PHYSICS I

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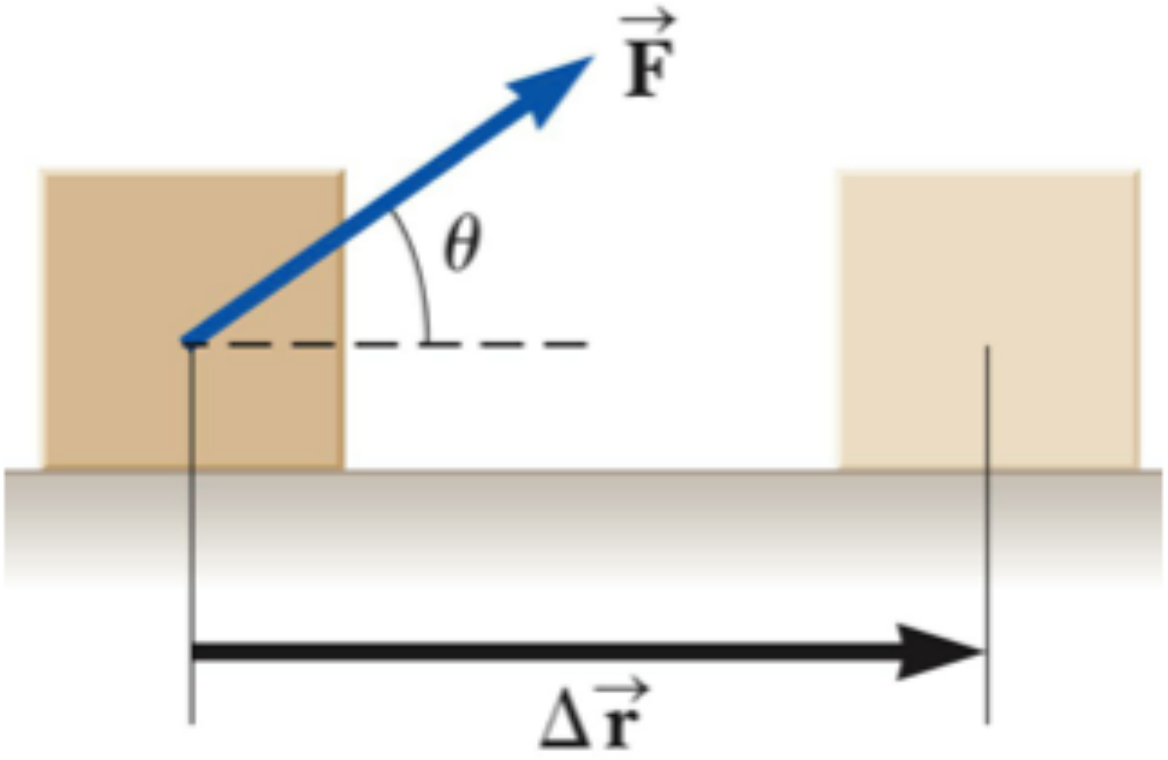
ENERGY & WORK

Work

$W = F \Delta r \cos \theta$

- The displacement is that of the point of application of the force.
- A force does no work on the object if the force does not move through a displacement.
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.



Displacement in the Work Equation

The displacement is that of the point of application of the force.

If the force is applied to a rigid object that can be modeled as a particle, the displacement is the same as that of the particle.

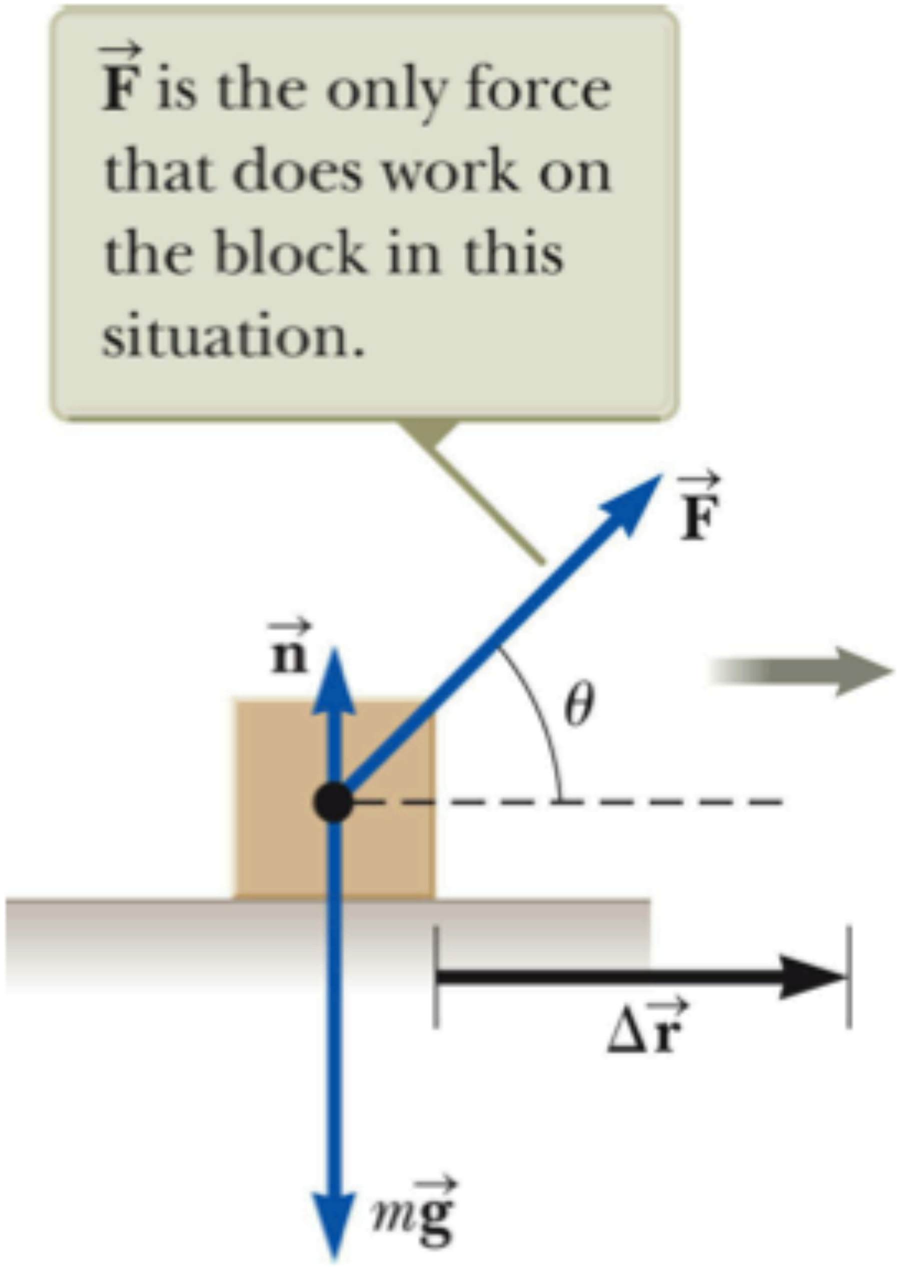
For a deformable system, the displacement of the object generally is not the same as the displacement associated with the forces applied.

Work Example

The normal force and the gravitational force do no work on the object.

- $\cos\theta = \cos 90^\circ = 0$

The force $\vec{\mathbf{F}}$ is the only force that does work on the object.



Scalar Product of Two Vectors

The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

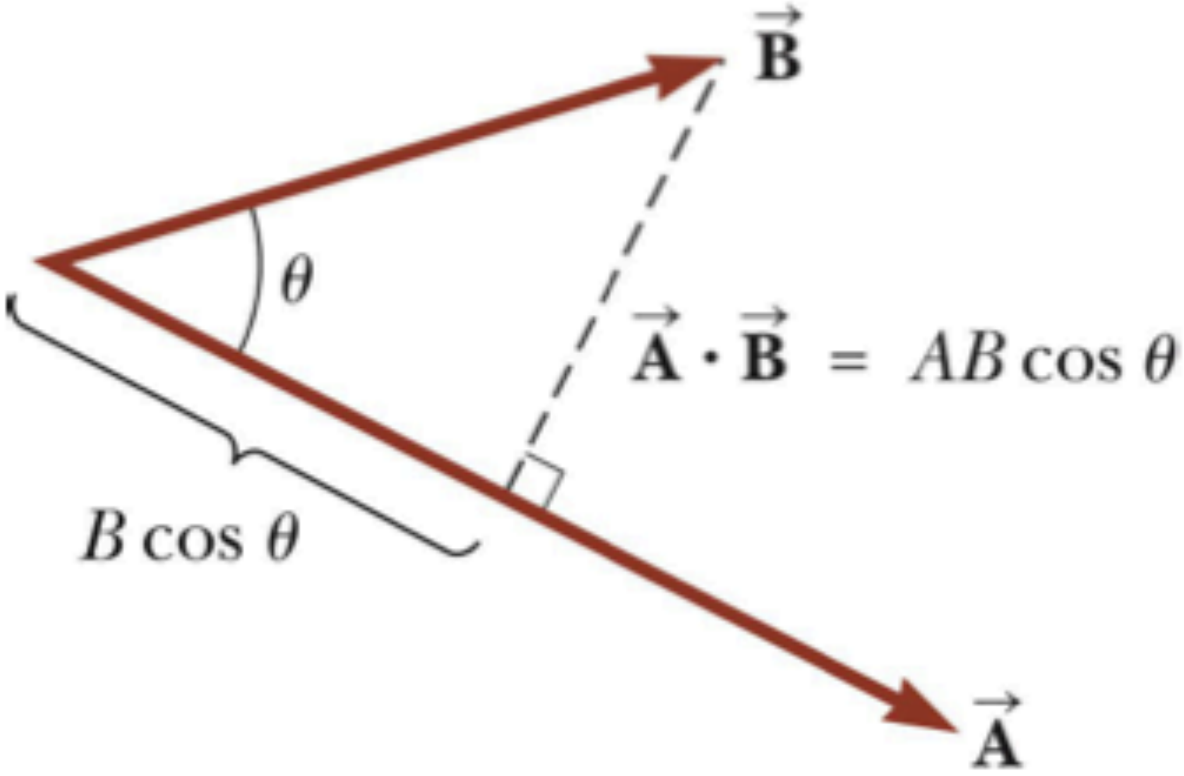
- It is also called the dot product.

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\,\cos\theta$$

- θ is the angle *between* A and B

Applied to work, this means

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$



Work Done by a Varying Force

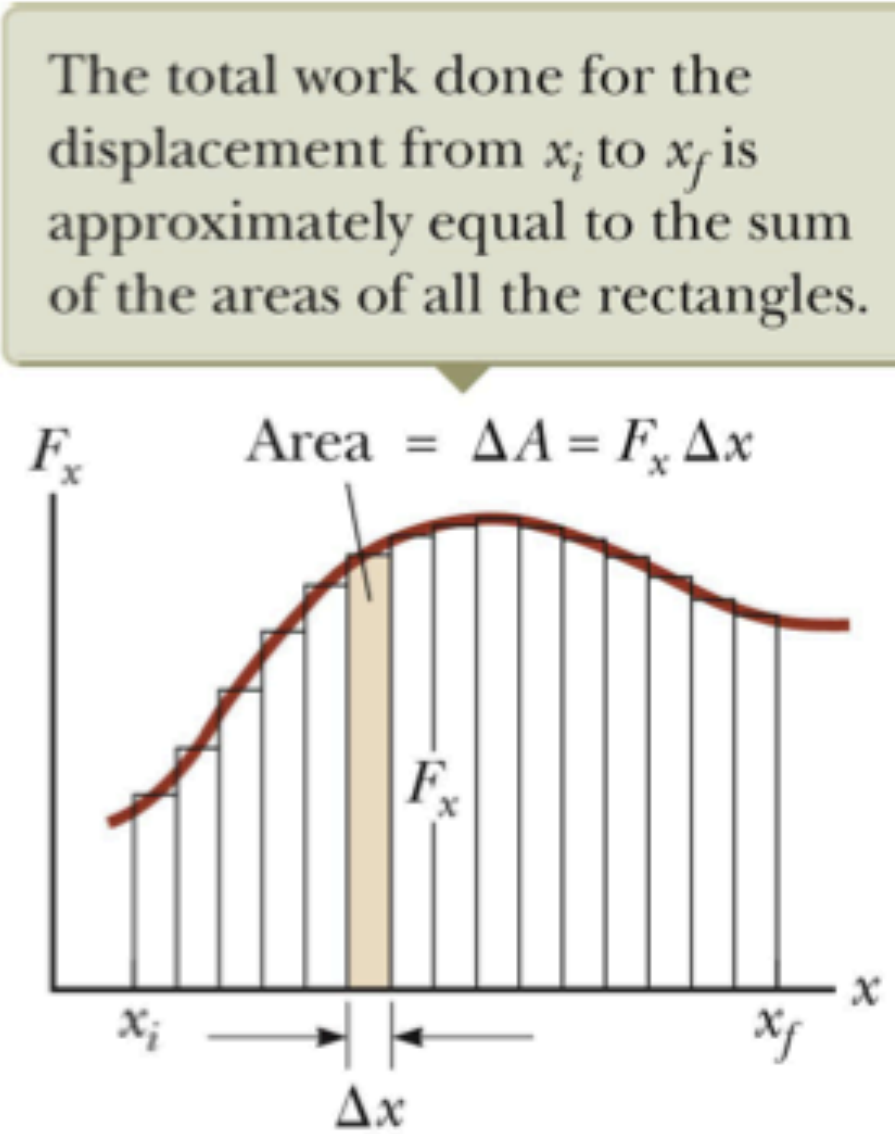
To use W = F Δ r cos θ, the force must be constant, so the equation cannot be used to calculate the work done by a varying force.

Assume that during a very small displacement, Δ*x*, *F* is constant.

For that displacement, $W \sim F \Delta x$

For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont.

Let the size of the small displacements approach zero .

Since

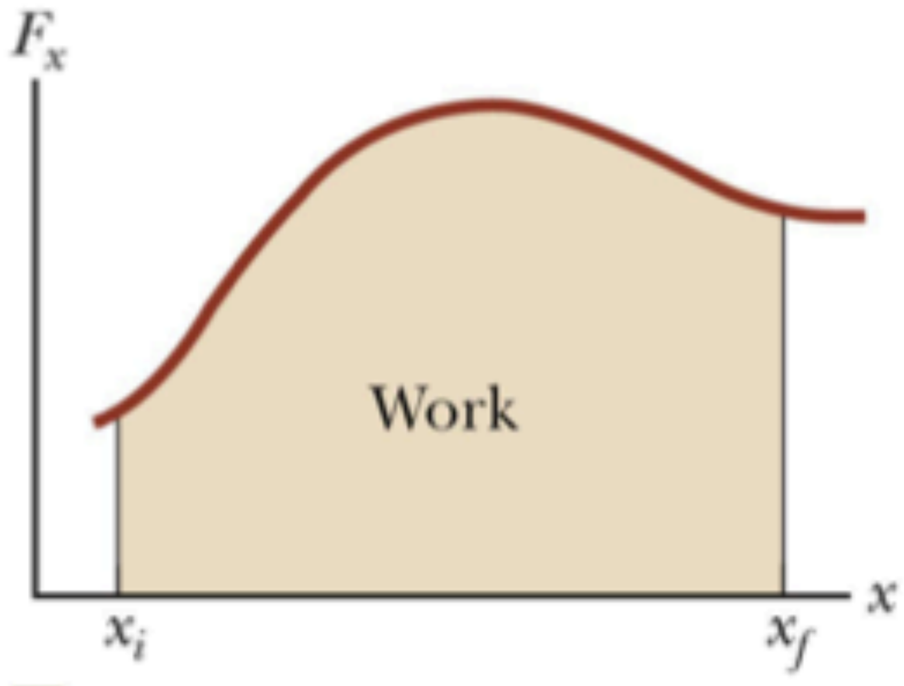
$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Therefore,

$$W = \int_{x_i}^{x_f} F_x dx$$

The work done is equal to the area under the curve between x_i and x_f.

The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under the curve.

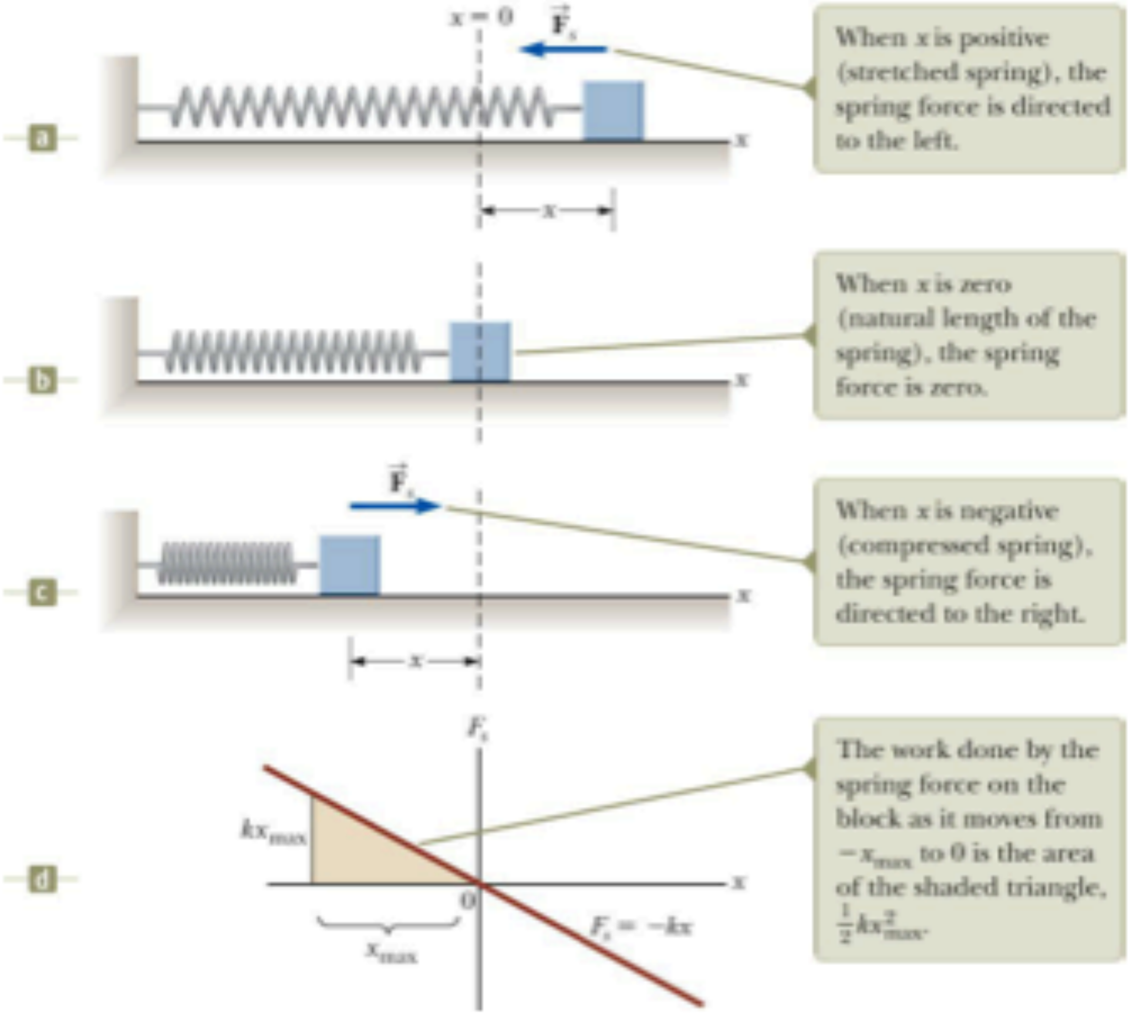


Work Done By A Spring

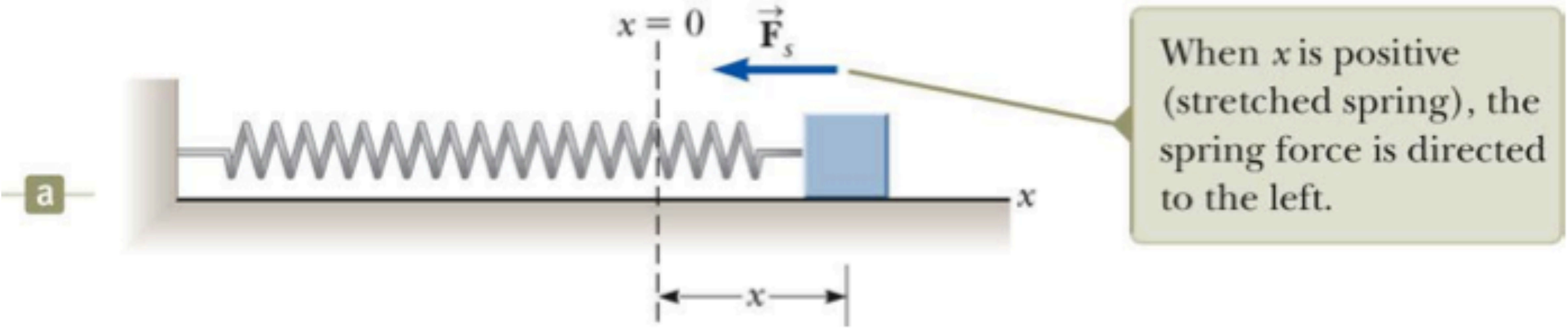
A model of a common physical system for which the force varies with position.

The block is on a horizontal, frictionless surface.

Observe the motion of the block with various values of the spring constant.



Spring Force (Hooke's Law)



The force exerted by the spring is

$$F_s = -kx$$

- x is the position of the block with respect to the equilibrium position ($x = 0$).
- k is called the spring constant or force constant and measures the stiffness of the spring.
  - k measures the *stiffness* of the spring.

Work Done by a Spring, cont.

Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$.

The work done by the spring on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- If the motion ends where it begins, W = 0

Kinetic Energy

One possible result of work acting as an influence on a system is that the system changes its speed.

The system could possess *kinetic energy*.

Kinetic Energy is the energy of a particle due to its motion.

- $K = \frac{1}{2} mv^2$
  - K is the kinetic energy
  - m is the mass of the particle
  - v is the speed of the particle

A change in kinetic energy is one possible result of doing work to transfer energy into a system.

Kinetic Energy, cont

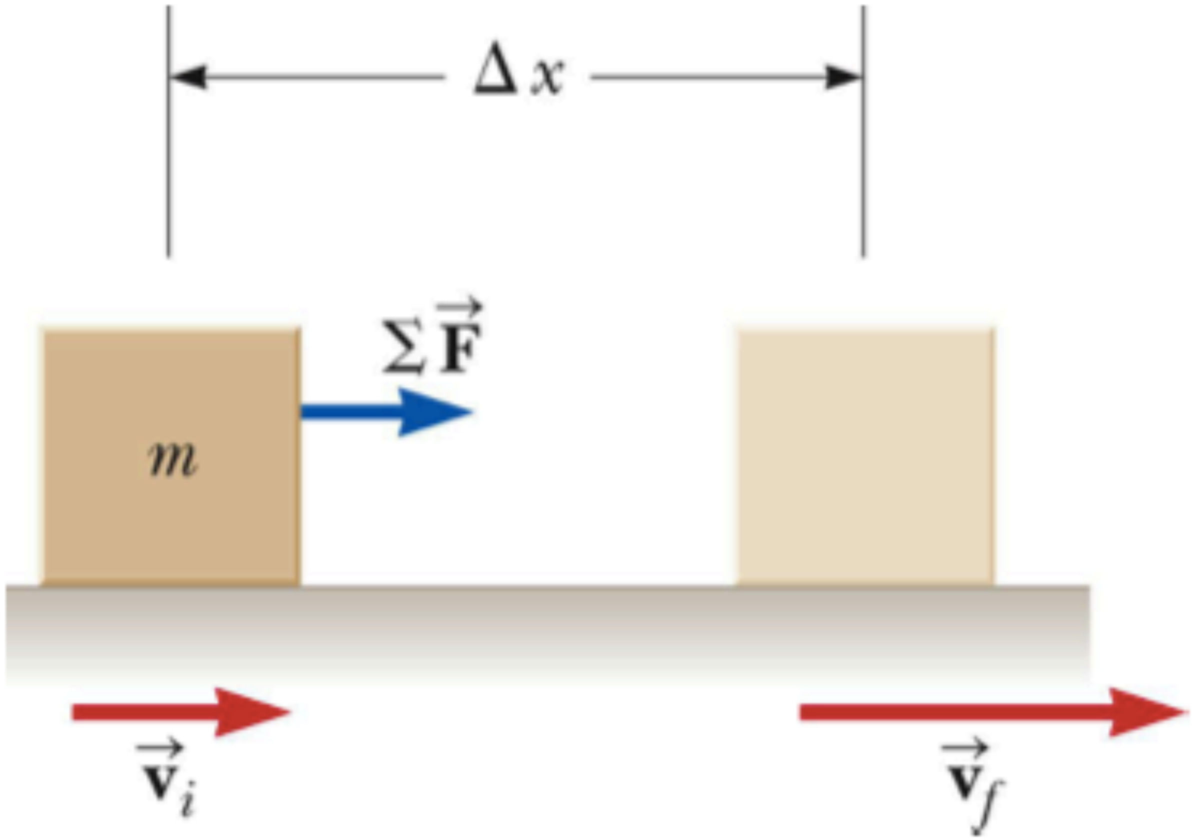
Calculating the work:

$$W_{ext} = \int_{x_i}^{x_f} \sum F \, dx = \int_{x_i}^{x_f} ma \, dx$$

$$W_{ext} = \int_{v_i}^{v_f} mv \, dv$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{ext} = K_f - K_i = \Delta K$$



Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem states $W_{ext} = K_f - K_i = \Delta K$

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

- The speed of the system increases if the work done on it is positive.
- The speed of the system decreases if the net work is negative.
- Also valid for changes in rotational speed

The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.

The work-kinetic energy theorem applies to the speed of the system, not its velocity.

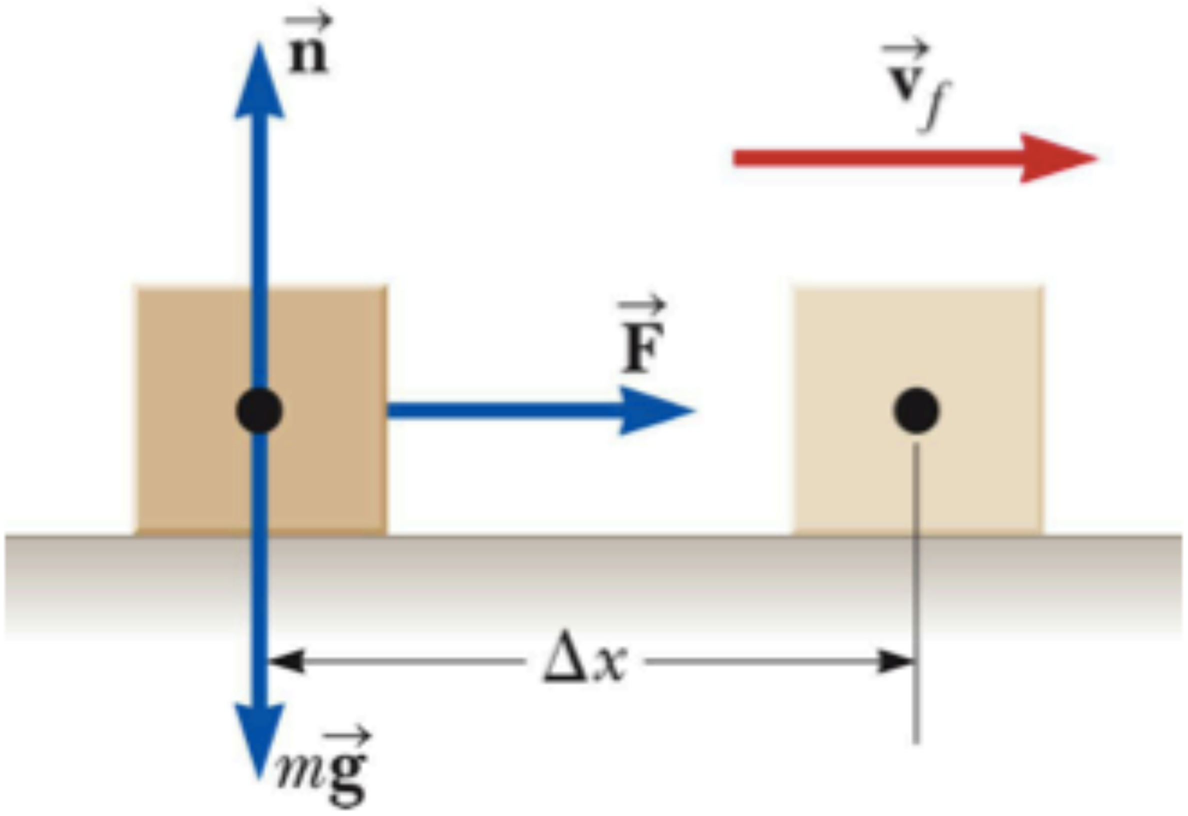
Work-Kinetic Energy Theorem – Example

The block is the system and three external forces act on it.

The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement.

$W_{ext} = \Delta K = ½\, m v_f^2 - 0$

The answer could be checked by modeling the block as a particle and using the kinematic equations.



Potential Energy

Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.

- The forces are internal to the system.
- Can be associated with only specific types of forces acting between members of a system

Gravitational Potential Energy

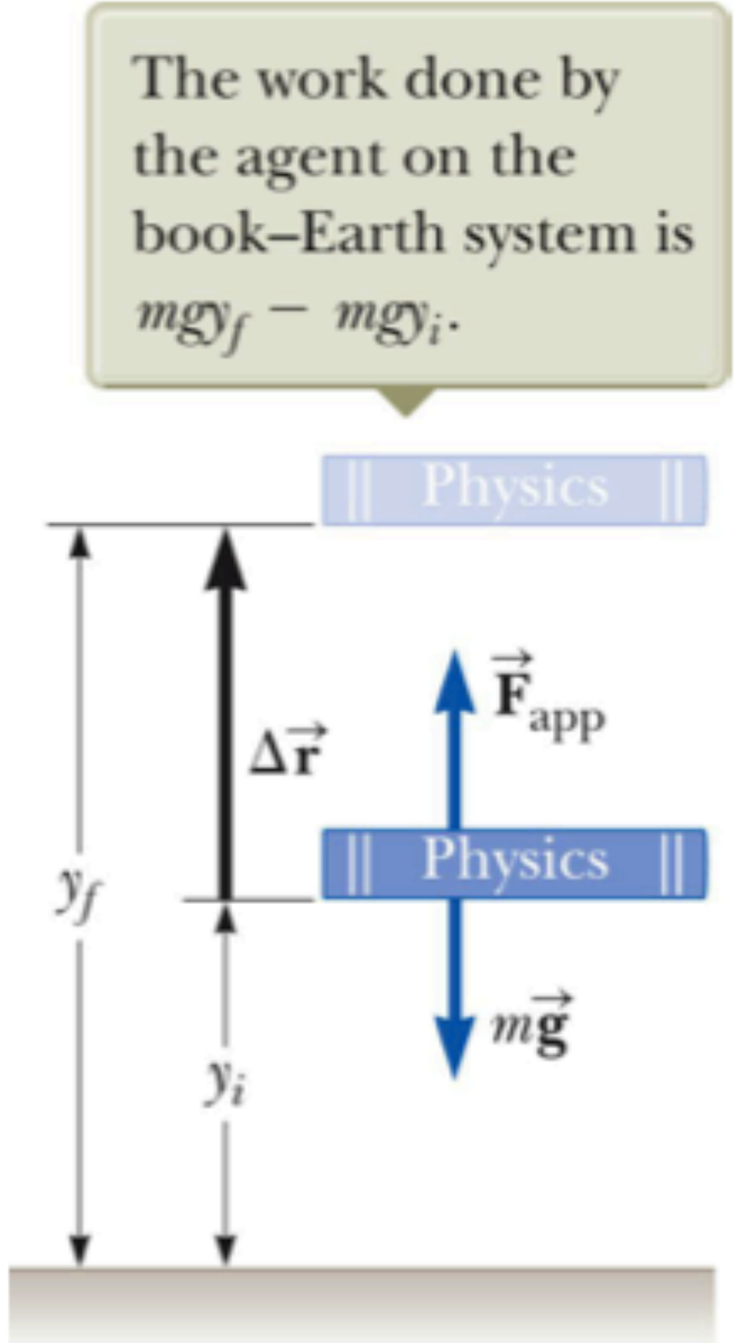
The system is the Earth and the book.

Do work on the book by lifting it slowly through a vertical displacement.

$$\Delta\vec{\mathbf{r}} = (y_f - y_i)\,\hat{\mathbf{j}}$$

The work done on the system must appear as an increase in the energy of the system.

The energy storage mechanism is called *potential energy*.



Gravitational Potential Energy, final

The quantity *mgy* is identified as the gravitational potential energy, U_g.

- $U_g = mgy$

Units are joules (J)

Is a scalar

Work may change the gravitational potential energy of the system.

- $W_{ext} = \Delta u_g$

Potential energy is always associated with a system of two or more interacting objects.

Elastic Potential Energy

Elastic Potential Energy is associated with a spring.

The force the spring exerts (on a block, for example) is $F_s = - kx$

The work done by an external applied force on a spring-block system is

- $W = ½ kx_f^2 – ½ kx_i^2$
- The work is equal to the difference between the initial and final values of an expression related to the configuration of the system.

Elastic Potential Energy, cont.

This expression is the elastic potential energy:

$$U_s = \frac{1}{2} kx^2$$

The elastic potential energy can be thought of as the energy stored in the deformed spring.

The stored potential energy can be converted into kinetic energy.

Observe the effects of different amounts of compression of the spring.

Conservative Forces

The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

The work done by a conservative force on a particle moving through any closed path is zero.

- A closed path is one in which the beginning and ending points are the same.

Examples of conservative forces:

- Gravity
- Spring force

Conservative Forces, cont

We can associate a potential energy for a system with any conservative force acting between members of the system.

- This can be done only for conservative forces.
- In general: $W_{int} = - \Delta U$
  - W_{int} is used as a reminder that the work is done by one member of the system on another member and is internal to the system.
- Positive work done by an outside agent on a system causes an increase in the potential energy of the system.
- Work done on a component of a system by a conservative force internal to an isolated system causes a decrease in the potential energy of the system.

Non-conservative Forces

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

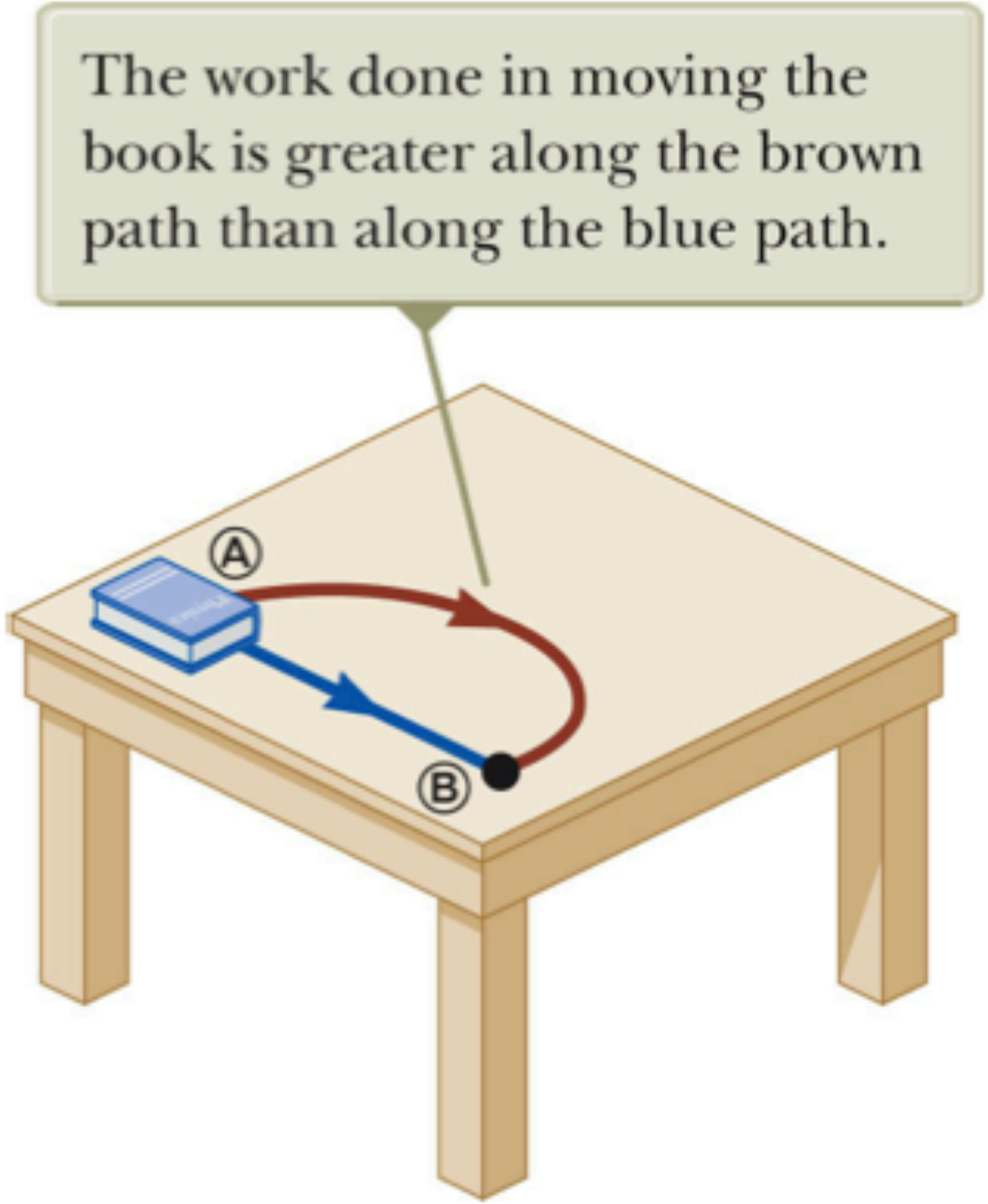
$E_{mech} = K + U$

- K includes the kinetic energy of all moving members of the system.
- U includes all types of potential energy in the system.

Non-conservative Forces, cont.

The work done against friction is greater along the brown path than along the blue path.

Because the work done depends on the path, friction is a non-conservative force.



Conservative Forces and Potential Energy

Define a potential energy function, *U*, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, *F*, is

$$W_{\text{int}} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

- ΔU is negative when F and x are in the same direction

Conservative Forces and Potential Energy

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x*.

- Can be extended to three dimensions

Conservative Forces and Potential Energy – Check

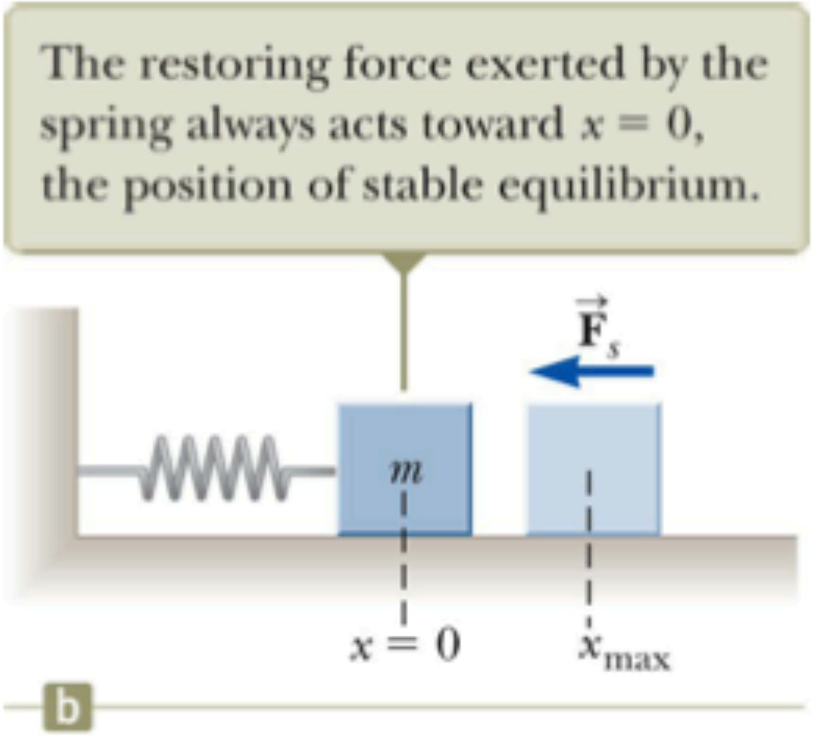
Look at the case of a deformed spring:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

- This is Hooke's Law and confirms the equation for U

U is an important function because a conservative force can be derived from it.

Energy Diagrams and Equilibrium



Motion in a system can be observed in terms of a graph of its position and energy.

In a spring-mass system example, the block oscillates between the turning points, $x = \pm x_{max}$.

The block will always accelerate back toward $x = 0$.

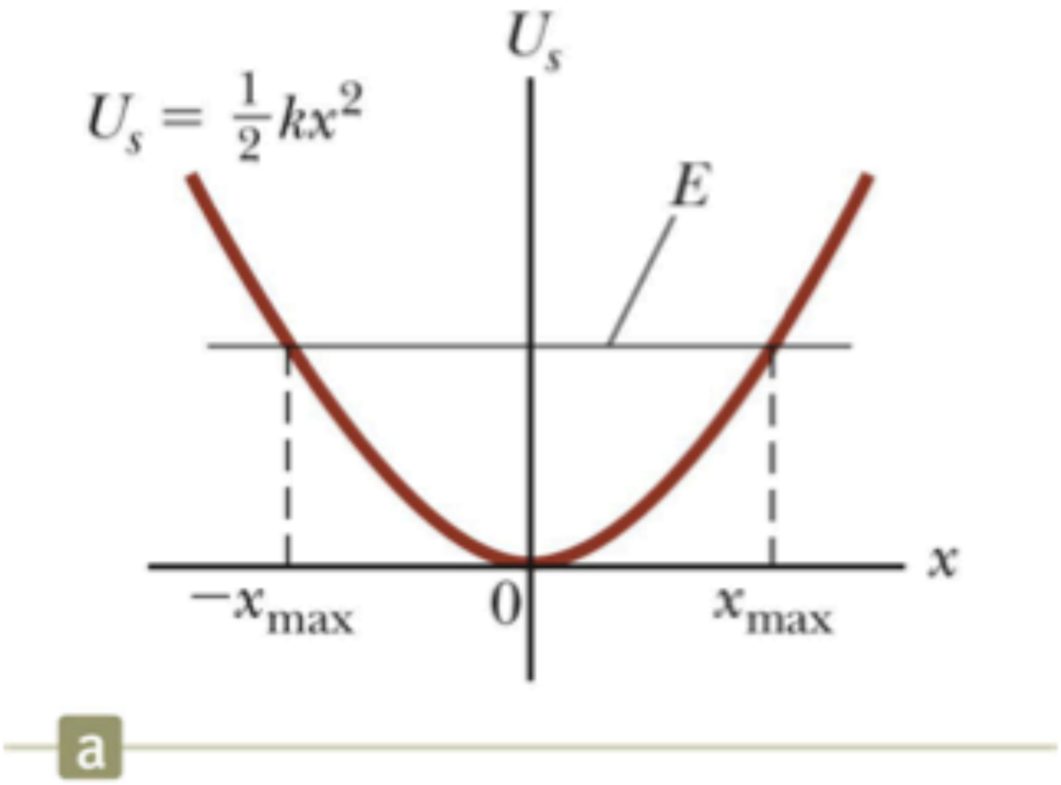
Energy Diagrams and Stable Equilibrium

The $x = 0$ position is one of ***stable equilibrium.***

- Any movement away from this position results in a force directed back toward $x = 0$.

Configurations of stable equilibrium correspond to those for which $U(x)$ is a minimum.

$x = x_{max}$ and $x = -x_{max}$ are called the turning points.



Energy Diagrams and Unstable Equilibrium

$F_x = 0$ at $x = 0$, so the particle is in equilibrium.

For any other value of x, the particle moves away from the equilibrium position.

This is an example of ***unstable equilibrium.***

Configurations of unstable equilibrium correspond to those for which $U(x)$ is a maximum.