

# **FEEDBACK CONTROL SYSTEMS**

LECTURE NOTES-8

$$F(s) = \frac{\prod_{i=1}^m (s + z_i)}{\prod_{i=1}^n (s + p_i)} = \frac{\text{numerator's complex factors}}{\text{denominator's complex factors}}$$

M=# of zeros  
N=# of poles

$$M = \frac{\prod \text{zero lengths}}{\prod \text{pole lengths}} = \frac{\prod_{i=1}^m |(s + z_i)|}{\prod_{i=1}^n |(s + p_i)|}$$

M=magnitude of vector F(s) at any point s

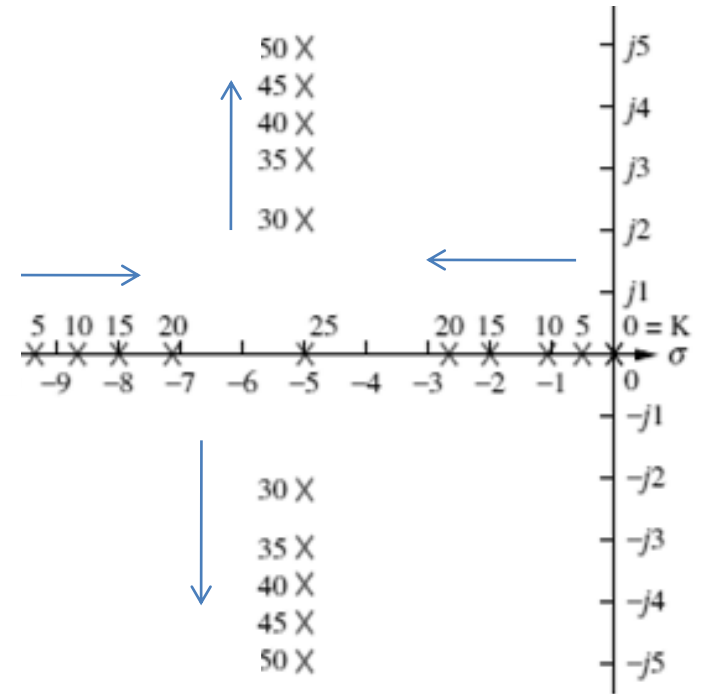
$$\theta = \sum \text{zero angles} - \sum \text{pole angles}$$

$$\theta = \sum_{i=1}^m \angle(s - z_i) - \sum_{i=1}^n \angle(s - p_i)$$

Θ=The angle of F(s) at any point s

$$\frac{K}{s^2 + 10s + K}$$

K	Pole 1	Pole 2
0	-10	0
5	-9.47	-0.53
10	-8.87	-1.13
15	-8.16	-1.84
20	-7.24	-2.76
25	-5	-5
30	-5+j2.24	-5-j2.24
35	-5+j3.16	-5-j3.16
40	-5+j3.87	-5-j3.87
45	-5+j4.47	-5-j4.47
50	-5+j5	-5-j5



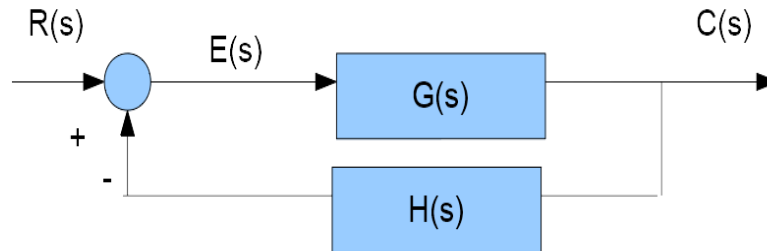
## Sketching the Root Locus

$$KG(s)H(s) = -1 = 1 \angle (2k + 1)180$$

$$k = 0 \pm 1, \pm 2, \pm 3, \dots$$

Poles makes the characteristic eq. zero

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

Angle Condition

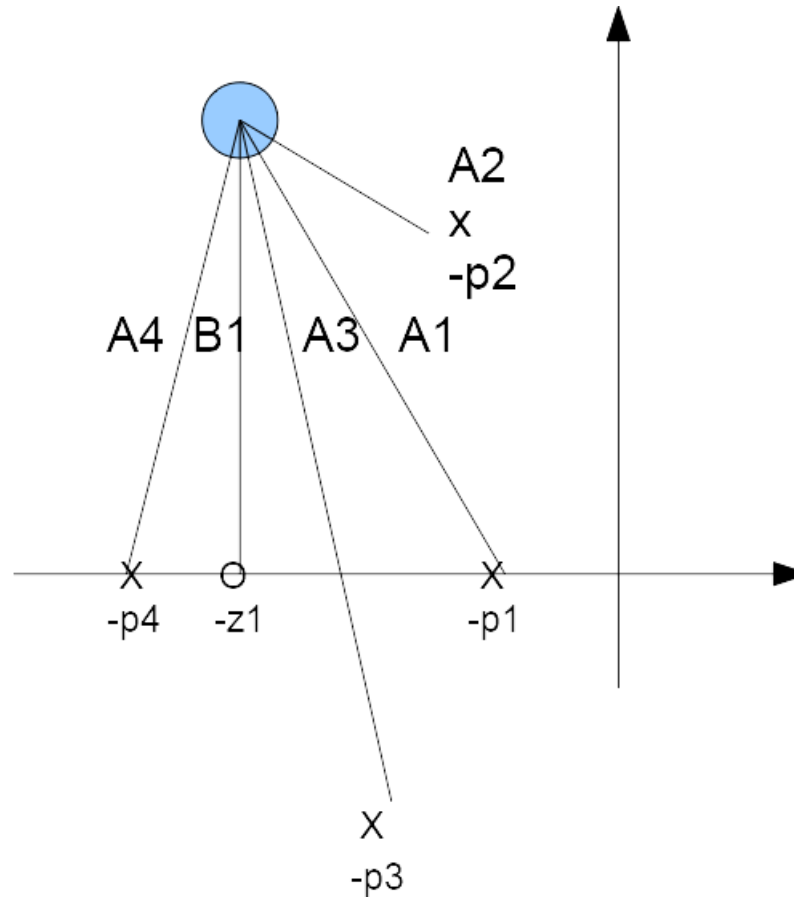
$$\angle KG(s)H(s) = (2k + 1)180$$

Magnitude Condition

$$|KG(s)H(s)| = 1$$

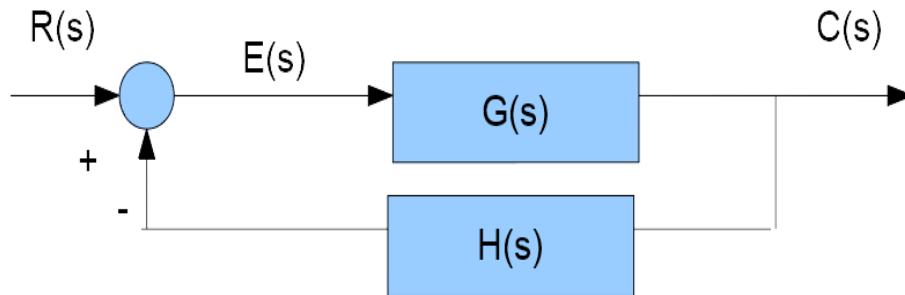
$$K = \frac{1}{|G(s)H(s)|}$$

$$G(s)H(s) = \frac{K(s + z_1)}{(s + p_1)(s + p_2)(s + p_3)(s + p_4)}$$



$$\angle G(s)H(s) = \phi_1 - \theta_1 - \theta_2 - \theta_3 - \theta_4$$

$$|G(s)H(s)| = \frac{KB_1}{A_1 A_2 A_3 A_4}$$



$$G(s) = \frac{K}{s(s+1)(s+2)}$$

$$H(s) = 1$$

$$\angle G(s) = \angle \frac{K}{s(s+1)(s+2)}$$

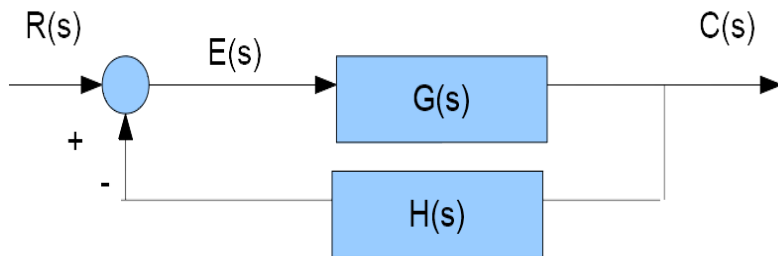
$$\angle G(s) = -\angle s - \angle(s+1) - \angle(s+2)$$

$$\angle G(s) = \pm 180(2k+1)$$

$$k = 0, 1, 2, \dots$$

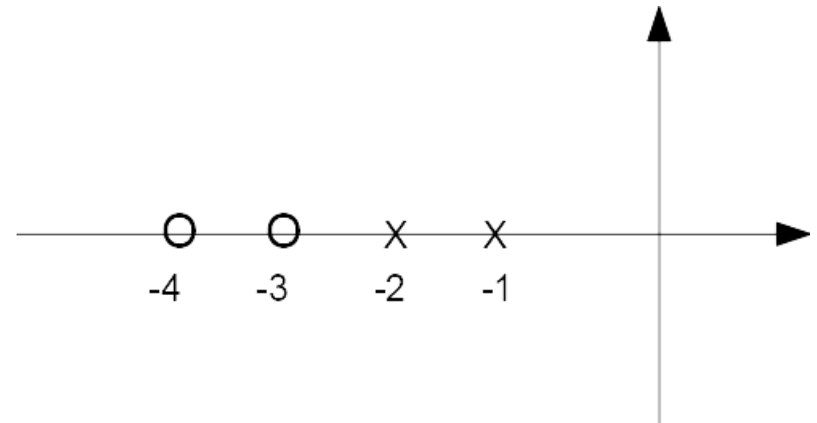
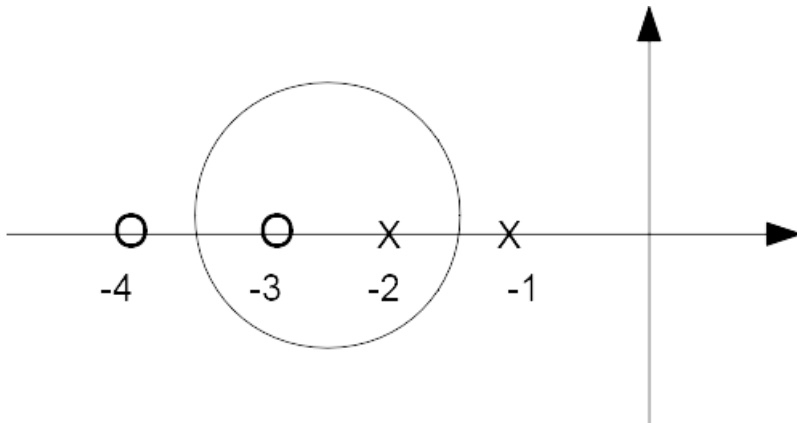
$$|G(s)| = \left| \frac{K}{s(s+1)(s+2)} \right| = 1$$

The root locus begins at the infinite/finite poles of  $G(s)H(s)$  and ends at infinite/finite zeros of  $G(s)H(s)$  (RL begins at poles and ends at zeros)



$$G(s) = \frac{K(s+3)(s+4)}{(s+1)(s+2)}$$

$$H(s) = 1$$



The system has equal number of poles and zeros if not  $\rightarrow$  assumed at infinite

The root locus approaches straight lines as asymptotes as the locus approaches to infinity. Moreover, the equation of the asymptotes is given by the real axis intercept and angle.

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\text{number of finite poles} - \text{number of finite zeros}}$$
$$\theta_a = \frac{(2k + 1)\pi}{\text{number of finite poles} - \text{number of finite zeros}}$$

Where,  $k=0,+1, -1,+2,-2,+3,-3,\dots$  and the angle is given in radius with respect to the positive extension of the real axis.



## Real Axis Break-in and Breakaway points

$$KG(s)H(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)} = \frac{K(s^2 - 8s + 15)}{(s^2 + 3s + 2)}$$

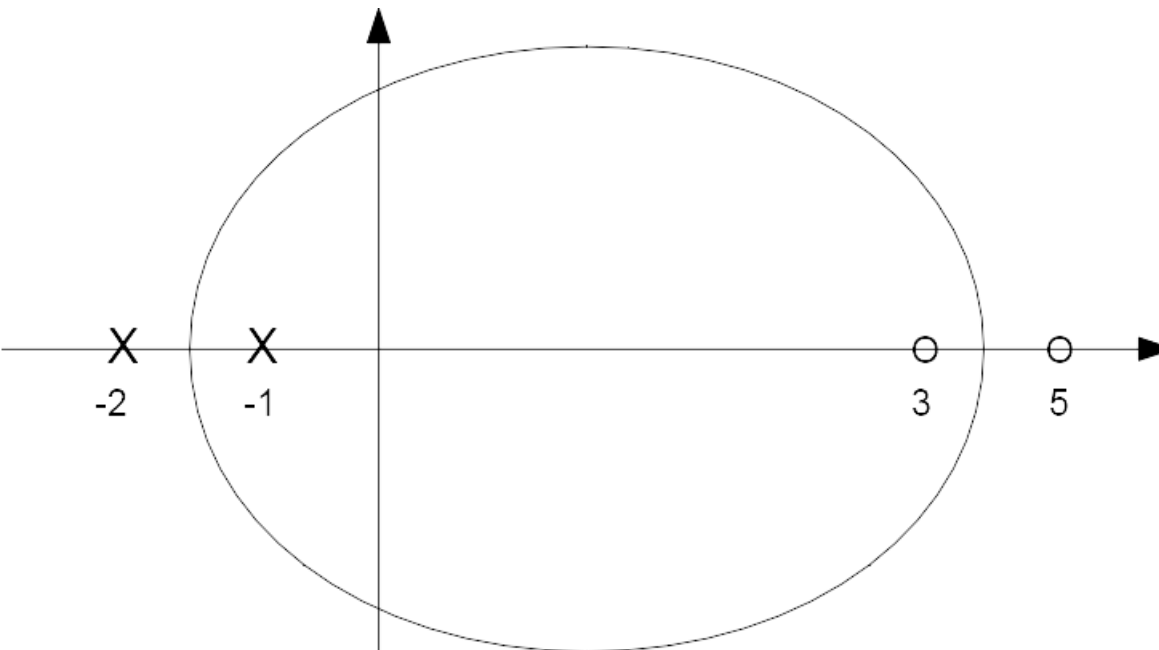
$$KG(s)H(s) = -1$$
$$s = \sigma$$

$$\frac{K(\sigma^2 - 8\sigma + 15)}{(\sigma^2 + 3\sigma + 2)} = -1$$

$$K = \frac{-(\sigma^2 + 3\sigma + 2)}{(\sigma^2 - 8\sigma + 15)}$$

$$\frac{dK}{d\sigma} = \frac{(11\sigma^2 - 26\sigma - 61)}{(\sigma^2 - 8\sigma + 15)^2} = 0$$

$$\sigma = -1.45 \text{ and } 3.82$$



## Imaginary Axis Crossing (Transition to undamped and instability region)

Use the Routh-Hurwitz criterion such that a row of zeros will yield the gain

$$T(s) = \frac{K(s+3)}{s^4 + 7s^3 + 14s^2 + (8+K)s + 3K}$$

$$-K^2 - 65K + 720 = 0$$

$$K = 9.65$$

$s^4$	1	14	3K
$s^3$	7	8+K	0
$s^2$	90-K	21K	0
$s^1$	$(-K^2-65K+720)/(90-K)$	0	0
$s^0$	21K	0	0

$$(90 - K)s^2 + 21K = 80.35s^2 + 202.7 = 0$$

$$s = \pm j1.59$$

the system is stable for  $0 \leq K < 9.65$