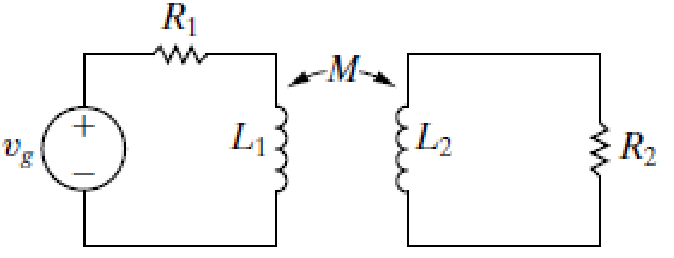
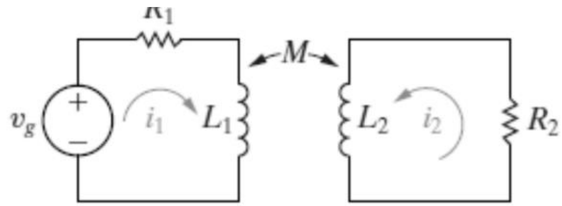
Inductance is the parameter that relates a voltage to a time-varying current in the same circuit; thus, inductance is more precisely referred to as self-inductance. We now consider the situation in which two circuits are linked by a magnetic field. In this case, the voltage induced in the second circuit can be related to the time-varying current in the first circuit by a parameter known as mutual inductance.



The self-inductances of the two coils are labeled L1 and L2, the mutual inductance is labeled *M*

The easiest way to analyze circuits containing mutual inductance is to use mesh currents



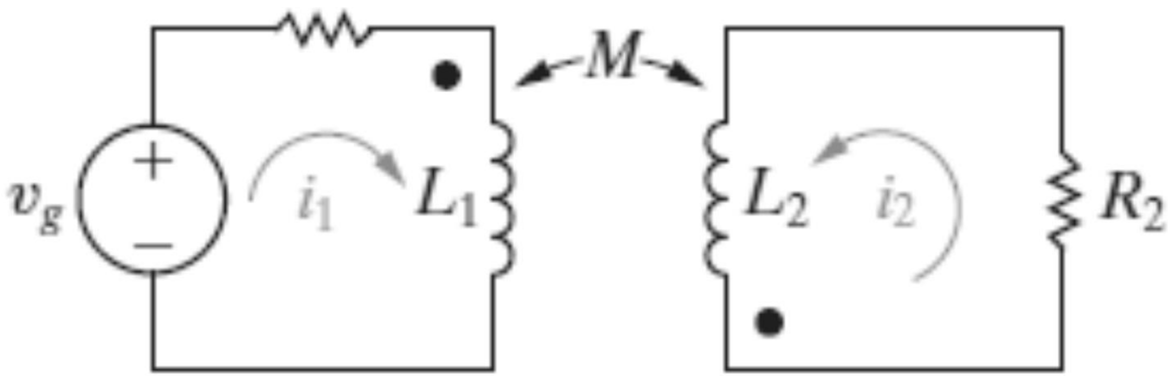
There will be two voltages across each coil, namely, a self-induced voltage and a mutually induced voltage

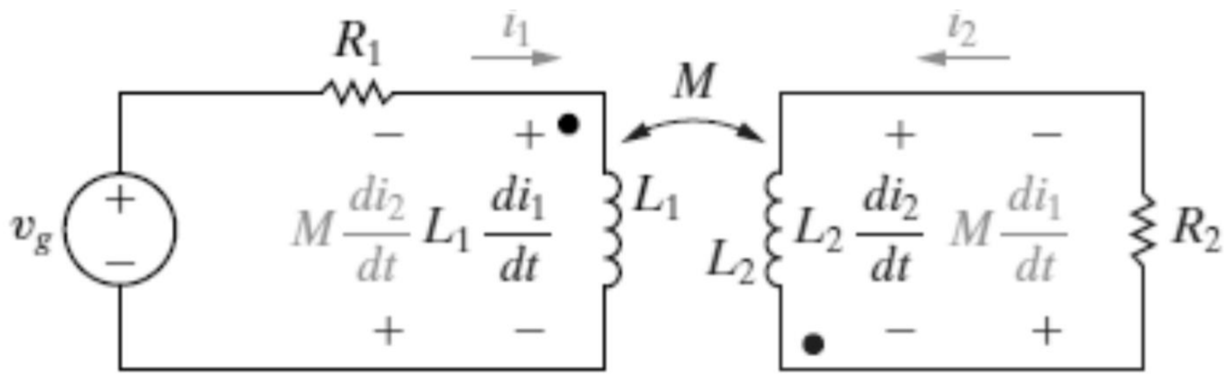
The self induced voltage is the product of the self-inductance of the coil and the first derivative of the current in that coil.

The mutually induced voltage is the product of the mutual inductance of the coils and the first derivative of the current in the other coil

The self-induced voltage across this coil is L1(di1/dt) and the mutually induced voltage across this coil is M(di2/dt). But what about the polarities of these two voltages?

The polarity of the mutually induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents. We keep track of the polarities by a method known as the **dot convention**, in which a dot is placed on one terminal of each winding

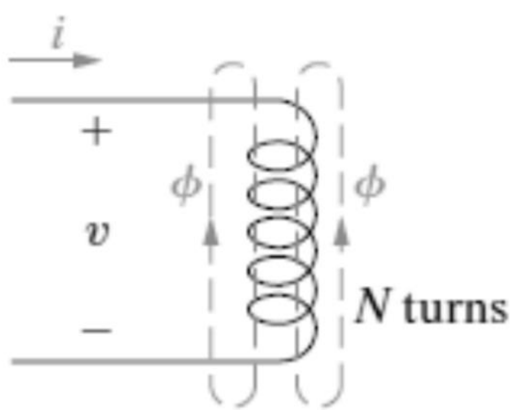




$$-v_g + i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0,$$

$$i_2R_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0.$$

A magnetic field consists of lines of force surrounding the current-carrying conductor. Visualize these lines of force as energy-storing elastic bands that close on themselves. As the current increases and decreases, the elastic bands (that is, the lines of force) spread and collapse about the conductor. The voltage induced in the conductor is proportional to the number of lines that collapse into, or cut, the conductor. This image of induced voltage is expressed by what is called Faraday's law; that is

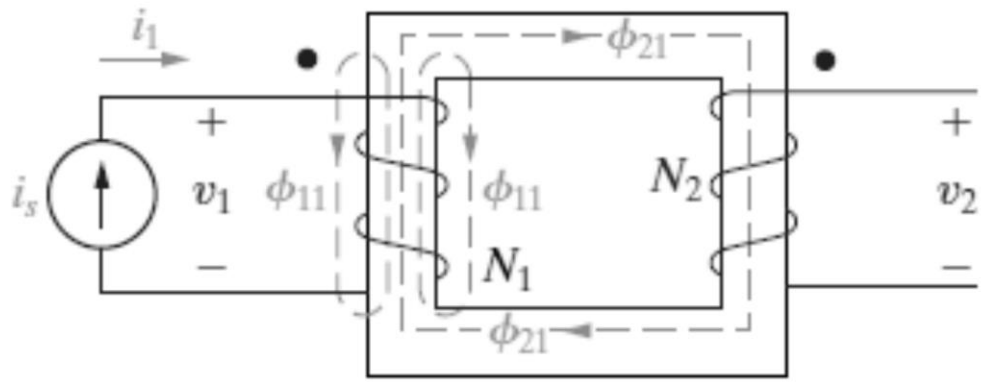


$$v = \frac{d\lambda}{dt},$$

$$v = \frac{d\lambda}{dt} = \frac{d(N\phi)}{dt}$$

$$= N\frac{d\phi}{dt} = N\frac{d}{dt}(\mathcal{P}Ni)$$

$$= N^2\mathcal{P}\frac{di}{dt} = L\frac{di}{dt},$$



$$\phi_1 = \mathcal{P}_1 N_1 i_1,$$

$$\phi_{11} = \mathcal{P}_{11} N_1 i_1,$$

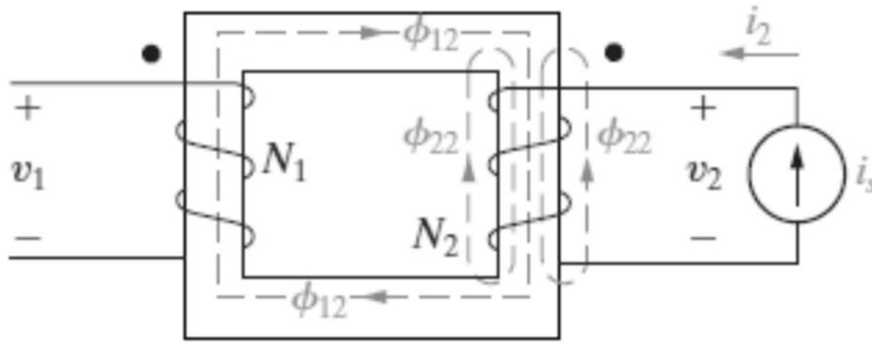
$$\phi_{21} = \mathcal{P}_{21} N_1 i_1,$$

$$v_1 = \frac{d\lambda_1}{dt} = \frac{d(N_1\phi_1)}{dt} = N_1\frac{d}{dt}(\phi_{11} + \phi_{21})$$

$$= N_1^2(\mathcal{P}_{11} + \mathcal{P}_{21})\frac{di_1}{dt} = N_1^2\mathcal{P}_1\frac{di_1}{dt} = L_1\frac{di_1}{dt},$$

$$v_2 = \frac{d\lambda_2}{dt} = \frac{d(N_2\phi_{21})}{dt} = N_2\frac{d}{dt}(\mathcal{P}_{21}N_1 i_1)$$

$$= N_2 N_1 \mathcal{P}_{21}\frac{di_1}{dt}.$$



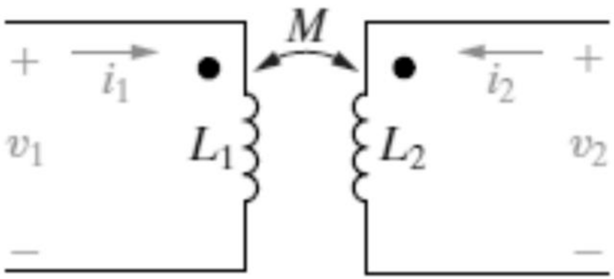
$$\phi_2 = \mathcal{P}_2 N_2 i_2,$$

$$\phi_{22} = \mathcal{P}_{22} N_2 i_2,$$

$$\phi_{12} = \mathcal{P}_{12} N_2 i_2.$$

$$v_2 = \frac{d\lambda_2}{dt} = N_2^2 \mathcal{P}_2 \frac{di_2}{dt} = L_2 \frac{di_2}{dt},$$

$$v_1 = \frac{d\lambda_1}{dt} = \frac{d}{dt}(N_1 \phi_{12}) = N_1 N_2 \mathcal{P}_{12} \frac{di_2}{dt}.$$

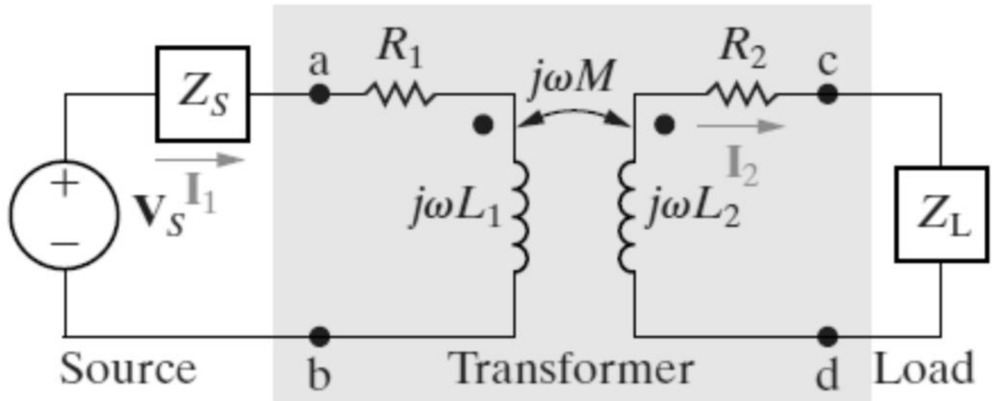


$$\int_0^{W_1} dw = L_1 \int_0^{I_1} i_1 di_1,$$

$$W_1 = \frac{1}{2} L_1 I_1^2.$$

$$\begin{aligned} W &= W_1 + I_1 I_2 M_{12} + \frac{1}{2} L_2 I_2^2, \\ &= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 I_2 M_{12}. \end{aligned}$$

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 I_2 M_{21}.$$



$$\mathbf{V}_s = (Z_s + R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2,$$

$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + Z_L)\mathbf{I}_2.$$

$$Z_{11} = Z_s + R_1 + j\omega L_1,$$

$$Z_{22} = R_2 + j\omega L_2 + Z_L,$$

$$\mathbf{I}_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2}\mathbf{V}_s,$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2}\mathbf{V}_s = \frac{j\omega M}{Z_{22}}\mathbf{I}_1.$$

$$\frac{\mathbf{V}_s}{\mathbf{I}_1} = Z_{\text{int}} = \frac{Z_{11}Z_{22} + \omega^2 M^2}{Z_{22}} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}.$$

$$Z_{\text{ab}} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s = R_1 + j\omega L_1 + \underbrace{\frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_{\text{L}})}}_{\textbf{reflected impedance } (Z_r)}.$$