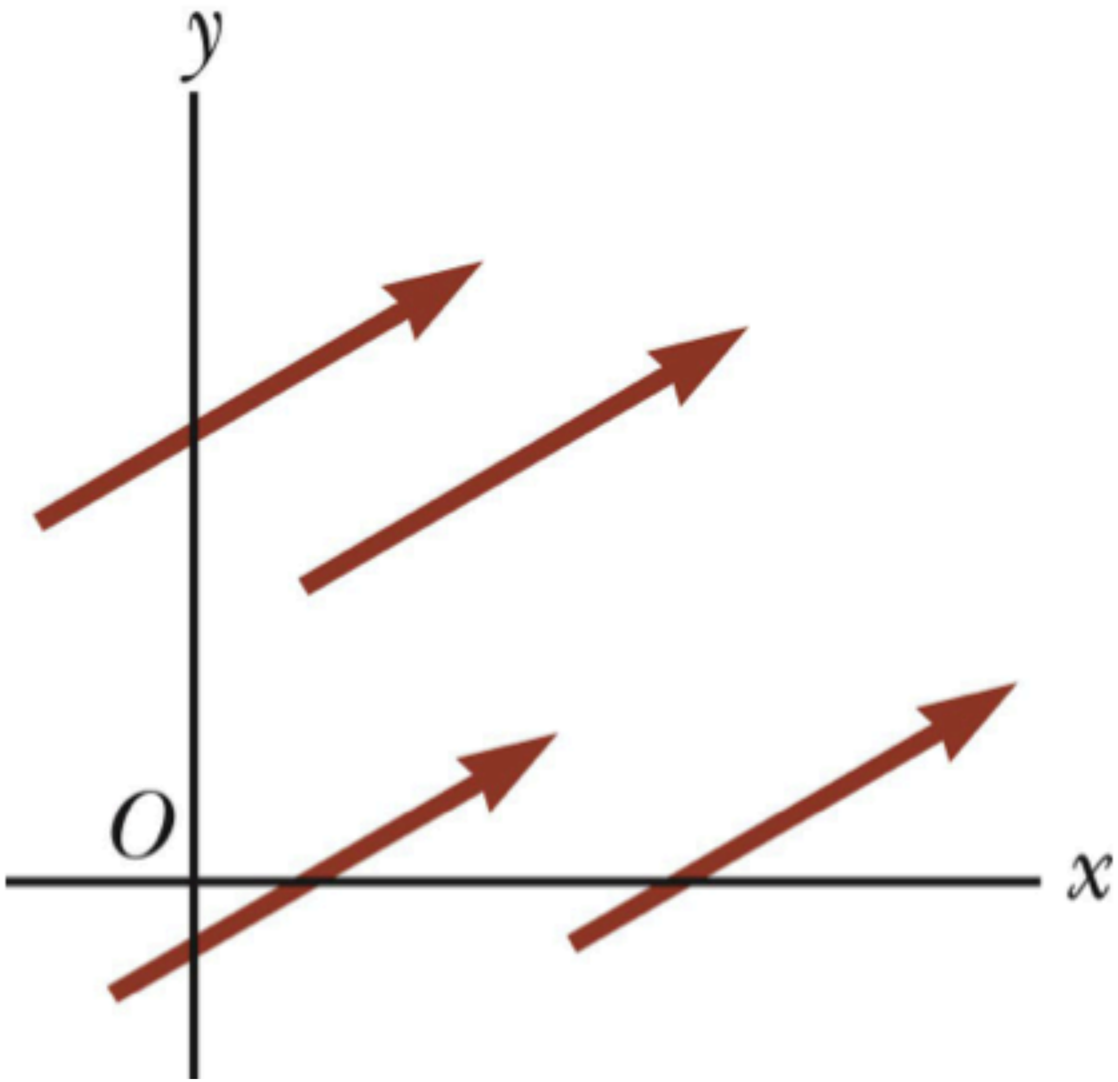
Equality of Two Vectors

Two vectors are ***equal*** if they have the same magnitude and the same direction.

$\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if $A = B$ and they point along parallel lines

All of the vectors shown are equal.

Allows a vector to be moved to a position parallel to itself



Adding Vectors

Vector addition is very different from adding scalar quantities.

When adding vectors, their directions must be taken into account.

Units must be the same

Graphical Methods

- Use scale drawings

Algebraic Methods

- More convenient

Adding Vectors Graphically

Choose a scale.

Draw the first vector, $\vec{\mathbf{A}}$, with the appropriate length and in the direction specified, with respect to a coordinate system.

Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{\mathbf{A}}$ and parallel to the coordinate system used for $\vec{\mathbf{A}}$.

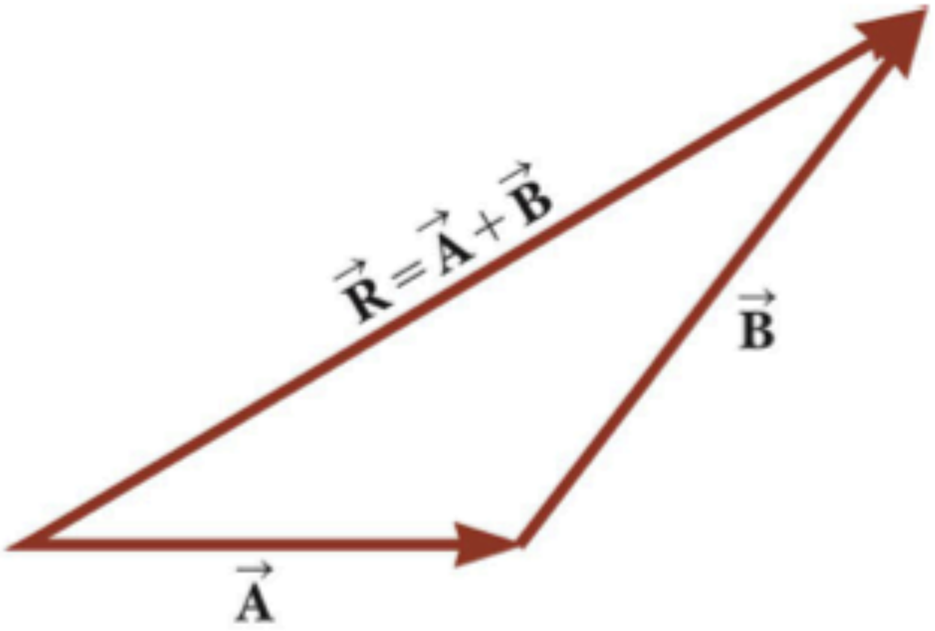
Adding Vectors Graphically, cont.

Continue drawing the vectors "tip-to-tail" or "head-to-tail".

The resultant is drawn from the origin of the first vector to the end of the last vector.

Measure the length of the resultant and its angle.

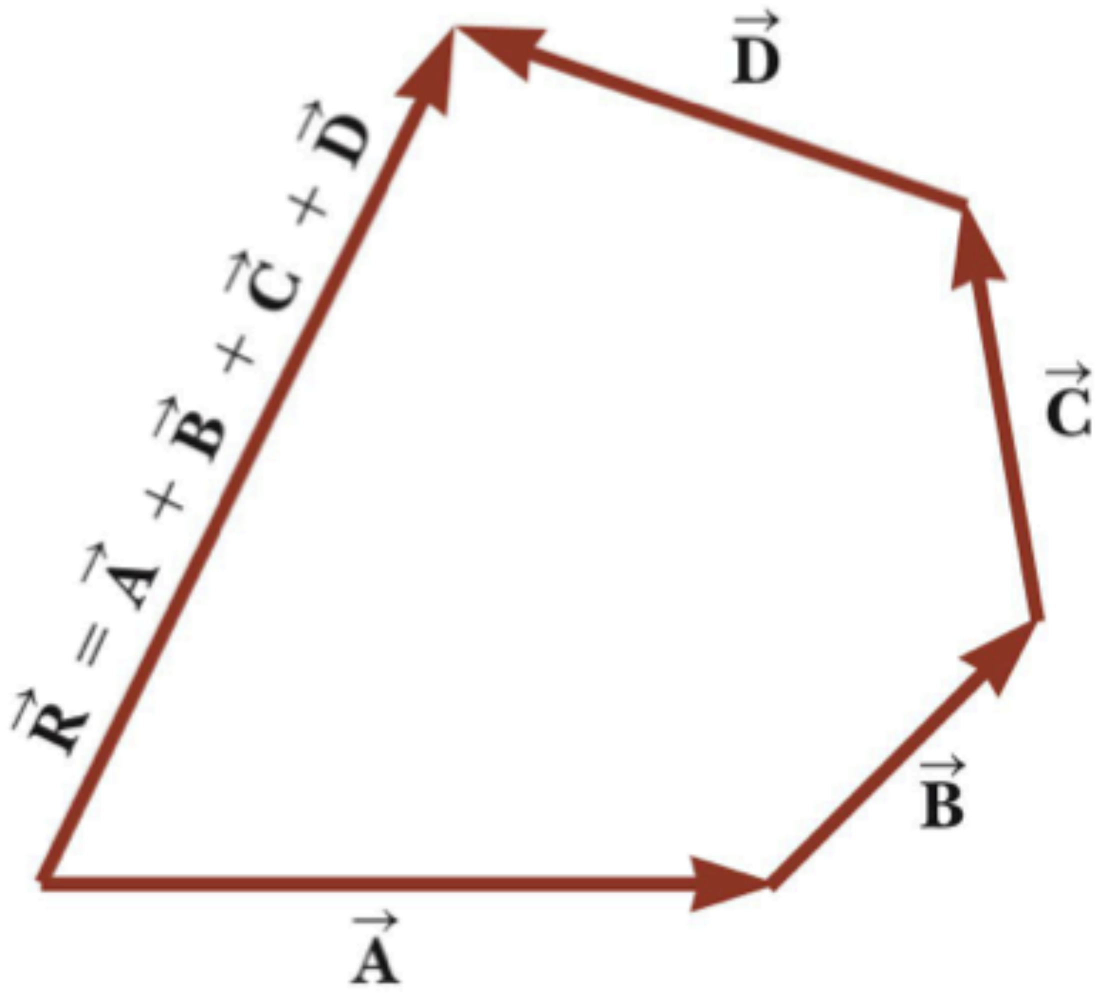
- Use the scale factor to convert length to actual magnitude.



Adding Vectors Graphically, final

When you have many vectors, just keep repeating the process until all are included.

The resultant is still drawn from the tail of the first vector to the tip of the last vector.

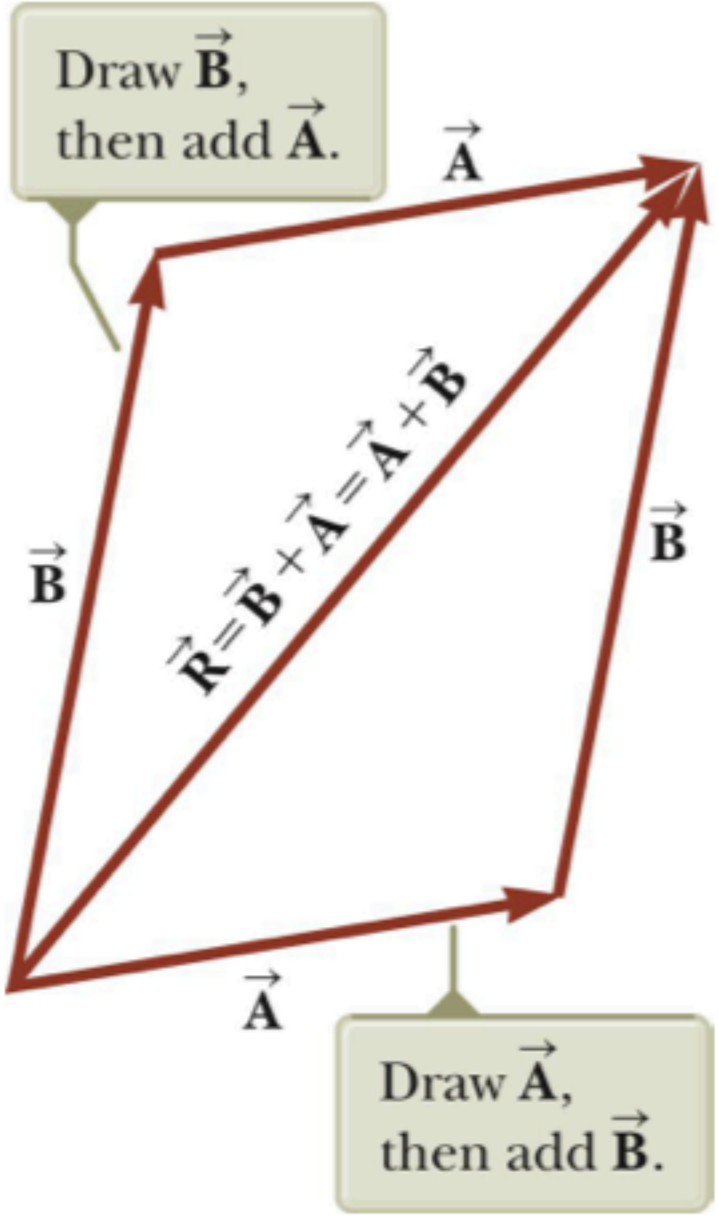


Adding Vectors, Rules

When two vectors are added, the sum is independent of the order of the addition.

- This is the ***Commutative Law of Addition.***

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

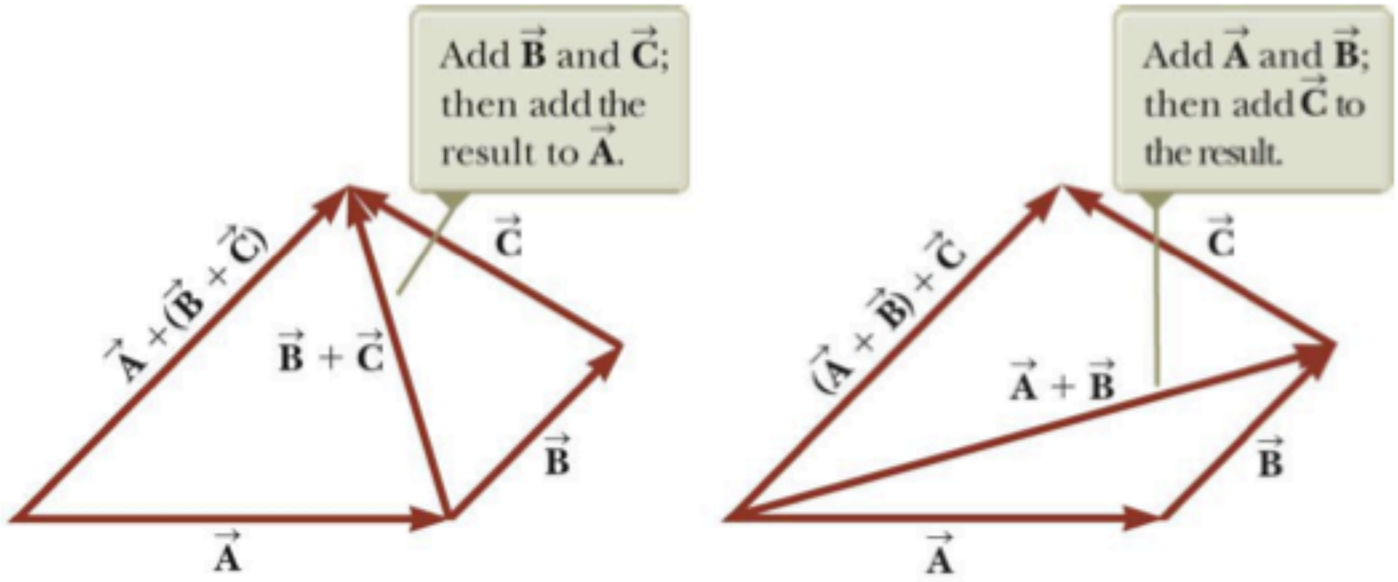


Adding Vectors, Rules cont.

When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped.

- This is called the ***Associative Property of Addition.***

$$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}$$



Adding Vectors, Rules final

When adding vectors, all of the vectors must have the same units.

All of the vectors must be of the same type of quantity.

- For example, you cannot add a displacement to a velocity.

Negative of a Vector

The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero.

- Represented as $-\vec{\mathbf{A}}$
- $\vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$

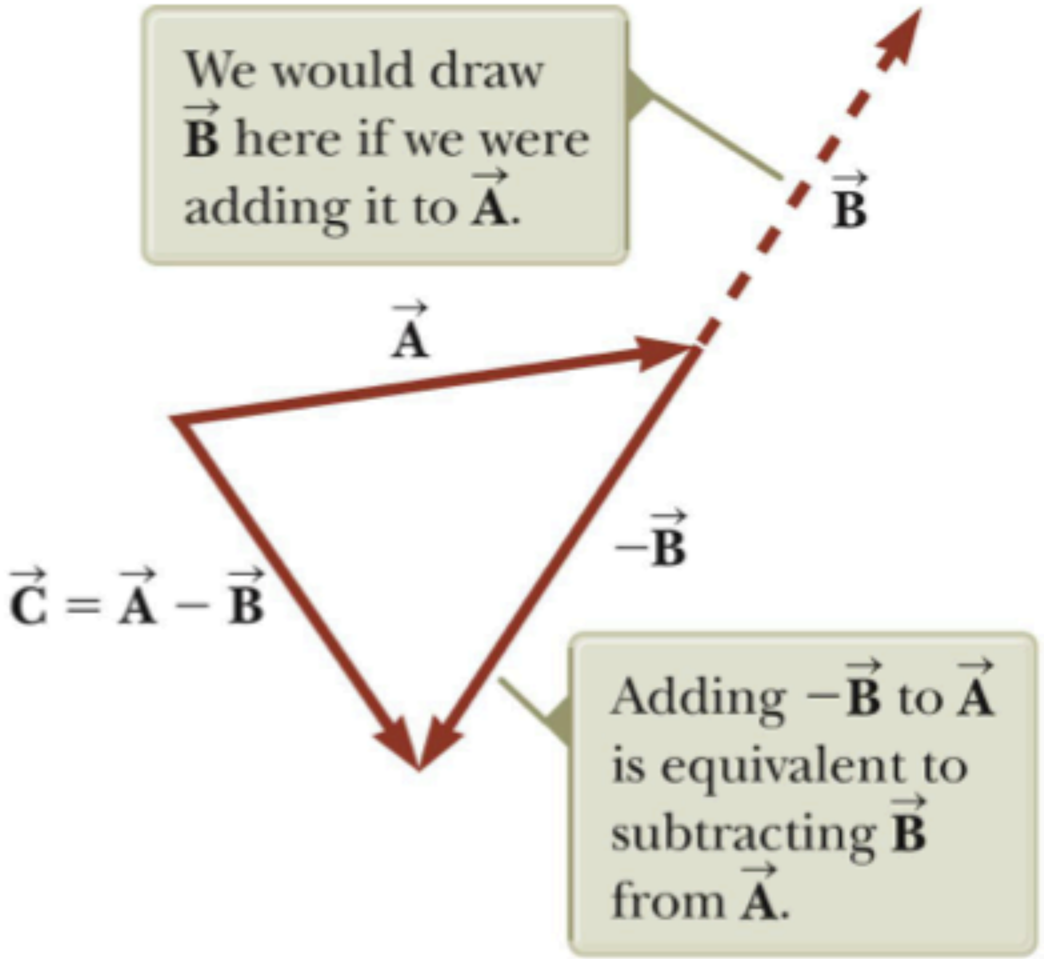
The negative of the vector will have the same magnitude, but point in the opposite direction.

Subtracting Vectors

Special case of vector addition:

If $\vec{A}-\vec{B}$, then use $\vec{A}+\left(-\vec{B}\right)$

Continue with standard vector addition procedure.

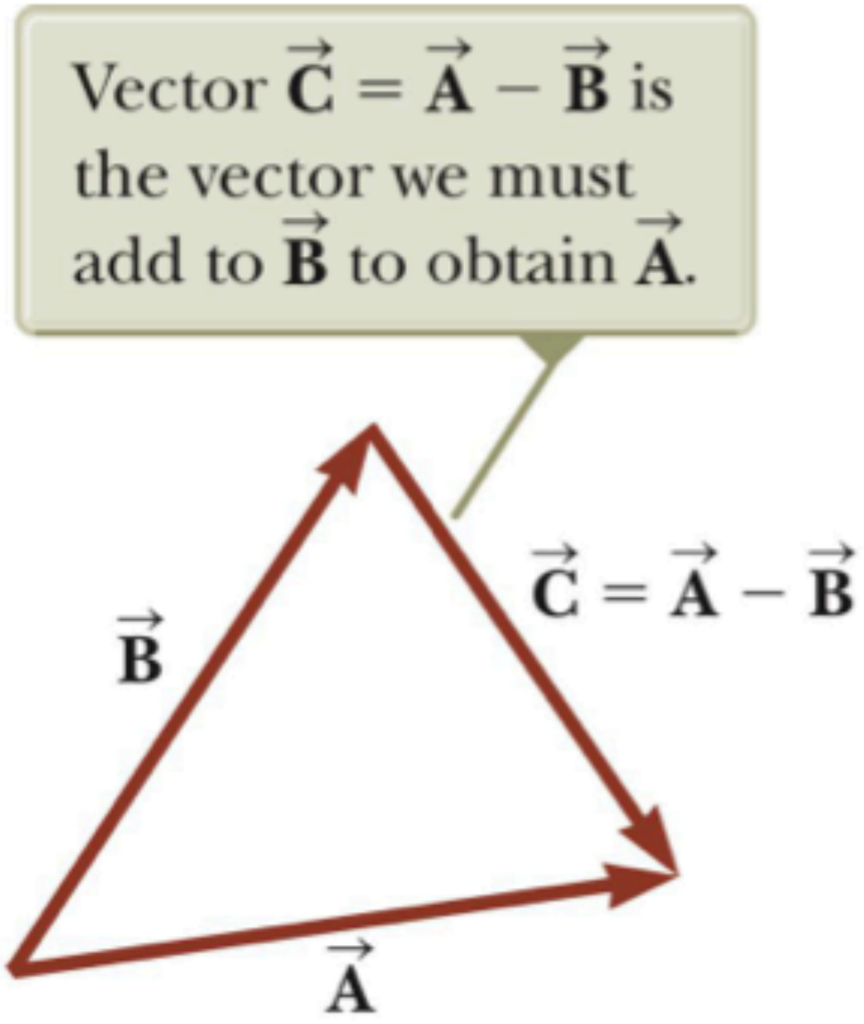


Subtracting Vectors, Method 2

Another way to look at subtraction is to find the vector that, added to the second vector gives you the first vector.

$$\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right) = \vec{\mathbf{C}}$$

- As shown, the resultant vector points from the tip of the second to the tip of the first.



Multiplying or Dividing a Vector by a Scalar

The result of the multiplication or division of a vector by a scalar is a vector.

The magnitude of the vector is multiplied or divided by the scalar.

If the scalar is positive, the direction of the result is the same as of the original vector.

If the scalar is negative, the direction of the result is opposite that of the original vector