- Ankara Üniversitesi BLM bölümü

BLM433 Sayısal Analiz Teknikleri

BLM433-2

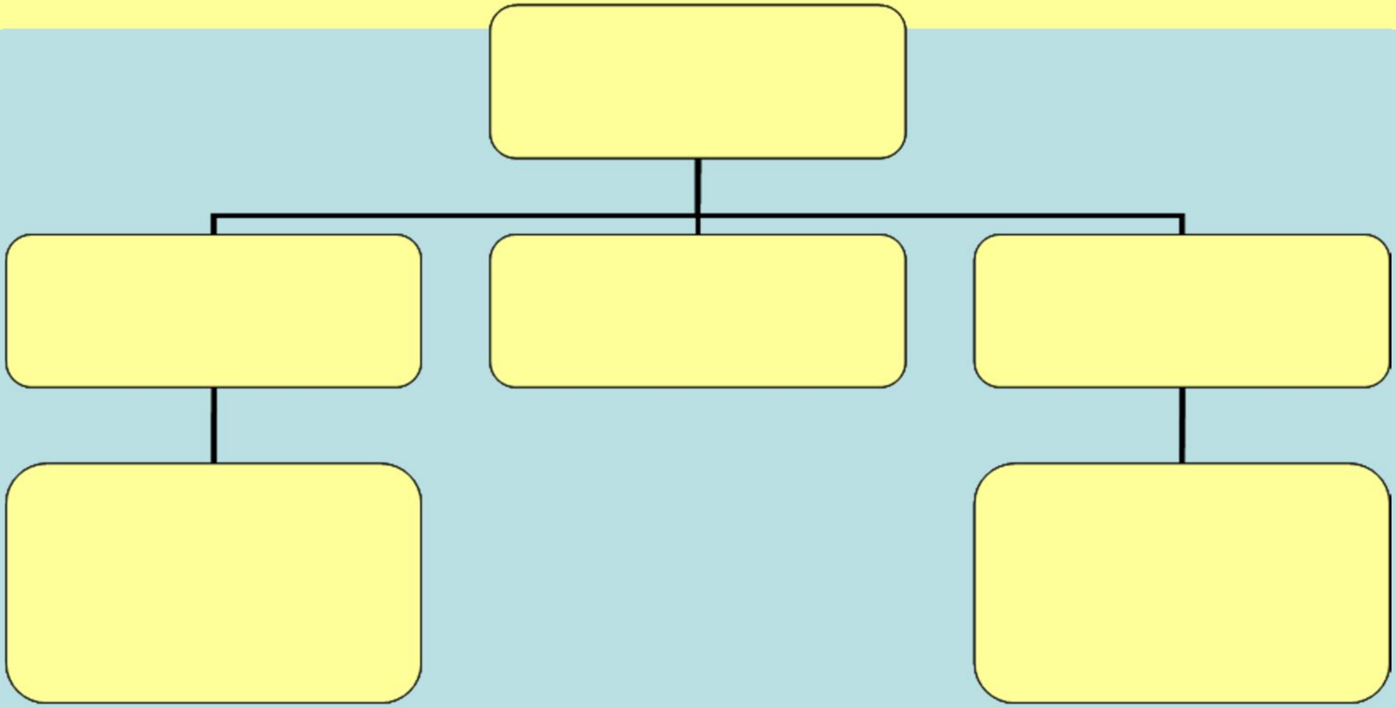
- Denklemlerin kökleri

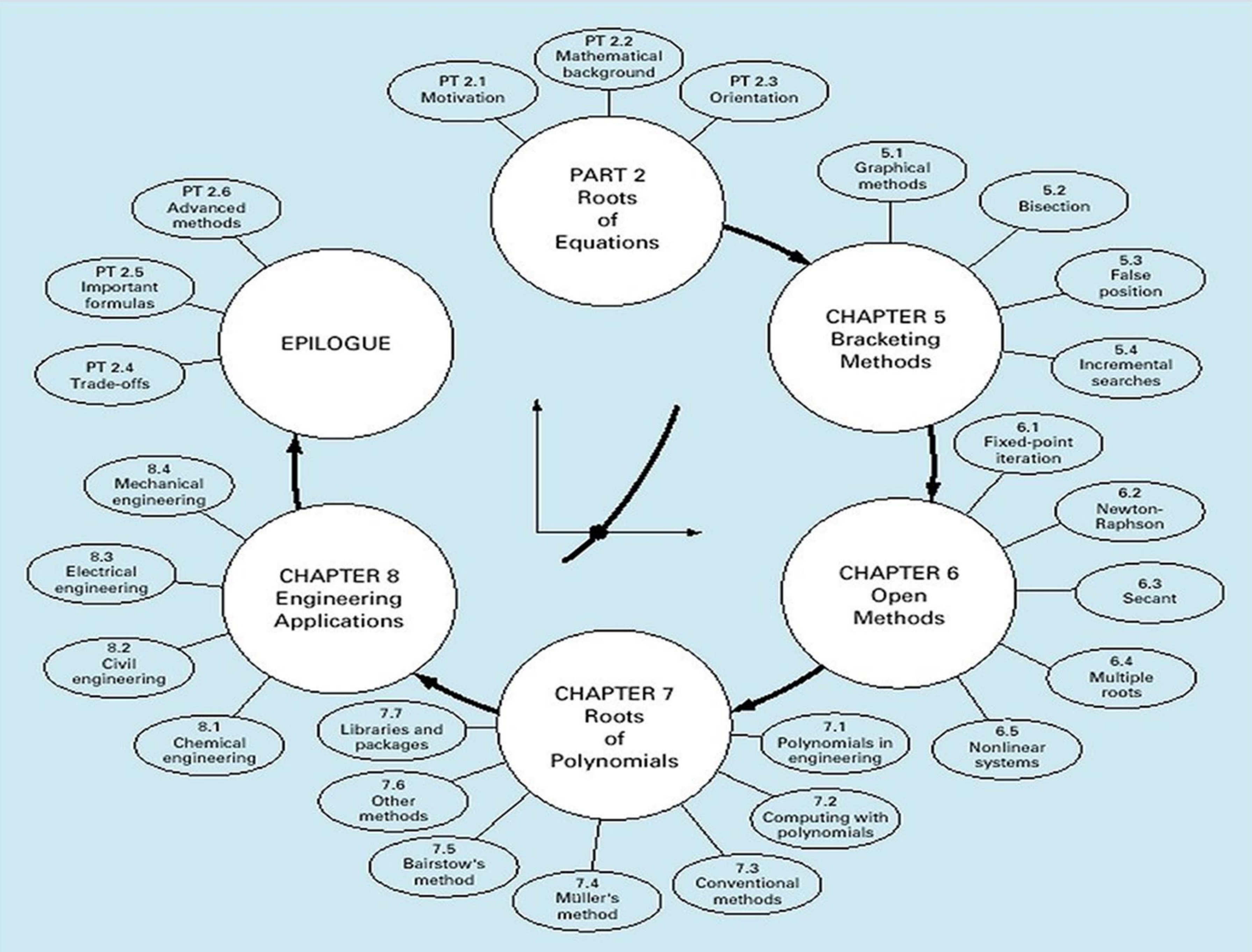
$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0 \quad \Rightarrow x = ?$$

$$\sin x + x = 0 \quad \Rightarrow x = ?$$

Denklemlerin kökleri

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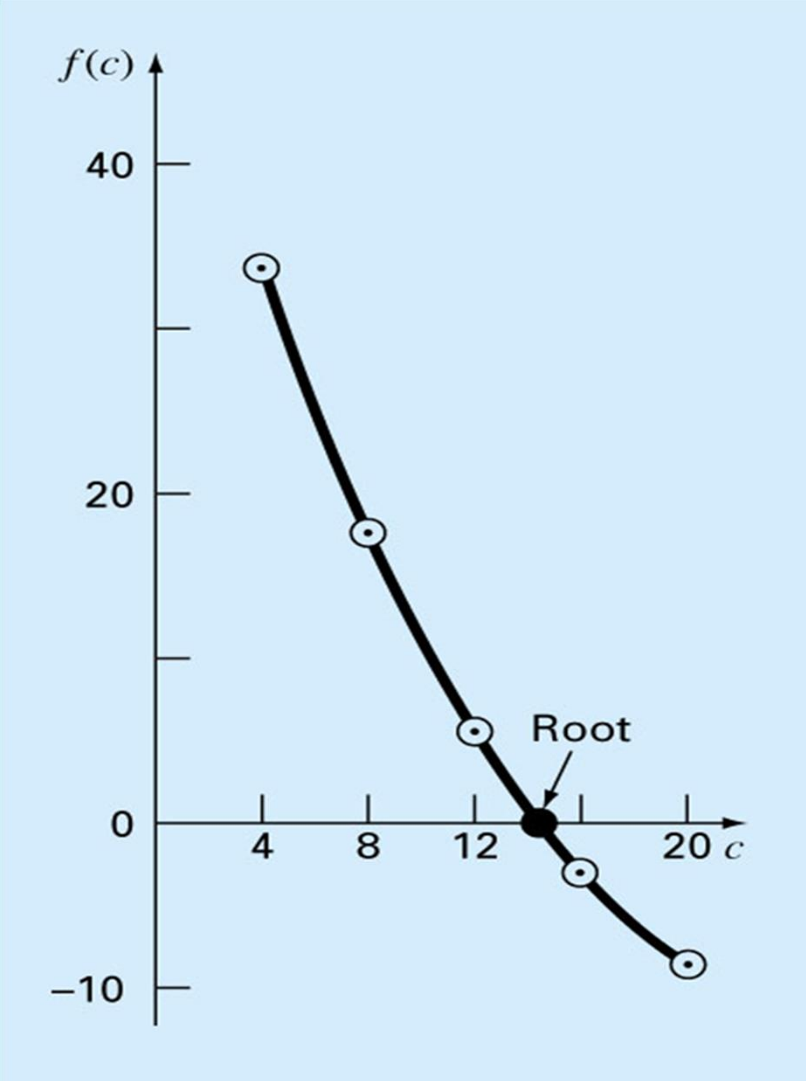
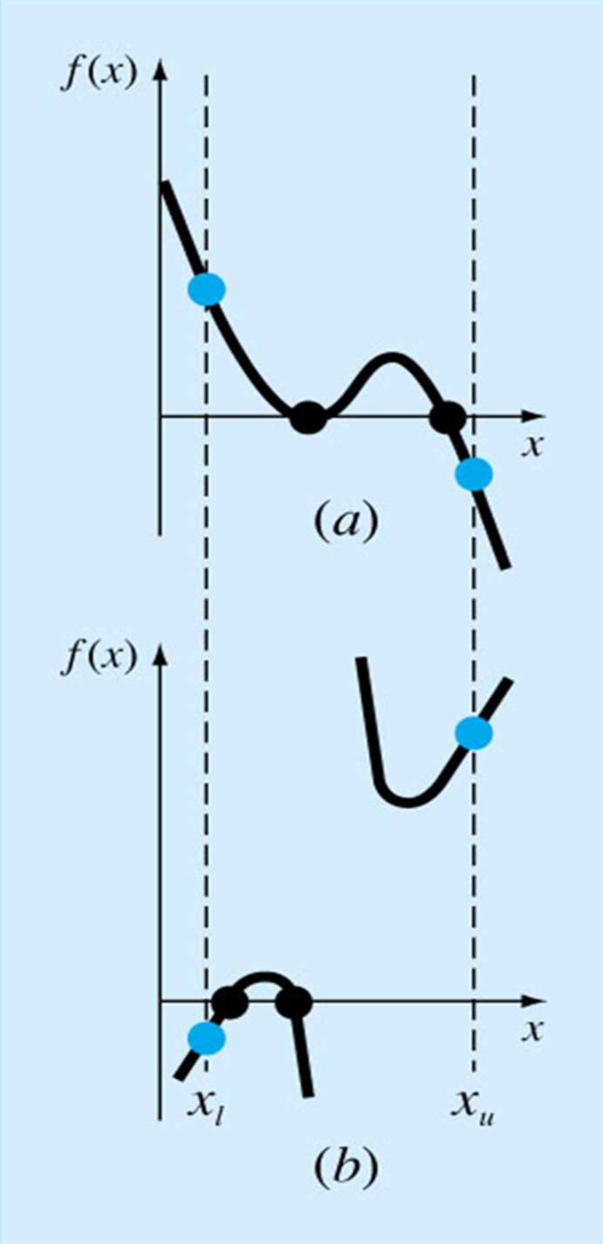
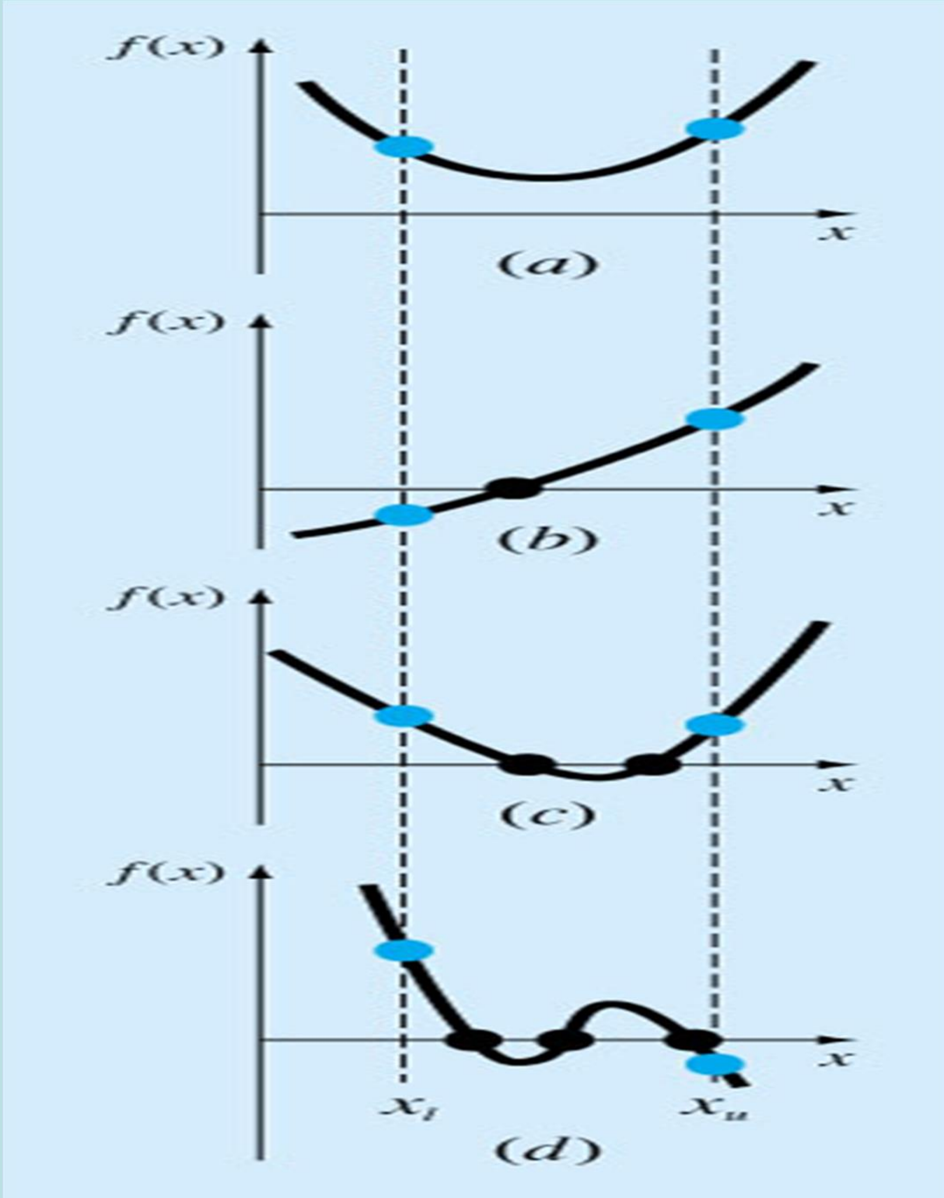
Bracketing Methods

f one root of a real and continuous function, f(x)=0, is bounded by values x=xl, x =xu then
f(xl) . f(xu) <0. (The function changes sign on opposite sides of the root)

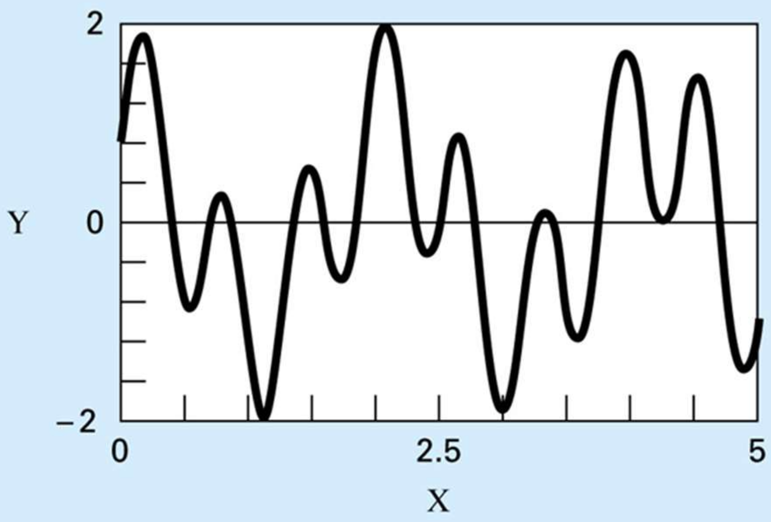


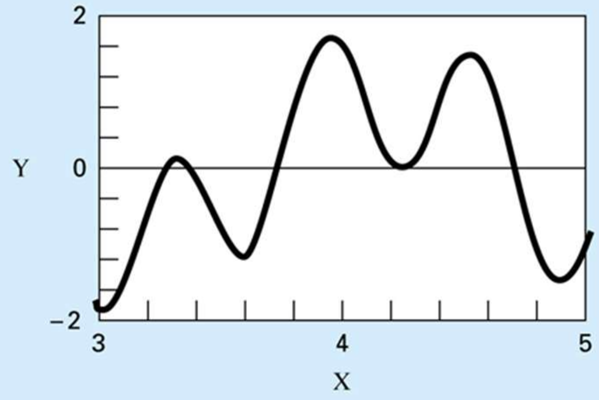


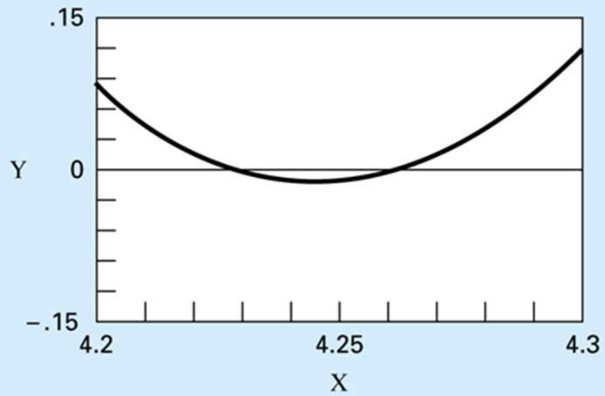
f(x)
x
(a)
f(x)
x
x_l
x_u
(b)



f(x)
x
(a)
f(x)
x
(b)
f(x)
x
(c)
f(x)
x
x_l
x_u
(d)


2
Y
0
−2
0
2.5
5
X


2
Y
0
−2
3
4
5
X


.15
Y
0
−.15
4.2
4.25
4.3
X

Bisection method

For the arbitrary equation of one variable, f(x)=0

- Pick xl and xu such that they bound the root of interest, check if f(xl).f(xu) <0.
- Estimate the root by evaluating f[(xl+xu)/2].
- Find the pair
- If f(xl). f[(xl+xu)/2]<0, root lies in the lower interval, then xu=(xl+xu)/2 and go to step 2

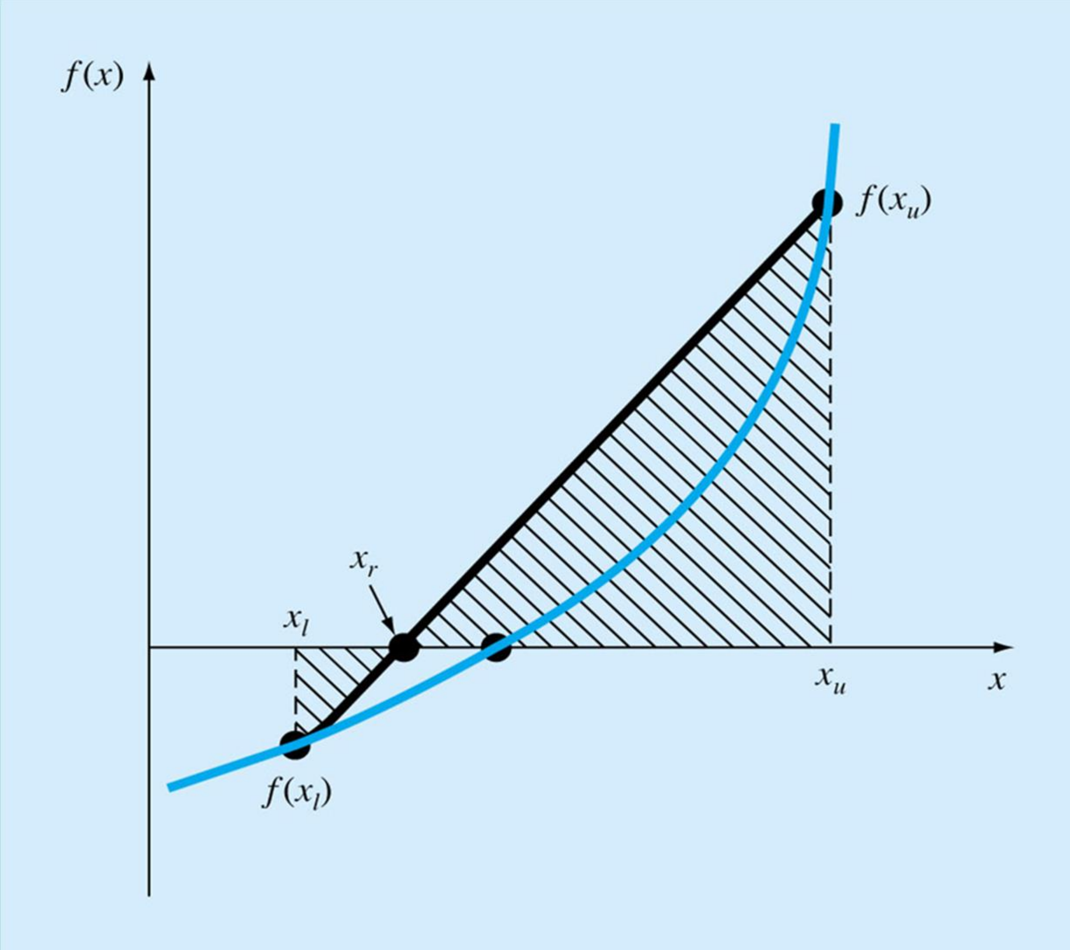
Number of steps

- Length of the first Interval Lo=b-a
- After 1 iteration L1=Lo/2
- After 2 iterations L2=Lo/4

- After k iterations Lk=Lo/2k

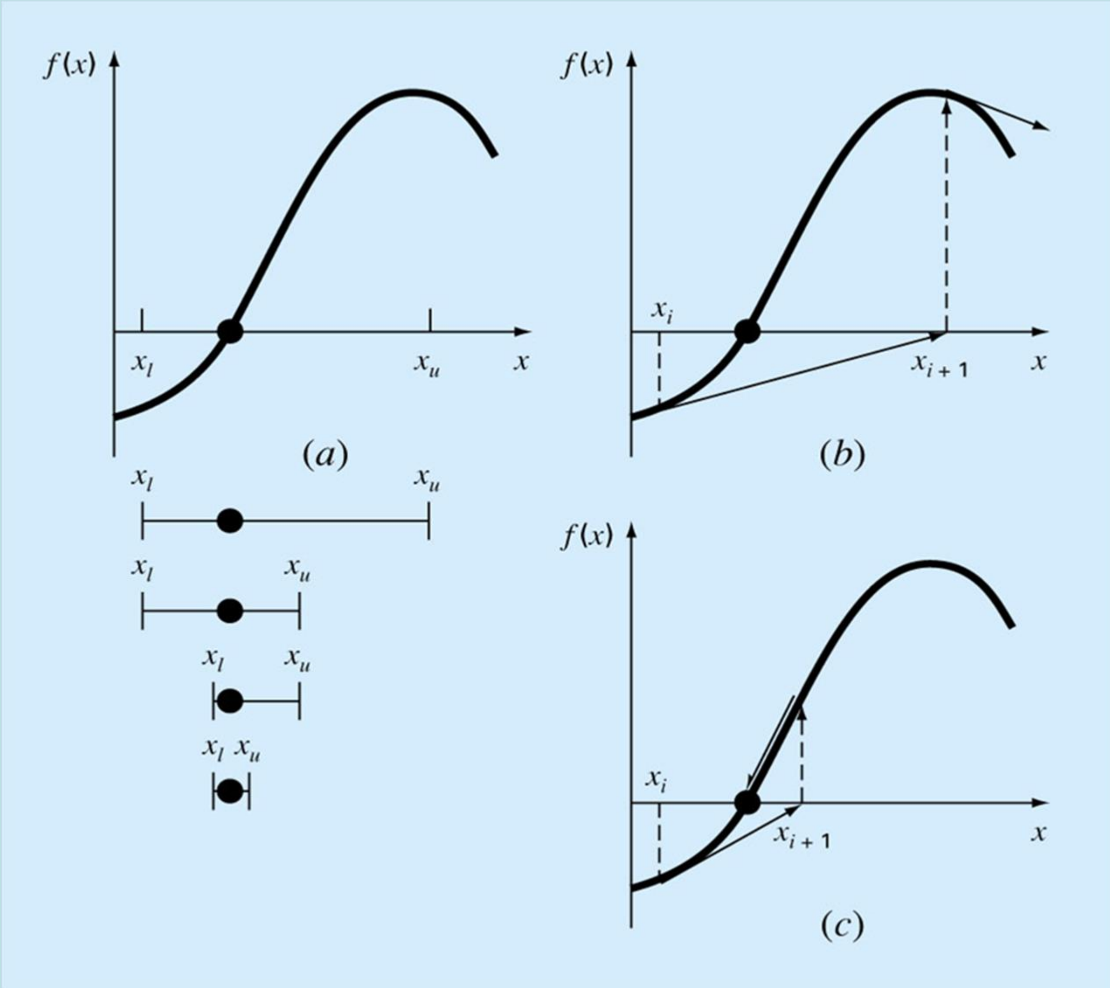
False-position method

$$x_r = \frac{x_l f_u - x_u f_l}{f_u - f_l}$$



Open methods

are based on formulas that require only a single starting value of x or two starting values that do not necessarily bracket the root



Fixed-point methods

may sometime "diverge", depending on the stating point (initial guess) and how the function behaves.

$$f(x) = 0 \quad \Rightarrow \quad g(x) = x$$

$$x_k = g(x_{k-1}) \qquad x_0 \text{ given, k} = 1, 2, \ldots$$

$$f(x) = x^2 - x - 2 \qquad x \succ 0$$

$$g(x) = x^2 - 2$$

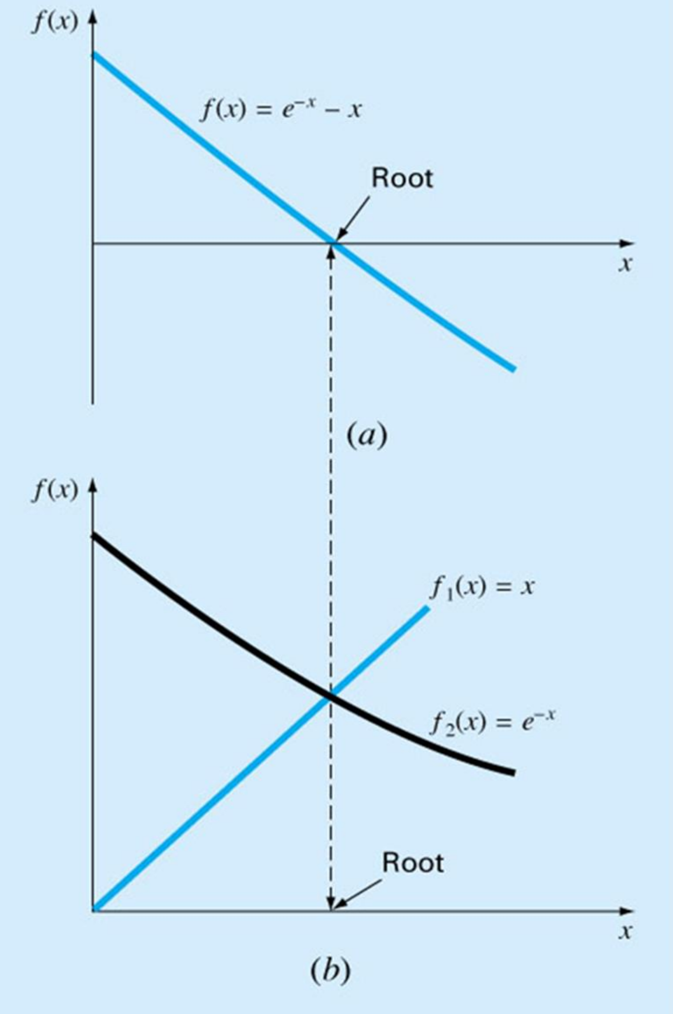
or

$$g(x) = \sqrt{x+2}$$

or

$$g(x) = 1 + \frac{2}{x}$$

$\vdots$



Newton-Raphson Method

$$f(x_{i+1}) = f(x_i) + f'(x_i)\Delta x + f''(x_i)\frac{\Delta x^2}{2!} + O\Delta x^3$$

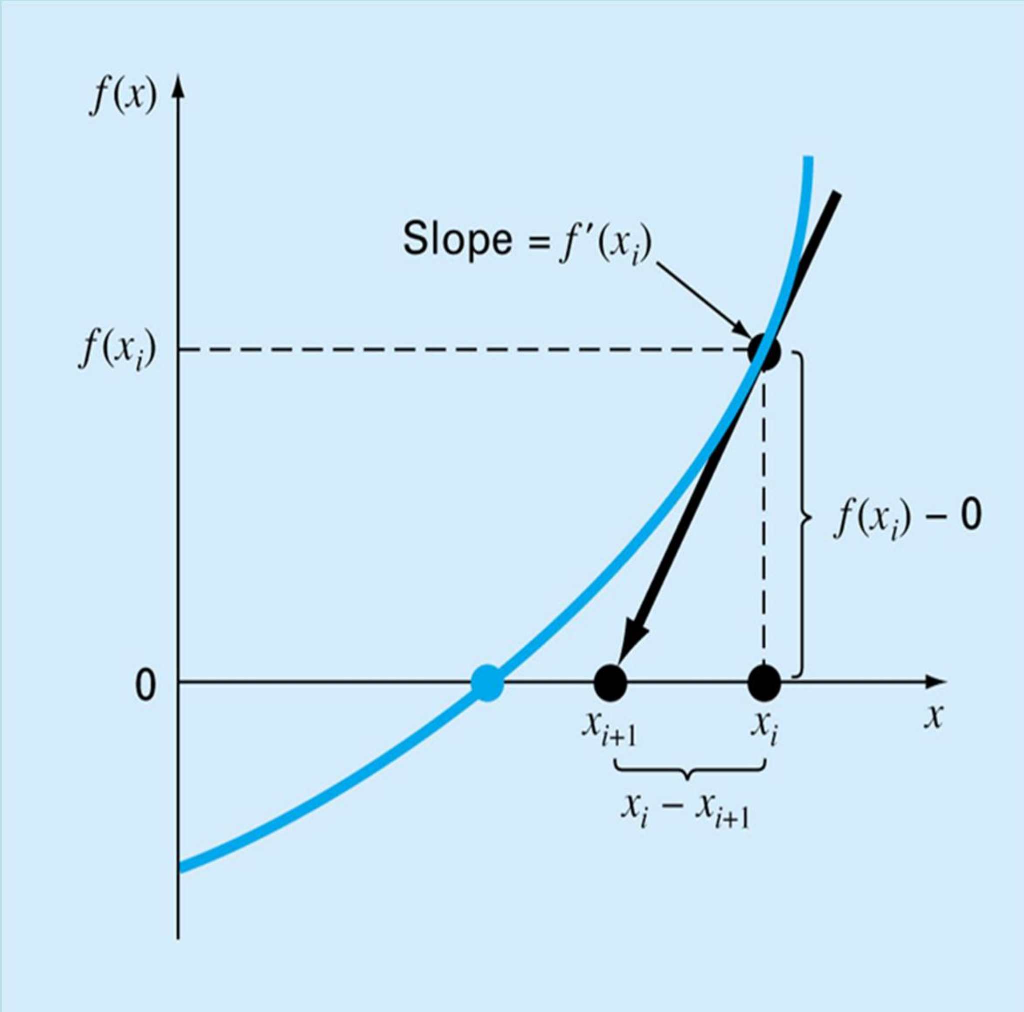
The root is the value of x_{i+1} when $f(x_{i+1}) = 0$

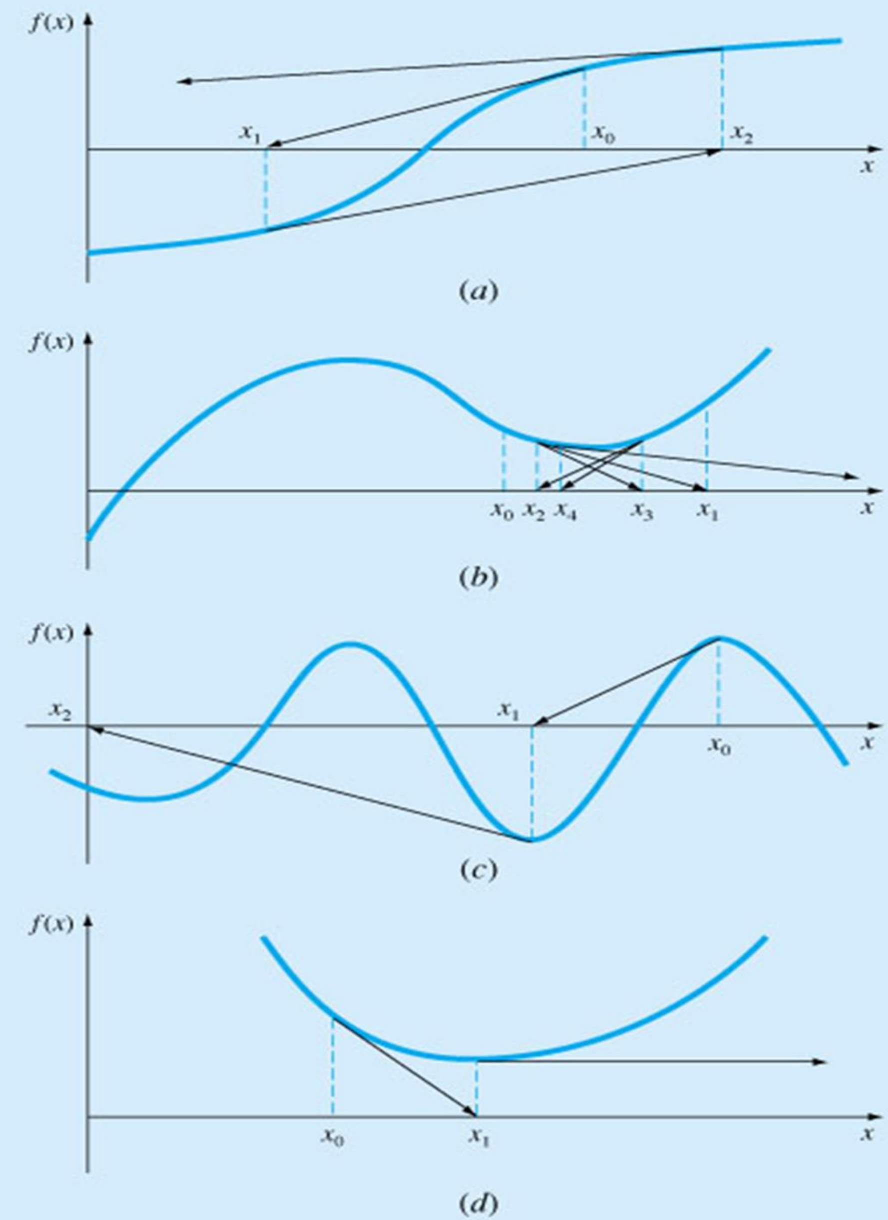
Rearranging,

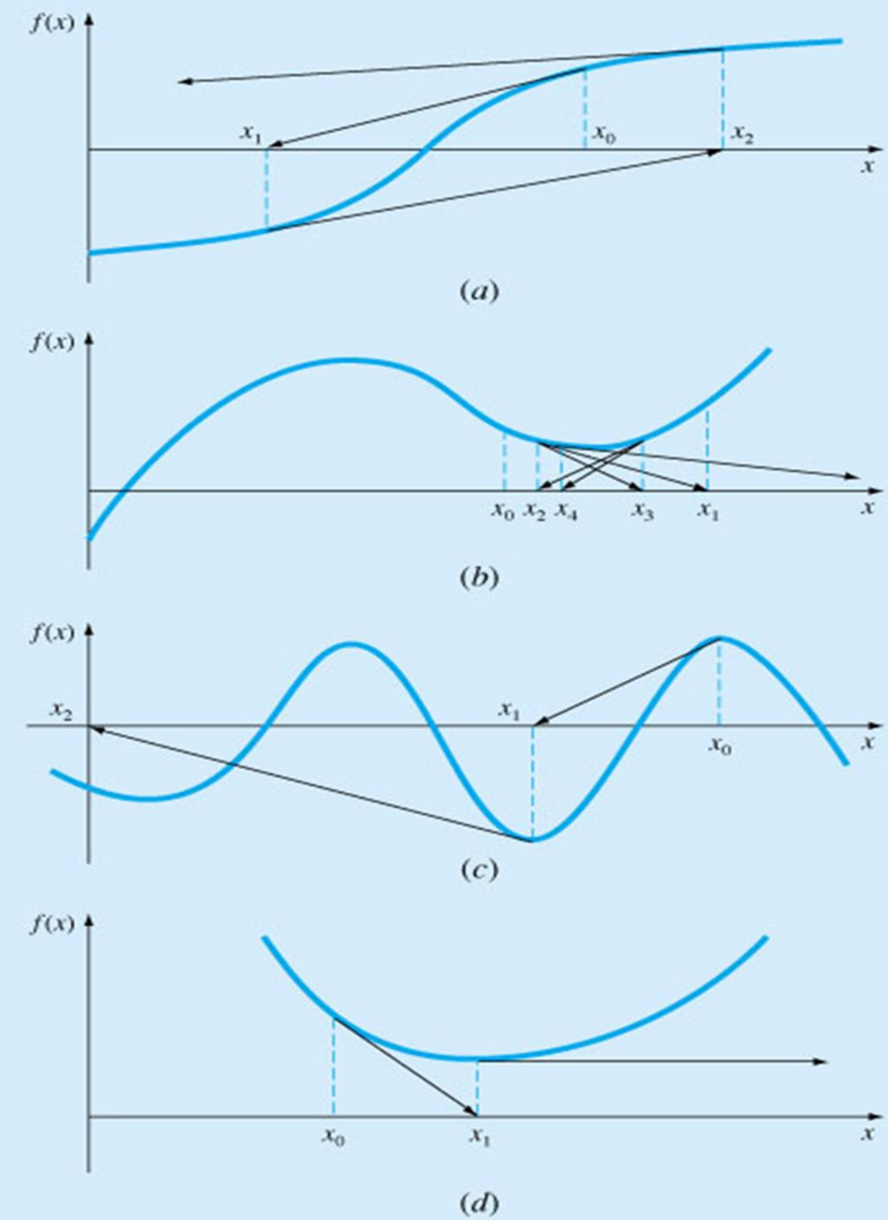
$$0 = f(x_i) + f'(x_i)(x_{i+1} - x_i)$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

A convenient method for functions whose derivatives can be evaluated analytically. It may not be convenient for functions whose derivatives cannot be evaluated analytically




f(x)
x
x1
x0
x2
(a)
f(x)
x
x0 x2 x4 x3 x1
(b)
f(x)
x
x2
x1
x0
(c)
f(x)
x
x0
x1
(d)

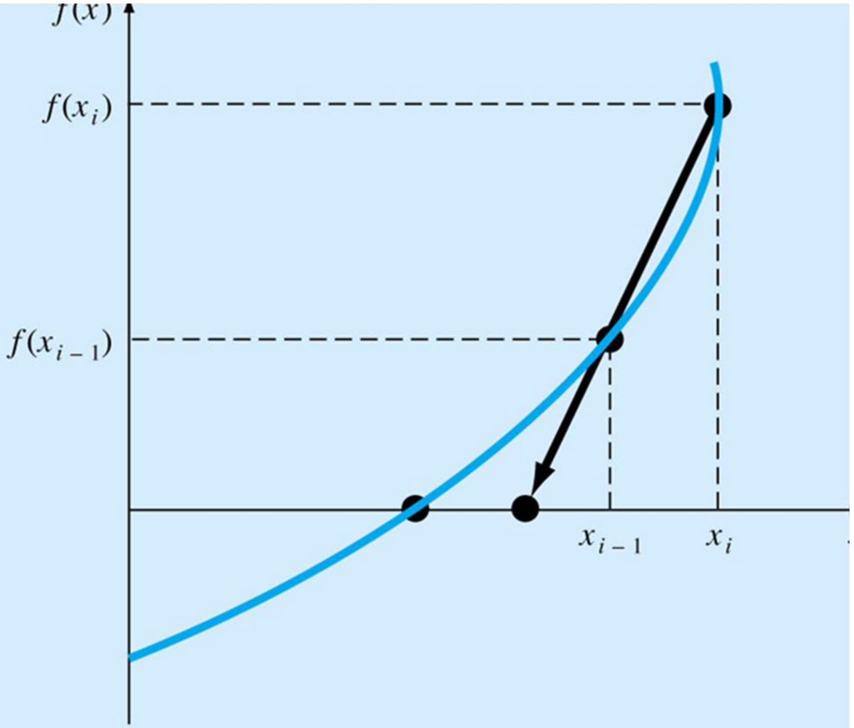

f(x)
x
x1
x0
x2
(a)
f(x)
x
x0 x2 x4 x3 x1
(b)
f(x)
x
x2
x1
x0
(c)
f(x)
x
x0
x1
(d)

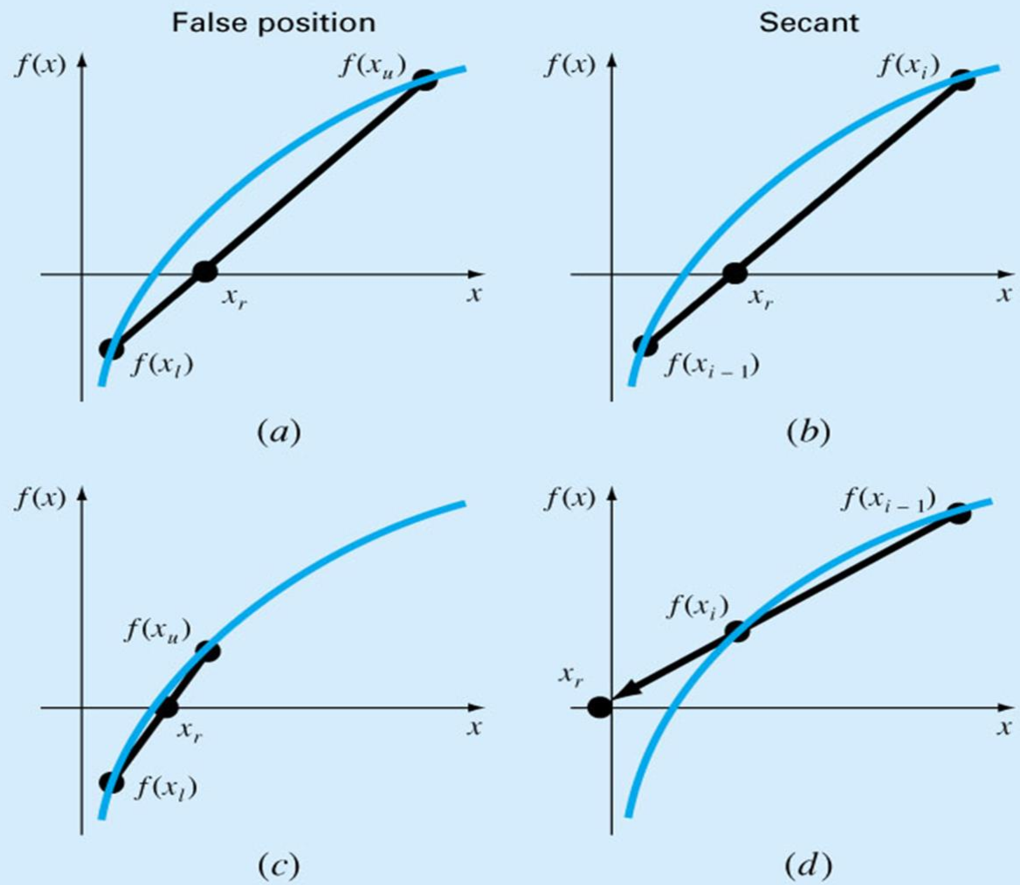
Secant method

A slight variation of Newton's method for functions whose derivatives are difficult to evaluate. For these cases the derivative can be approximated by a backward finite divided difference.

$$f'(x_i) \cong \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

$$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} \qquad i = 1,2,3,\ldots$$

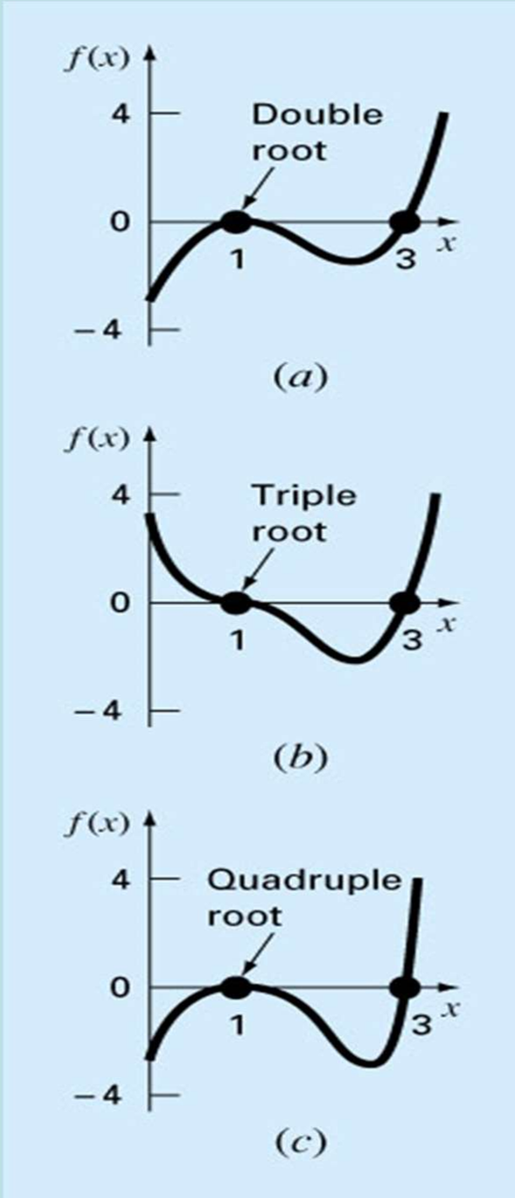

$f(x)$
$f(x_i)$
$f(x_{i-1})$
x_{i-1}
x_i


False position
Secant
$f(x)$
$f(x_u)$
x_r
x
$f(x_l)$
(a)
$f(x)$
$f(x_i)$
x_r
x
$f(x_{i-1})$
(b)
$f(x)$
$f(x_u)$
x_r
x
$f(x_l)$
(c)
$f(x)$
$f(x_{i-1})$
$f(x_i)$
x_r
x
(d)

Multiple Roots

Set $u(x_i) = \frac{f(x_i)}{f'(x_i)}$

Then find $x_i + 1 - \frac{u(x_i)}{u'(x_i)}$



Systems of Equations

$$f_1(x_1, x_2, x_3, \ldots, x_n) = 0$$

$$f_2(x_1, x_2, x_3, \ldots, x_n) = 0$$

$$\vdots$$

$$f_n(x_1, x_2, x_3, \ldots, x_n) = 0$$

$$u_{i+1} = u_i + \frac{\partial u_i}{\partial x}(x_{1i+1} - x_{1i}) + \frac{\partial u_i}{\partial y}(y_{i+1} - y_i)$$

$$v_{i+1} = v_i + \frac{\partial v_i}{\partial x}(x_{1i+1} - x_{1i}) + \frac{\partial v_i}{\partial y}(y_{i+1} - y_i)$$

$$\frac{\partial u_i}{\partial x} x_{i+1} + \frac{\partial u_i}{\partial y} y_{i+1} = -u_i + x_i \frac{\partial u_i}{\partial x} + y_i \frac{\partial u_i}{\partial y}$$

$$\frac{\partial v_i}{\partial x} x_{i+1} + \frac{\partial v_i}{\partial y} y_{i+1} = -v_i + x_i \frac{\partial v_i}{\partial x} + y_i \frac{\partial v_i}{\partial y}$$

$$x_{i+1} = x_i - \frac{u_i \frac{\partial v_i}{\partial y} - v_i \frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}}$$

$$y_{i+1} = y_i - \frac{u_i \frac{\partial v_i}{\partial x} - v_i \frac{\partial u_i}{\partial x}}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}}$$