

Precision Measurement Experiment



I. INTRODUCTION

1.1. Accuracy, Precision and Uncertainty of a Measurement

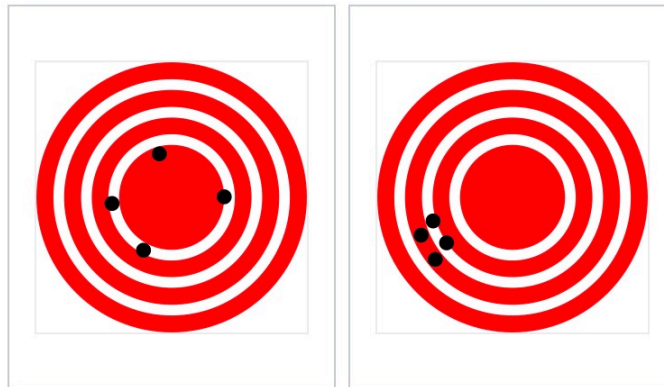
Science is based on observation and experiment that is, on measurements. Accuracy is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring your length. You measure your length three times and obtain the following measurements: 160 cm, 161 cm, and 159 cm. These measurements are quite accurate because they are very close to the correct value of 160 cm. In contrast, if you had obtained a measurement of 168 cm, your measurement would not be very accurate.

In measurement of a set, **accuracy** is closeness of the measurements to a specific value, while **precision** is the closeness of the measurements to each other.

Accuracy has two definitions:

- 1 More commonly, it is a description of *systematic errors*, a measure of statistical bias; low accuracy causes a difference between a result and a "true" value.
- 2 Alternatively, accuracy as describing a combination of both types of observational error above (random and systematic), so high accuracy requires both high precision and high trueness.

The **uncertainty** estimate associated with a measurement should account for both the accuracy and precision of the measurement.



Low accuracy due to poor precision

Low accuracy due to poor trueness

1.2. Types of Errors

Measurement errors may be classified as either **random** or **systematic**, depending on how the measurement was obtained (an instrument could cause a random error in one situation and a systematic error in another).

Random errors are statistical fluctuations (in either direction) in the measured data due to the precision limitations of the measurement device. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations (see standard error).

Systematic errors are reproducible inaccuracies that are consistently in the same direction. These errors are difficult to detect and cannot be analyzed statistically. If a systematic error is identified when calibrating against a standard, applying a correction or correction factor to compensate for the effect can reduce the bias. Unlike random errors, systematic errors cannot be detected or reduced by increasing the number of observations.

Measurement = (measured value \pm standard uncertainty) unit of measurement

1.2.1. Estimating Uncertainty

Suppose you time the period of oscillation of a pendulum using a digital instrument (that you assume is measuring accurately) and find: $T = 0.44$ seconds. This single measurement of the period suggests a precision of ± 0.005 s, but this instrument precision may not give a complete sense of the uncertainty. If you repeat the measurement several times and examine the variation among the measured values, you can get a better idea of the uncertainty in the period. For example, here are the results of 5 measurements, in seconds: 0.46, 0.44, 0.45, 0.44, 0.41.

$$\text{Average (mean)} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

Consider, as another example, the measurement of the width of a piece of paper using a meter stick. Being careful to keep the meter stick parallel to the edge of the paper (to avoid a systematic error which would cause the measured value to be consistently higher than the correct value), the width of the paper is measured at a number of points on the sheet, and the values obtained are entered in a data table. Note that the last digit is only a rough estimate, since it is difficult to read a meter stick to the nearest tenth of a millimeter (0.01 cm).

Observation	Width (cm)
#1	31.33
#2	31.15
#3	31.26
#4	31.02
#5	31.20

$$\text{Average} = \frac{\text{sum of observed widths}}{\text{no. of observations}} = \frac{155.96 \text{ cm}}{5} = 31.19 \text{ cm}$$

One way to express the variation among the measurements is to use the **average deviation**. This statistic tells us on average (with 50% confidence) how much the individual measurements vary from the mean. However, the **standard deviation** is the most common way to characterize the spread of a data set. The **standard deviation** is always slightly greater than the **average deviation**, and is used because of its association with the **normal distribution** that is frequently encountered in statistical analyses.

$$\bar{d} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_N - \bar{x}|}{N}$$

1.2.2. Standard Deviation

To calculate the standard deviation for a sample of N measurements:

- 1 Sum all the measurements and divide by N to get the **average**, or **mean**.
- 2 Now, subtract this **average** from each of the N measurements to obtain N "**deviations**".
- 3 **Square** each of these N **deviations** and add them all up.
- 4 Divide this result by $(N - 1)$ and take the square root.

$$s = \sqrt{\frac{(\delta x_1^2 + \delta x_2^2 + \dots + \delta x_N^2)}{(N-1)}} = \sqrt{\frac{\sum \delta x_i^2}{(N-1)}}$$

In our previous example, the average width \bar{x} is 31.19 cm. The deviations are:

Observation	Width (cm)	Deviation (cm)
#1	31.33	+0.14 = 31.33 - 31.19
#2	31.15	-0.04 = 31.15 - 31.19
#3	31.26	+0.07 = 31.26 - 31.19
#4	31.02	-0.17 = 31.02 - 31.19
#5	31.20	+0.01 = 31.20 - 31.19

The **average** deviation is: $\bar{d} = 0.086$ cm.

The **standard** deviation is: $s = \sqrt{\frac{(0.14)^2 + (0.04)^2 + (0.07)^2 + (0.17)^2 + (0.01)^2}{5-1}} = 0.12$ cm.

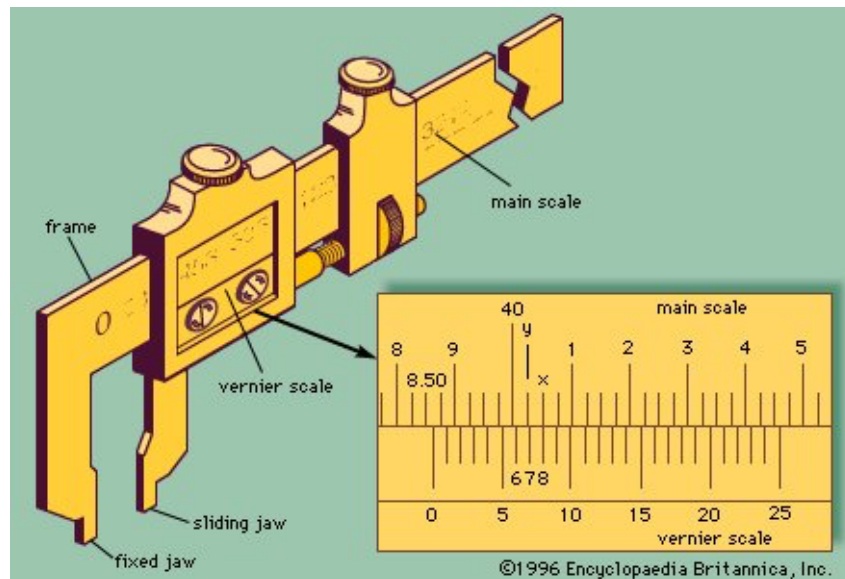
Experimental error is defined as the difference between an experimental value and the actual value of a quantity. This difference indicates the accuracy of the measurement. The accuracy is a measure of the degree of closeness of a measured or calculated value to its actual value. The percent error is the ratio of the error to the actual value multiplied by 100.

$$\% \text{ error} = \frac{\text{error}}{\text{actual value}} \times 100$$

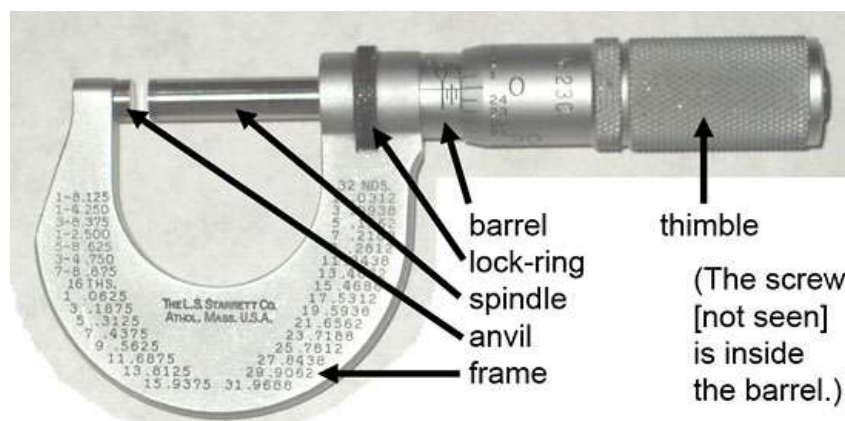
II. APPARATUS

Ruler, vernier caliper, micrometer, solid cylinder, steel ball

Vernier caliper, instrument for making very accurate linear measurements introduced in 1631 by Pierre Vernier of France. It utilizes two graduated scales: a main scale similar to that on a ruler and an especially graduated auxiliary scale, the vernier, that slides parallel to the main scale and enables readings to be made to a fraction of a division on the main scale. Vernier calipers are widely used in scientific laboratories and in manufacturing for quality control measurements. In the figure, the vernier scale has 25 divisions, whereas the main scale has 24 divisions in the same length. This means that the divisions on the vernier scale are shorter than those on the main scale by $1/25$ of a division on the main scale. In the figure, line 8 on the vernier coincides with line x on the main scale. To align lines 7 and y the vernier would have to be moved to the left by $1/25$ of a main-scale division; to align lines 6 and 40, the movement would be $2/25$, and so on. By similar reasoning, the 0 line on the vernier would have to be moved a distance equal to $8/25$ of a main-scale division to align it with the 8.50 line on the main scale. This means that in the position shown in the figure the 0 line is $8/25$ of a main-scale division to the right of the 8.50 line. The reading of the vernier is therefore $30 + 8.50 + 0.08 = 38.58$.



Micrometer, in full **micrometer caliper**, instrument for making precise linear measurements of dimensions such as diameters, thicknesses, and lengths of solid bodies; it consists of a C-shaped frame with a movable jaw operated by an integral screw. The fineness of the measurement that can be made depends on the lead of the screw—i.e., the amount the spindle moves toward or away from the anvil in one revolution and the means provided for indicating fractional parts of a revolution. The accuracy of the measurements depends on the accuracy of the screw-nut combination.



III. EXPERIMENTAL PROCEDURE

1. Measure the length of the solid cylinder, which is given to you, 10 times with a ruler and Vernier caliper, and fill the table.

Table1				
Observation	Ruler		Vernier caliper	
	$L_i(\text{cm})$	$a_i = L_i - L_{\text{avg}}(\text{cm})$	$L_i(\text{cm})$	$a_i = L_i - L_{\text{avg}}(\text{cm})$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

2. Measure the diameter of the steel ball 20 times with a micrometer and fill in the table.

Table2		
Observation	Ruler	
	$d_i(\text{mm})$	$a_i = d_i - d_{\text{avg}}(\text{mm})$
1		
2		
3		
4		
5		
.		
.		
20		

Ref.

- 1) https://www.webassign.net/question_assets/unccolphysmechl1/measurements/manual.html (12th of May 2020)
- 2) https://en.wikipedia.org/wiki/Accuracy_and_precision (12th of May 2020)
- 3) [https://phys.libretexts.org/Bookshelves/College_Physics/Book%3A_College_Physics_\(OpenStax\)/01%3A_The_Nature_of_Science_and_Physics/1.04%3A_Accuracy%2C_Precision%2C_and_Significant_Figures](https://phys.libretexts.org/Bookshelves/College_Physics/Book%3A_College_Physics_(OpenStax)/01%3A_The_Nature_of_Science_and_Physics/1.04%3A_Accuracy%2C_Precision%2C_and_Significant_Figures) (12th of May 2020)
- 4) <https://www.britannica.com/technology/micrometer>(12th of May 2020)
- 5) <https://www.britannica.com/technology/vernier-caliper>(12th of May 2020).