PHY404- Solid State Physics II

Superconductivity- PartII

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- BCS theory

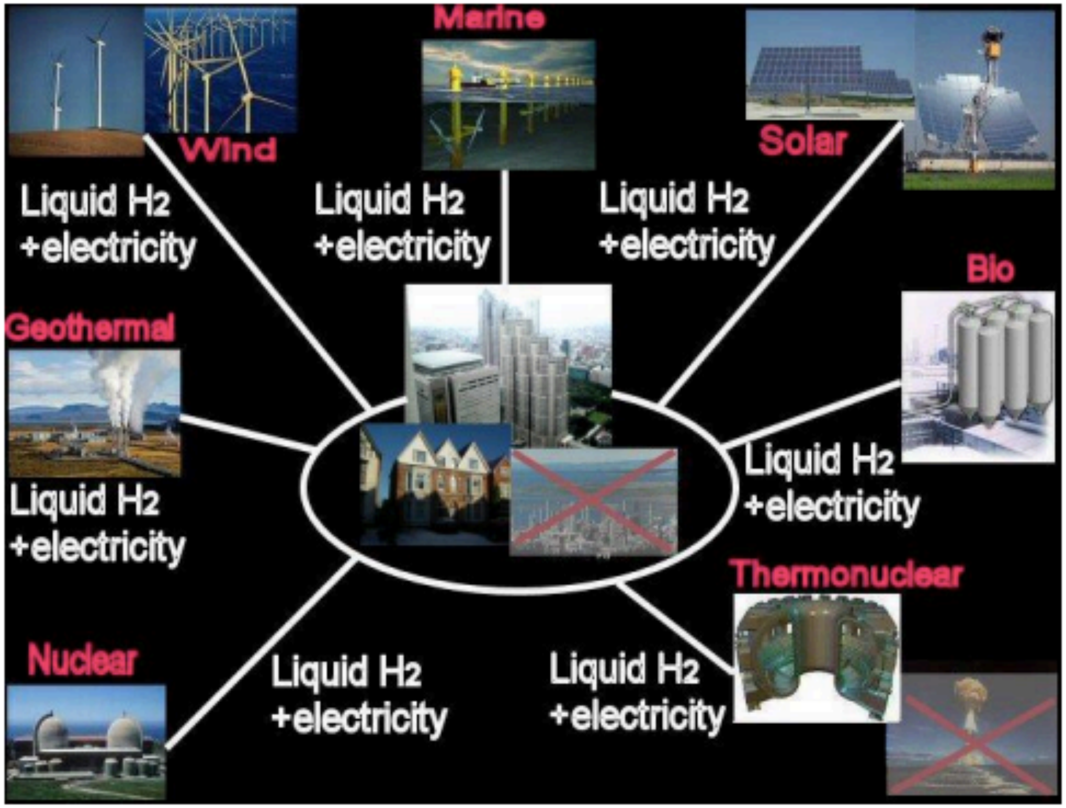
Brief summary for superconductivity

What is a superconductor?

1. Zero resistance
2. Complete expulsion of magnetic flux

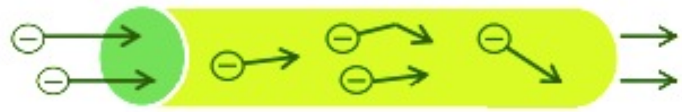
Brief summary for superconductivity

For renewable energy systems



- Hydrogen and electricity can easily be produced by renewable energy sources solving simultaneously problem of energy storage.
- Hydrogen can release full potential of superconductivity starting with building infrastructure for hydrogen economy.

Brief summary for superconductivity

Type of material	What happens in a wire?	Result
Conductor	 Electrons flow easily (like water through a garden hose)	Collisions cause dissipation (heat)
Insulator	 Electrons are tightly bound no flow (like a hose plugged with cement)	No current flow at all
Superconductor	 Electrons bind into pairs and cannot collide (a frictionless hose)	No collisions No dissipation No heat No resistance

Brief summary for superconductivity

HOW SMALL IS THE RESISTANCE?

Copper Cylinder

1) Induce current
2) Current decays in about 1/1000 second

Superconducting Cylinder

1) Induce current
2) Current does not decay
   (less than 0.1% in a year)
   so, resistance is smaller than copper
   by $\frac{1000 \text{ years}}{1/1000 \text{ second}}$
   i.e., at least 1 trillion times!

Brief summary for superconductivity

Why Superconductivity is so fascinating ?

- Fundamental SC mechanism
- Novel collective phenomenon at low temp
- Applications

  Bulk: - Persistent current, power storage
  - Magnetic levitation
  - High field magnet, MRI

  Electronics:
  - SQUID magnetometer
  - Josephson junction electronics

Brief summary for superconductivity

POSSIBLE IMPACT OF SUPERCONDUCTIVITY

- **Energy**
  - Superconductivity generators & motors
  - Power transmission & distribution
  - Energy storage systems
  - Magnets for fusion power
  - Magnets for magneto-hydrodynamic power
- **Transportation**
  - Magnets for levitated trains
  - Electro-magnetic powered ships
  - Magnets for automobiles
- **Health care**
  - Magnetic resonance imaging

Theory

Phenomenological:

- F & H. London (1935)
- Ginzburg& Landau (1950)

Quantum:

- Fröhlich (1950)
- Bardeen, Cooper & Schrieffer, BCS (1957)

London model

- Using two fluid model of Gorter and Casimir:
  Assume only a fraction of electrons $n_s(T)/n$ participate in supercurrent
- $n_s(T)$ is the 'density of superconducting electrons:
  $n_s \sim n$ at $T \ll Tc$, $n_s \to 0$ at $T \to T_c$
- $n - n_s$ electrons exhibit normal dissipation
- Current and supercurrent flow in parallel $\Rightarrow$ superconducting electrons carry all current, normal current is inert and can be ignored

London equations

In an electric field $\mathbf{E}$, S/C electrons will accelerate without dissipation, so we can relate the mean velocity v_s to the current density $\mathbf{j}$:

$$m\frac{d\mathbf{v}_s}{dt} = -e\mathbf{E} \quad \text{using} \quad \mathbf{j} = -e\mathbf{v}_s n_s \quad \text{get} \quad \boxed{\frac{d}{dt}\mathbf{j} = \frac{n_s e^2}{m}\mathbf{E}}$$

1st London equation

In a steady state, $\mathbf{j} = \text{const} \Rightarrow \mathbf{E} = 0$ Electric field inside a S/C vanishes

Maxwell's equation: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = 0 \Rightarrow \mathbf{B} = \text{const}$

These equations describe the magnetic fields and current densities within a perfect conductor, but they are incompatible with the Meissner effect.

From the above, have

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{m}{n_s e^2}\nabla \times \frac{\partial \mathbf{j}}{\partial t}$$

London assumed that

(2nd London equation)

$$\boxed{\mathbf{B} = -\frac{m}{n_s e^2}\nabla \times \mathbf{j}}$$

i.e. to successfully predict the Meissner effect the constant of integration must be chosen to be zero

Combining equation $\mathbf{B} = -\frac{m}{n_s e^2}\nabla \times \mathbf{j}$ and $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

and using $\nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - (\nabla \cdot \nabla)\mathbf{B} = -\nabla^2 \mathbf{B}$

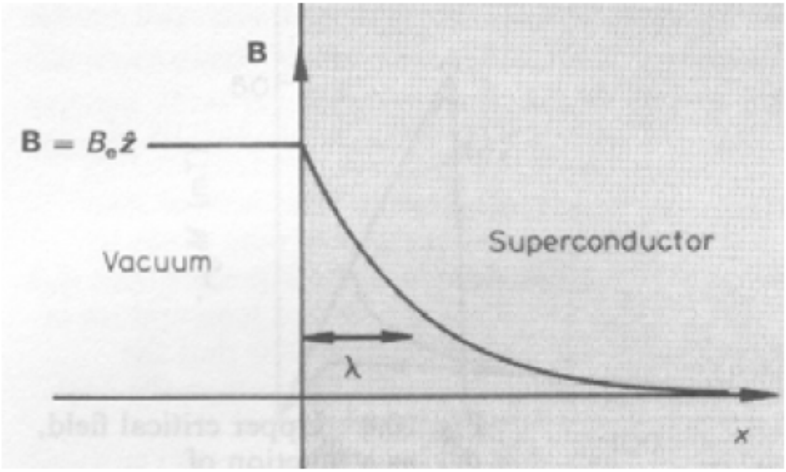
get $\nabla^2 \mathbf{B} = \frac{\mu_0 n_s e^2}{m}\mathbf{B}$ and $\nabla^2 \mathbf{j} = \frac{\mu_0 n_s e^2}{m}\mathbf{j}$

Solution (one-dimensional case): $B(x) = B_o\, e^{-x/\lambda}$

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}$$

- the London penetration depth

i.e. the Meissner effect is predicted



Solution for **j** gives a *surface current* – exponentially decaying into a S/C

Fundamental Mechanism

The superconducting state is an ordered state of the conduction electrons of the metal.

Electron-Phonon Coupling

Cooper Pair formed by two electrons ***k*,** and ***–k*** with opposite spins near the Fermi level, as coupled through **phonons** of the lattice

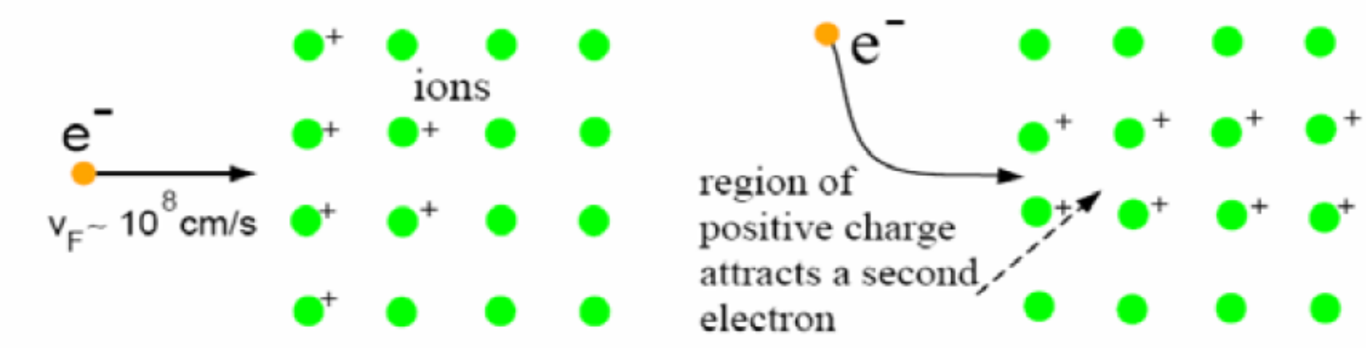
The nature and origin of the ordering was explained by Bardeen, Cooper, and Schrieffer.[3]

BCS Theory, 1957

J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957); **108**, 1175 (1957).

BCS theory

- Fröhlich (1950): e-e attraction via phonons (…. Isotope effect)
- Cooper (1956): electrons just above the Fermi surface form bound pairs
- Most stable when center of mass is at rest and total spin = 0,
  So, $+\boldsymbol{k}\uparrow$ and $-\boldsymbol{k}\downarrow$
- Attractive interaction is provided by lattice vibrations – phonons
- First electron deforms the lattice and second electron is then attracted by the deformation (i.e. the changed positive charge distribution)



- Time-scales: electron motion ~ 10^{-16} s; lattice deformed for ~10^{-13} s
  In this time, first electron has traveled $\sim v_F \tau \sim 10^6 ms^{-1} \times 10^{-13} s \sim 1000$Å

Lattice deformation attracts 2nd electron without it feeling the Coulomb repulsion of the 1st

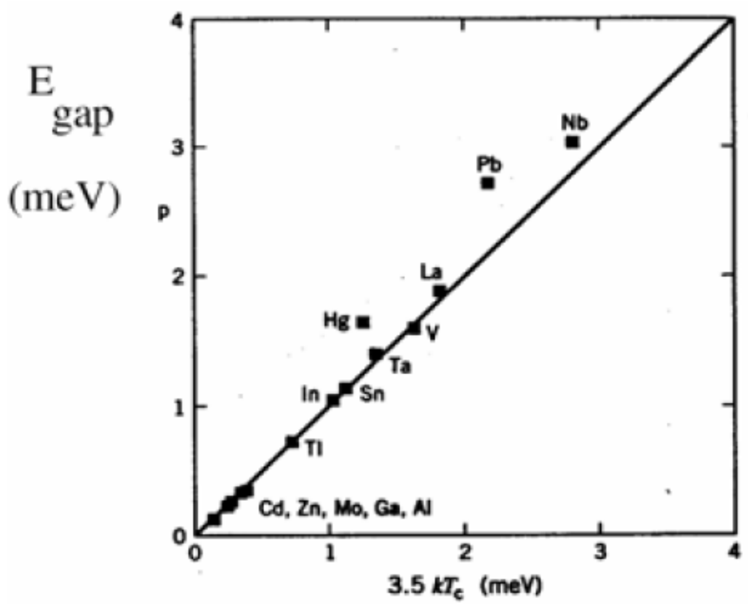
- Cooper calculation: solve Schrödinger eq.for 2 interacting electrons in the presence of a Fermi sphere of non-interacting electrons. Only effect of *N-2* electrons - restrict $\boldsymbol{k}$ values of e-e pair to be $> \boldsymbol{k}_F$, i.e. outside the Fermi sphere
- Cooper pair – boson.
- A single, coherent wave function extending over entire system. Can't change momentum of a pair without changing all pairs
- Bardeen, Cooper and Schrieffer (BCS) → extend Cooper's theory, construct a ground state where all electrons form bound pairs
- Each electron now has 2 roles:
  - Provide restriction on allowed wavevectors via Pauli principle
  - Participate in bound pair (called a Cooper pair)
- Electron-phonon interactions: responsible for resistance of metals *and* superconductivity
- Superconductors are generally poor conductors in normal state

Energy gap: a Cooper pair has a lower energy than 2 individual electrons. BCS gives

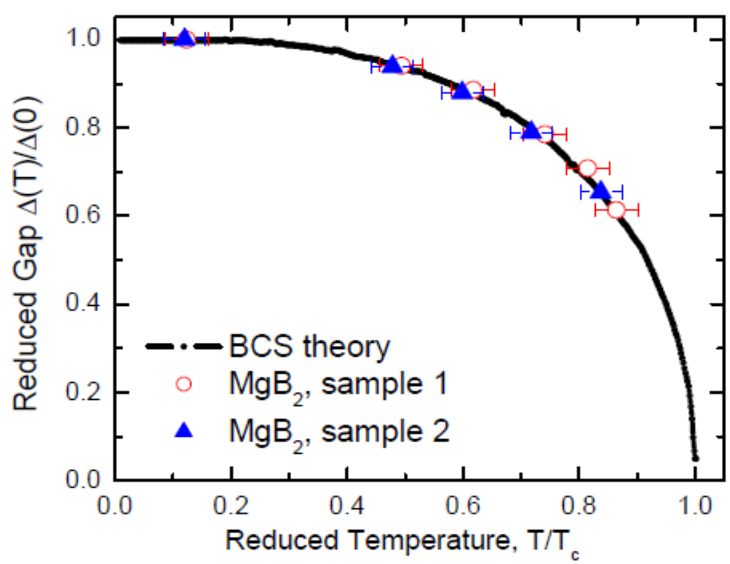
$$E_{gap}(T=0) = 3.53\,k_B T_c$$

$$E_{gap} \sim 10^{-4}\,E_F$$

BCS calculated gap



T-dependence of energy gap



Summary

- When a superconductor is cooled below the critical temperature (T_C), it enters a new state, in which its resistance vanishes.
- Superconductors expel magnetic field completely when in superconducting phase – the Meissner effect
- When a magnetic field higher than a certain value called *the critical field* (B_c) is applied to a superconductor, it reverts to a normal state
- Type I and type II superconductors are distinguished by their behavior in a magnetic field. In a type II S/C there are 2 critical fields. At intermediate fields, the material has both superconducting and normal regions
- Electrodynamics of superconductors is described by phenomenological London equations
- BCS theory – microscopic mechanism for superconductivity through the formation of *e-e* Cooper pairs via electron-phonon interaction.

  A Cooper pair has a lower energy than 2 individual electrons. The energy difference is 2Δ - energy gap.