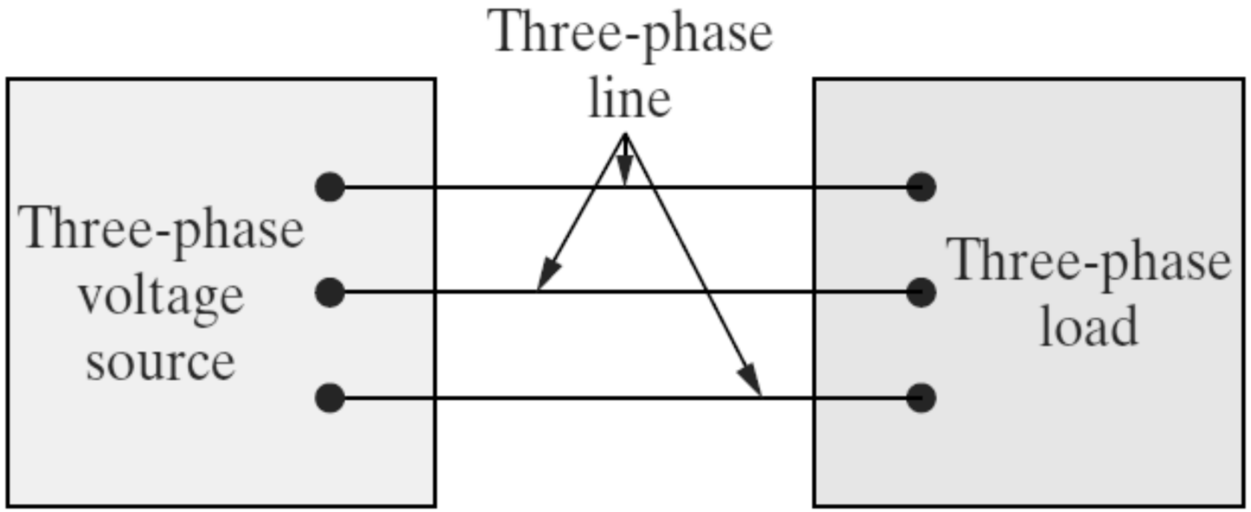
Generating, transmitting, distributing, and using large blocks of electric power is accomplished with three-phase circuits

The basic structure of a three-phase system consists of voltage sources connected to loads by means of transformers and transmission lines.

Transmission and Distribution of Electric Power

Circuits that are designed to handle large blocks of electric power. These are the circuits that are used to transport electric power from the generating plants to both industrial and residential customers.



Balanced Three-Phase Voltages

A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequencies but are out of phase with each other by exactly 120.

Standard practice is to refer to the three phases as a, b, and c, and to use the a-phase as the reference phase. The three voltages are referred to as the **a-phase voltage**, the **b-phase voltage**, and the **c-phase voltage**.

Only two possible phase relationships can exist between the a-phase voltage and the b- and c-phase voltages.

i) One possibility is for the b-phase voltage to lag the a-phase voltage by 120 in which case the c-phase voltage must lead the a-phase voltage by 120 This phase relationship is known as the **abc** (or **positive**) **phase sequence**.
ii) The only other possibility is for the b-phase voltage to lead the a-phase voltage by 120 in which case the c-phase voltage must lag the a-phase voltage by 120 This phase relationship is known as the **acb** (or **negative**) **phase sequence**. In phasor notation, the two possible sets of balanced phase voltages are

$$\mathbf{V}_a = V_m \angle 0^\circ, \quad \mathbf{V}_b = V_m \angle -120^\circ, \quad \mathbf{V}_c = V_m \angle +120^\circ,$$

$$\mathbf{V}_a = V_m \angle 0^\circ, \quad \mathbf{V}_b = V_m \angle +120^\circ, \quad \mathbf{V}_c = V_m \angle -120^\circ.$$

Another important characteristic of a set of balanced three-phase voltages is that the sum of the voltages is zero

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0.$$

Because the sum of the phasor voltages is zero, the sum of the instantaneous voltages also is zero; that is,

$$v_a + v_b + v_c = 0.$$

If we know **the phase sequence** and **one voltage** in the set, we know the entire set. Thus for a balanced three phase system, we can focus on determining the voltage (or current) in one phase, because once we know one phase quantity, we know the others

Three-Phase Voltage Sources

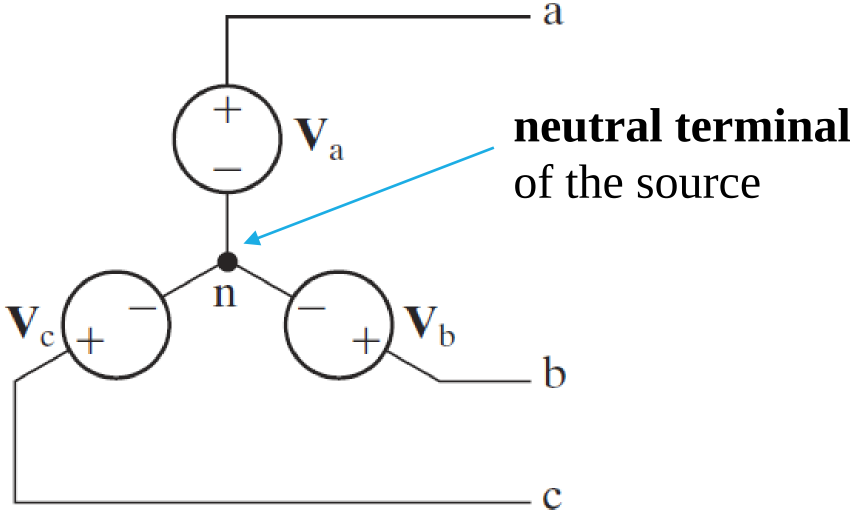
Three separate windings distributed around the periphery of the stator.

Rotation of the electromagnet induces a sinusoidal voltage in each winding

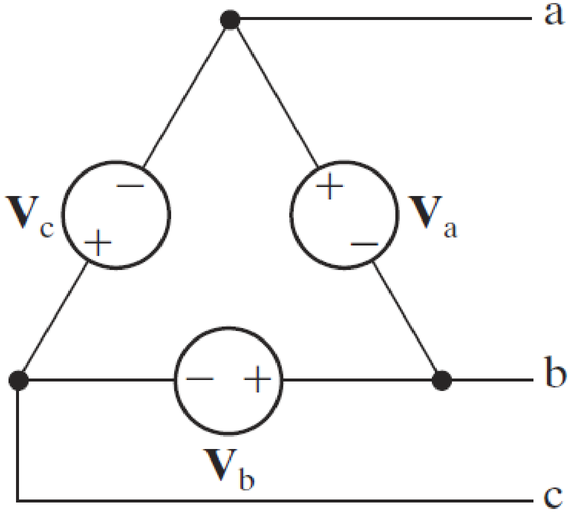
The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by 120

The phase windings are stationary with respect to the rotating electromagnet, so the frequency of the voltage induced in each winding is the same

There are two ways of interconnecting the separate phase windings to form a three-phase source: in either a wye (Y) or a delta (Δ) configuration.



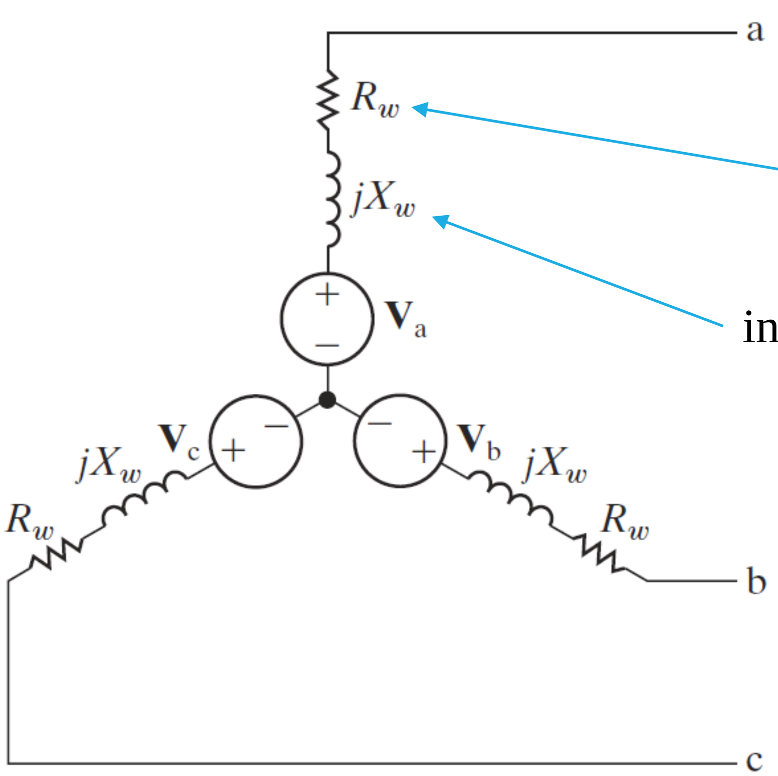
A Y-connected source.



A Δ-connected source.

The model consists solely of ideal voltage. However, if the impedance of each phase winding is not negligible, we place the winding impedance in series with an ideal sinusoidal voltage source. All windings on the machine are of the same construction, so we assume the winding impedances to be identical.

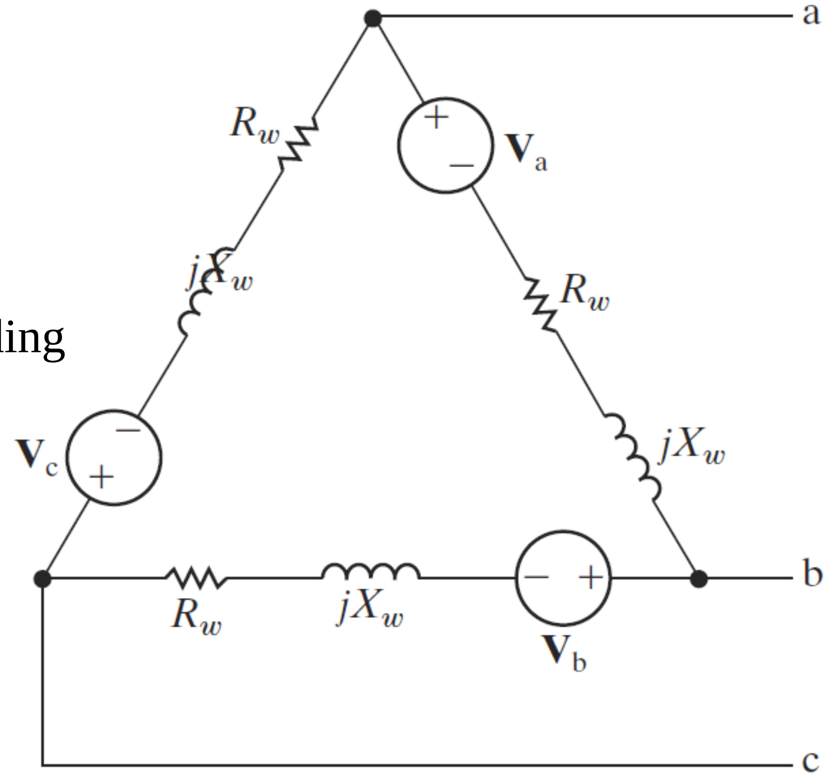
The winding impedance of a three-phase generator is inductive



winding resistance

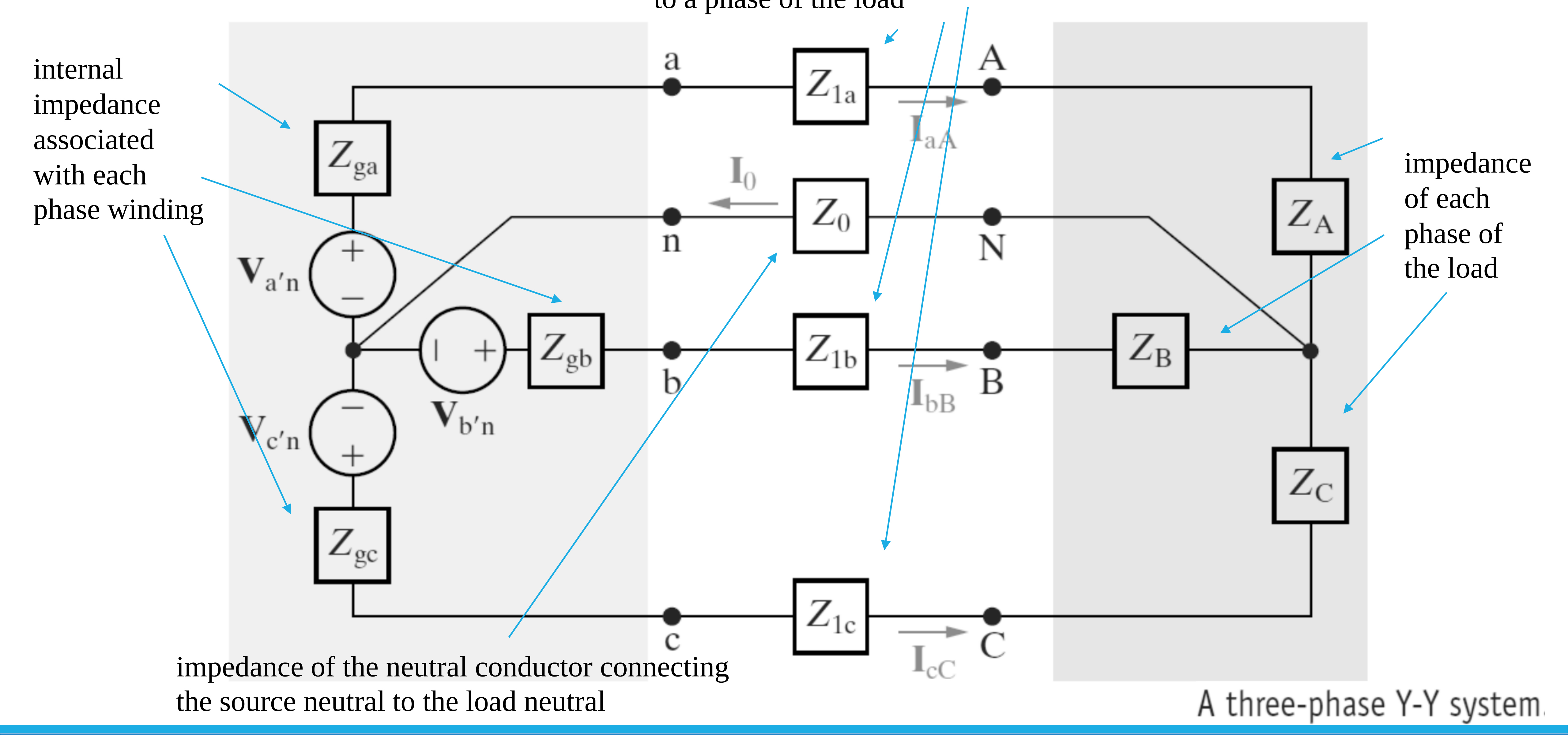
inductive reactance of the winding

Y-connected source;



Δ-connected source.

Analysis of the Wye-Wye Circuit



A three-phase Y-Y system.

There is no restriction on the impedance of a neutral conductor; its value has no effect on whether the system is balanced. If the circuit in Fig. is balanced, we may rewrite

$$\mathbf{V}_N\left(\frac{1}{Z_0} + \frac{3}{Z_\phi}\right) = \frac{\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n}}{Z_\phi},$$

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}.$$

The right-hand side of Eq. is zero, because by hypothesis the numerator is a set of balanced three-phase voltages and is not zero Z_ϕ only value of V_N that satisfies Eq. is zero. Therefore, for a balanced three phase circuit,

$$\mathbf{V}_N = 0.$$

If V_N is zero, there is no difference in potential between the source neutral, n, and the load neutral, N; consequently, the current in the neutral conductor is zero. Hence we may either remove the neutral conductor from a balanced Y-Y configuration (Io=0) or replace it with a perfect short circuit between the nodes n and N (V_N=0). Both equivalents are convenient to use when modeling balanced three-phase circuits.

We now turn to the effect that balanced conditions have on the three line currents

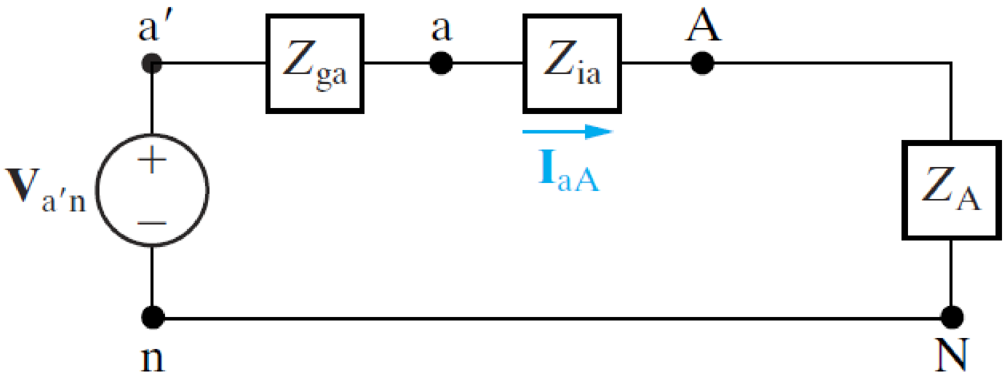
$$\mathbf{I}_{\mathrm{aA}} = \frac{\mathbf{V}_{\mathrm{a'n}} - \mathbf{V}_{\mathrm{N}}}{Z_{\mathrm{A}} + Z_{\mathrm{1a}} + Z_{\mathrm{ga}}} = \frac{\mathbf{V}_{\mathrm{a'n}}}{Z_{\phi}},$$

$$\mathbf{I}_{\mathrm{bB}} = \frac{\mathbf{V}_{\mathrm{b'n}} - \mathbf{V}_{\mathrm{N}}}{Z_{\mathrm{B}} + Z_{\mathrm{1b}} + Z_{\mathrm{gb}}} = \frac{\mathbf{V}_{\mathrm{b'n}}}{Z_{\phi}},$$

$$\mathbf{I}_{\mathrm{cC}} = \frac{\mathbf{V}_{\mathrm{c'n}} - \mathbf{V}_{\mathrm{N}}}{Z_{\mathrm{C}} + Z_{\mathrm{1c}} + Z_{\mathrm{gc}}} = \frac{\mathbf{V}_{\mathrm{c'n}}}{Z_{\phi}}.$$

the current in each line is equal in amplitude and frequency and is 120 out of phase with the other two line currents

Thus, if we calculate the current **I**aA and we know the phase sequence, we have a shortcut for finding IbB and IcC



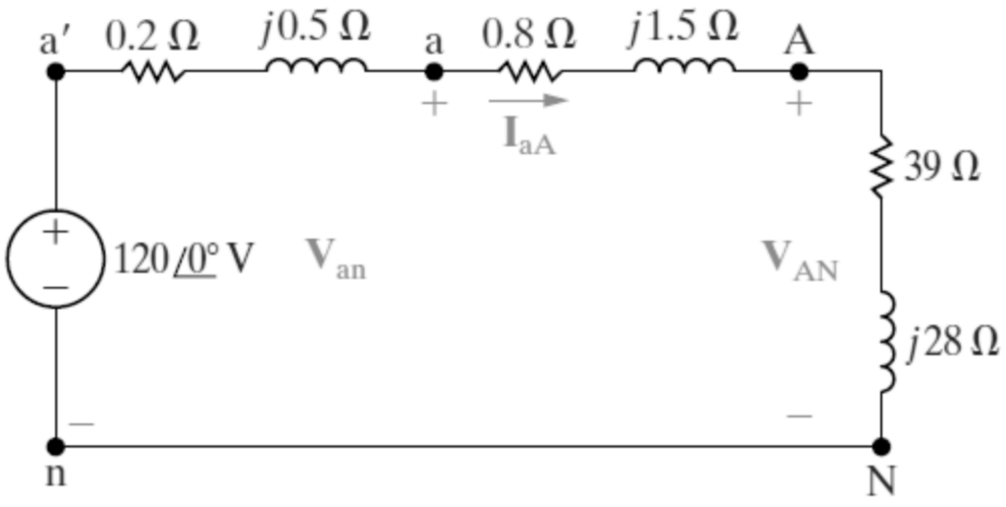
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_{N}}{Z_{A} + Z_{1a} + Z_{ga}} = \frac{\mathbf{V}_{a'n}}{Z_{\phi}},$$

single-phase equivalent circuit

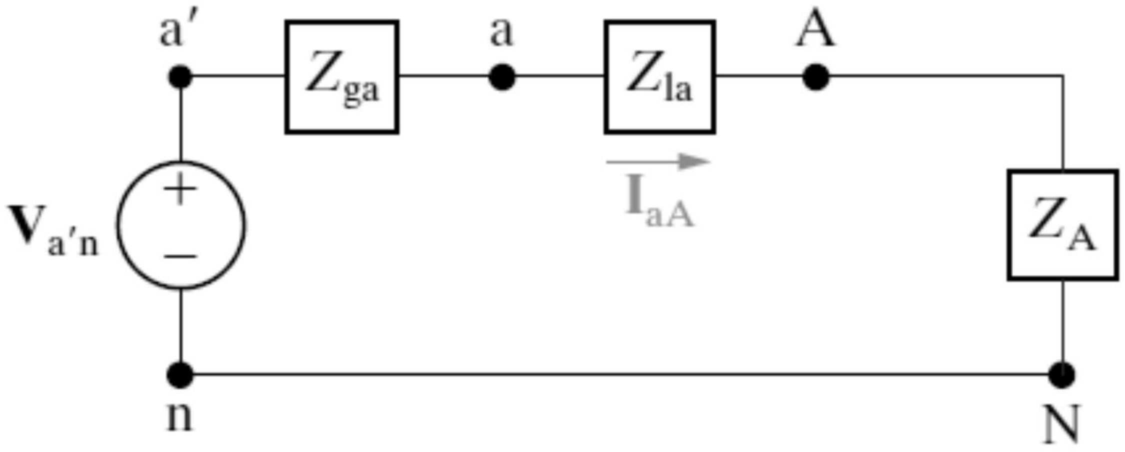
Construct an equivalent circuit for the a-phase of the balanced Y-Y circuit. From this equation, the current in the a-phase conductor line is simply the voltage generated in the a phase winding of the generator divided by the total impedance in the a phase of the circuit

Neutral conductor has been replaced by a perfect short circuit

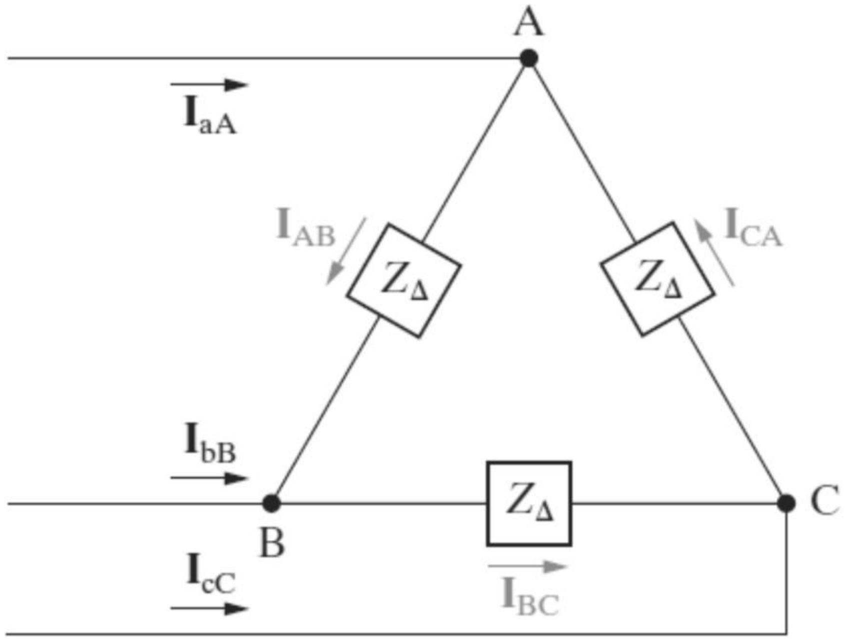
Because of the established relationships between phases, once we solve this circuit, we can easily write down the voltages and currents in the other two phases. Thus, drawing a single phase equivalent circuit is an important first step in analyzing a three phase circuit.



$$Z_Y = \frac{Z_\Delta}{3},$$



We use this circuit to calculate the line currents, and we then use the line currents to find the currents in each leg of the original Δ load. The relationship between the line currents and the currents in each leg of the delta can be derived using the circuit shown in



When a load (or source) is connected in a delta, the current in each leg of the delta is the phase current, and the voltage across each leg is the phase voltage. Figure shows that, in the Δ configuration, the phase voltage is identical to the line voltage.

Complex Power in a Balanced Wye Load

For a balanced load, the expressions for the reactive power are

For a balanced load

$$S_\phi = \mathbf{V}_{AN}\mathbf{I}^*_{aA} = \mathbf{V}_{BN}\mathbf{I}^*_{bB} = \mathbf{V}_{CN}\mathbf{I}^*_{cC} = \mathbf{V}_\phi \mathbf{I}^*_\phi,$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}^*_\phi,$$

$$S_T = 3S_\phi = \sqrt{3}V_L I_L \angle\theta^\circ_\phi.$$

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi,$$

$$Q_T = 3Q_\phi = \sqrt{3}V_L I_L \sin\theta_\phi.$$