PHY404- Solid State Physics II

MAGNETISM- PartI

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- Definition of fundamental quantities
- Classification of magnetic materials
- Diamagnetism

Magnetism

Black sand

Northern Lights

Peary's Compass

Superconducting Magnet

Photo by: National Geographic

What is magnetism and what causes magnetism?

Magnetism, phenomenon associated with magnetic fields, which arise from the motion of electric charges.

The magnetic moment of a free atom has three principal sources:

1- The spin
2- Angular momentum
→ Diamagnetism

3- Orbital moment → Paramagnetism

MAGNETISM

Definition of fundamental quantities

In vacuum, $\mathbf{B} = \mu_0\mathbf{H}$; $\mu_0 = 4\pi \times 10^7$ (SI units: $N \cdot A^{-2}$);

$\mathbf{B}$ – magnetic induction; $\mathbf{H}$ – magnetic field intensity

When a material medium is placed in a magnetic field, the medium is magnetized. This is described by the *magnetization vector* **M** - the magnetic dipole moment per unit volume.

$$\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$$

Magnetization is induced by the field $\Rightarrow$ assume that **M** is proportional to **H**:

$$\mathbf{M} = \chi\mathbf{H} \quad \rightarrow \mathbf{B} = \mu_0(1 + \chi)\mathbf{H} \quad \text{or} \quad \mathbf{B} = \mu\mathbf{H}; \quad \mu = \mu_0(1 + \chi); \quad \mu_r = 1 + \chi$$

χ - *magnetic susceptibility* of the medium (no physical relationship to the electric susceptibility).

This is justifiable in the case of paramagnetic and diamagnetic materials because **M** is very small compared to **H** (typically $\chi = \mathbf{M}/\mathbf{H} \sim 10^{-5}$), unlike the electric case, in which $\chi \sim 1$.

But when we deal with ferromagnetic materials, where **M** is quite large, the above effects must be included.

Note also that χ can be dependent on the applied magnetic field.

In this case, we can define the magnetic susceptibility as follows:

$$\chi = \frac{\partial \mathbf{M}}{\partial \mathbf{H}}$$

The magnetization can be defined as $\mathbf{M} = -\frac{\partial E}{\partial \mathbf{H}}$

Classification of magnetic materials

- ***diamagnetics***: the magnetic susceptibility is negative - the magnetization is opposite to the applied magnetic field. Usually its magnitude is ~ -10^{-6} to -10^{-5}.
In diamagnetic materials the susceptibility nearly has a constant value independent of temperature.
Example: Ionic crystals and inert gases.

- ***paramagnetics***: χ is positive, i.e. **M** is parallel to **H**. The susceptibility is also very small: 10^{-4} to 10^{-5}.
The best-known examples of paramagnetic materials are the ions of transition and rare-earth ions.

- ***ferromagnetics***: very large positive χ (e.g. 10^5), spontaneous magnetization below a certain temperature.

Quantum-mechanical calculation of atomic susceptibilities

In the presence of a uniform magnetic field the Hamiltonian of an ion (atom) is modified in the two major ways:

1) In the total kinetic energy term the electron momentum is replaced: $\mathbf{p} \rightarrow \mathbf{p} + e\mathbf{A}$, where $\mathbf{A}$ is the vector potential associated with the magnetic field: $\mathbf{B} = \nabla \times \mathbf{A}$.

We assume that the applied field is uniform so that $\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$

2) The interaction energy of the field with each electron spin must be added to the Hamiltonian: $H_{spin} = 2\mu_B \mathbf{BS}$

where μ_B is the Bohr magneton $\mu_B = \frac{e\hbar}{2m_0}$ $\mathbf{S}$ – spin momentum

As the result the total energy of electrons will have a form:

$$H = \frac{1}{2m}\sum_i \left(\mathbf{p}_i - \frac{e}{2}\mathbf{r}_i \times \mathbf{B}\right)^2 + 2\mu_B \mathbf{BS}$$

Have $$H = \frac{1}{2m}\sum_i \left(\mathbf{p}_i - \frac{e}{2}\mathbf{r}_i \times \mathbf{B}\right)^2 + 2\mu_B \mathbf{BS}$$

Let T_0 - the kinetic energy in the absence of the applied field: $T_0 = \frac{1}{2m}\sum_i \mathbf{p}_i^2$

The cross term is the brackets can be rewritten taking into account that $\mathbf{p}_i \cdot (\mathbf{r}_i \times \mathbf{B}) = -\mathbf{B} \cdot (\mathbf{r}_i \times \mathbf{p}_i)$

Note that although **r** and **p** are quantum-mechanical operators, here we can work with them as with classical variables because only non-diagonal components enter this product (i.e. there no terms which contain, e.g., x components of both **r** and **p** which do not commute).

Assume that **B** is along z direction → can rewrite $(\mathbf{r}_i \times \mathbf{B})^2 = B^2 (x_i^2 + y_i^2)$

Finally we find for the field-dependent correction to the total Hamiltonian:

$$\Delta H = H - T_0 = \mu_B (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B} + \frac{e^2}{8m} B^2 \sum_i (x_i^2 + y_i^2)$$

where **L** is the total orbital momentum: $\hbar \mathbf{L} = \sum_i (\mathbf{r}_i \times \mathbf{p}_i)$

$$\Delta H = H - T_0 = \mu_B(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B} + \frac{e^2}{8m} B^2 \sum_i \left(x_i^2 + y_i^2 \right)$$

This equation is the basis for theories of the magnetic susceptibility of individual atoms, ions, or molecules.

The energy correction due to the applied electric field is small compared to electron energies; $\mu_B = 5.8\times10^{-5}$ eV/T. $\rightarrow$ for $B = 1$T $\mu_B B = 5.8\times10^{-5}$ eV.

Langevin Diamagnetism

Such atoms have zero spin and orbital angular momentum in its ground state:

$$\langle 0|\mathbf{S}|0\rangle = \langle 0|\mathbf{L}|0\rangle = 0$$

$\Rightarrow$ only last term of ΔH contributes to the field-induced shift in the ground state energy:

$$E = \langle 0|H|0\rangle = \frac{e^2}{8m}B^2\langle 0|\sum_i \left(x_i^2 + y_i^2\right)|0\rangle = \frac{e^2}{12m}B^2\langle 0|\sum_i r_i^2|0\rangle$$

This follows from the spherical symmetry of the closed-shell ion:

$$\langle 0|\sum_i x_i^2|0\rangle = \langle 0|\sum_i y_i^2|0\rangle = \langle 0|\sum_i z_i^2|0\rangle = \frac{1}{3}\langle 0|\sum_i r_i^2|0\rangle$$

$$\langle r^2 \rangle = \frac{1}{Z} \langle 0 | \sum_i r_i^2 | 0 \rangle$$

where Z is the total number of electrons in an ion.

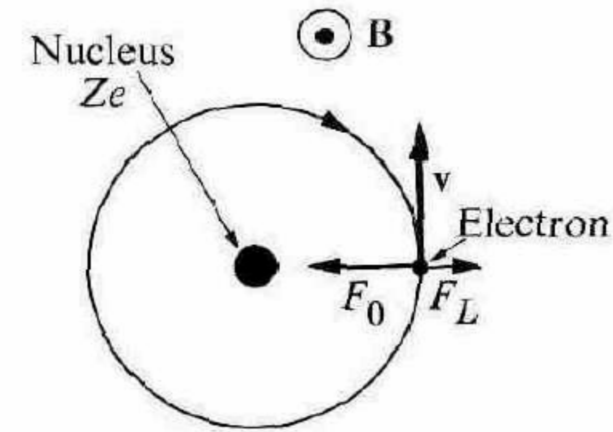
We obtain then for the magnetization induced by the applied magnetic field:

$$E = \frac{e^2}{12m} NZ \langle r^2 \rangle B^2 \qquad \mathbf{M} = -\frac{\partial E}{\partial H} = -\mu_0 \frac{\partial E}{\partial B} = -\frac{\mu_0 e^2 NZ \langle r^2 \rangle B}{6m}$$

where N is the number of atoms per unit volume.

$\Rightarrow$ a negative magnetic susceptibility: $\chi = -\dfrac{\mu_0 e^2 NZ \langle r^2 \rangle}{6m}$

Can obtain the same formula classically:



Consider an electron rotating about the nucleus in a circular orbit; let a magnetic field be applied.

Before this field is applied, we have, according to Newton's second law,

$$F_0 = m\omega_0^{\ 2} r$$

F_0 is the attractive Coulomb force between the nucleus and the electron, and ω_0 is the angular velocity.

Applied field → an additional force: the *Lorentz force* $F_L = -e(\mathbf{v} \times \mathbf{B})$

$$F_L = -eB\omega r \Rightarrow F_0 - eB\omega r = m\omega^2 r \qquad \omega = \omega_0 - \frac{eB}{2m}$$

$$\Delta I = -Ze\frac{1}{2\pi}\frac{eB}{2m}$$

The magnetic moment of a circular current is given by the product (current) x (area of orbit)

$$\Delta\mu = -e\frac{1}{2\pi}\frac{eB}{2m}\pi\left\langle r_{xy}{}^{2}\right\rangle = -\frac{e^{2}\left\langle r_{xy}{}^{2}\right\rangle}{4m}B$$

Here $\langle r_{xy}{}^{2}\rangle = \langle x^{2}\rangle + \langle y^{2}\rangle$. The mean square distance of the electrons from the nucleus is $\langle r^{2}\rangle = \langle x^{2}\rangle + \langle y^{2}\rangle + \langle z^{2}\rangle$.

For a spherically symmetrical charge distribution $\langle x^{2}\rangle = \langle y^{2}\rangle = \langle z^{2}\rangle = \langle r^{2}\rangle/3$

$$\Rightarrow \left\langle r_{xy}{}^{2}\right\rangle = \frac{2}{3}\left\langle r^{2}\right\rangle \quad \Rightarrow \Delta\mu = -\frac{e^{2}\left\langle r^{2}\right\rangle}{6m}B \quad \Rightarrow \quad \chi = -\frac{\mu_{0}e^{2}NZ\left\langle r^{2}\right\rangle}{6m}$$

Summary

- When a material medium is placed in a magnetic field, the medium is magnetized. Magnetisation is proportional to the magnetic field:

  $\mathbf{M} = \chi\mathbf{H}$; χ - magnetic susceptibility of the medium
  magnetic permittivity $\mu = \mu_0(1 + \chi)$; $\mu_r = 1 + \chi$

- Langevin diamagnetism - ions or atoms with all electronic shells filled

  negative magnetic susceptibility: $\chi = -\frac{\mu_0 e^2 NZ\langle r^2 \rangle}{6m}$