Ankara Ü. BLM bölümü

BLM 433 Sayısal Analiz Teknikleri

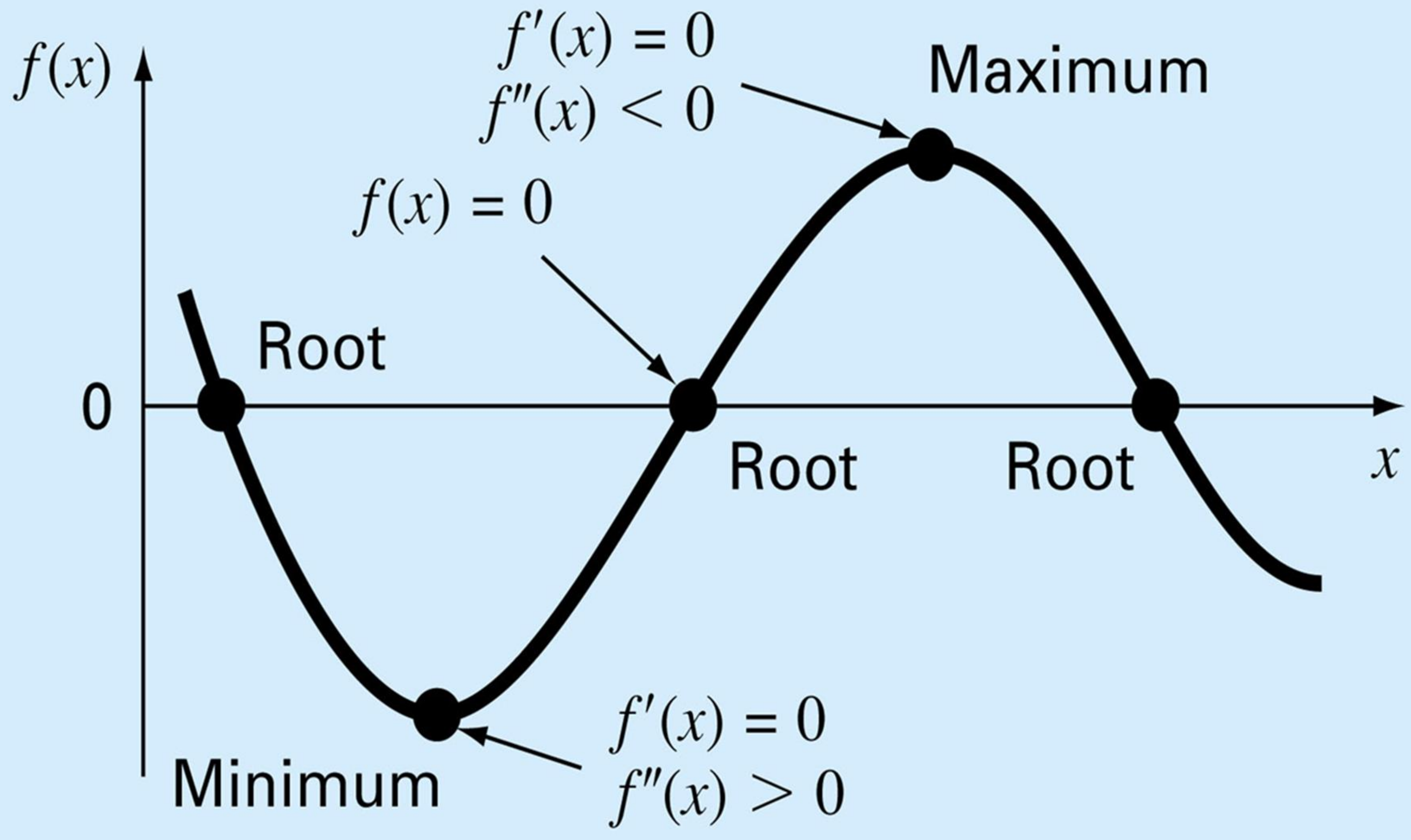
OPTIMIZATION

Root finding and optimization are related, both involve guessing and searching for a point on a function.

Fundamental difference is:

Root finding is searching for zeros of a function or functions

Optimization is finding the minimum or the maximum of a function of several variables.


$f(x)$
$f'(x) = 0$
$f''(x) < 0$
Maximum
$f(x) = 0$
Root
0
Root
Root
x
Minimum
$f'(x) = 0$
$f''(x) > 0$

Mathematical Foundation

$$d_i(x) \le a_i \quad i = 1,2,\ldots,m^*$$
$$e_i(x) = b_i \quad i = 1,2,\ldots,p^*$$

Where x is an n-dimensional design vector, f(x) is the objective function, di(x) are inequality constraints, ei(x) are equality constraints, and ai and bi are constants

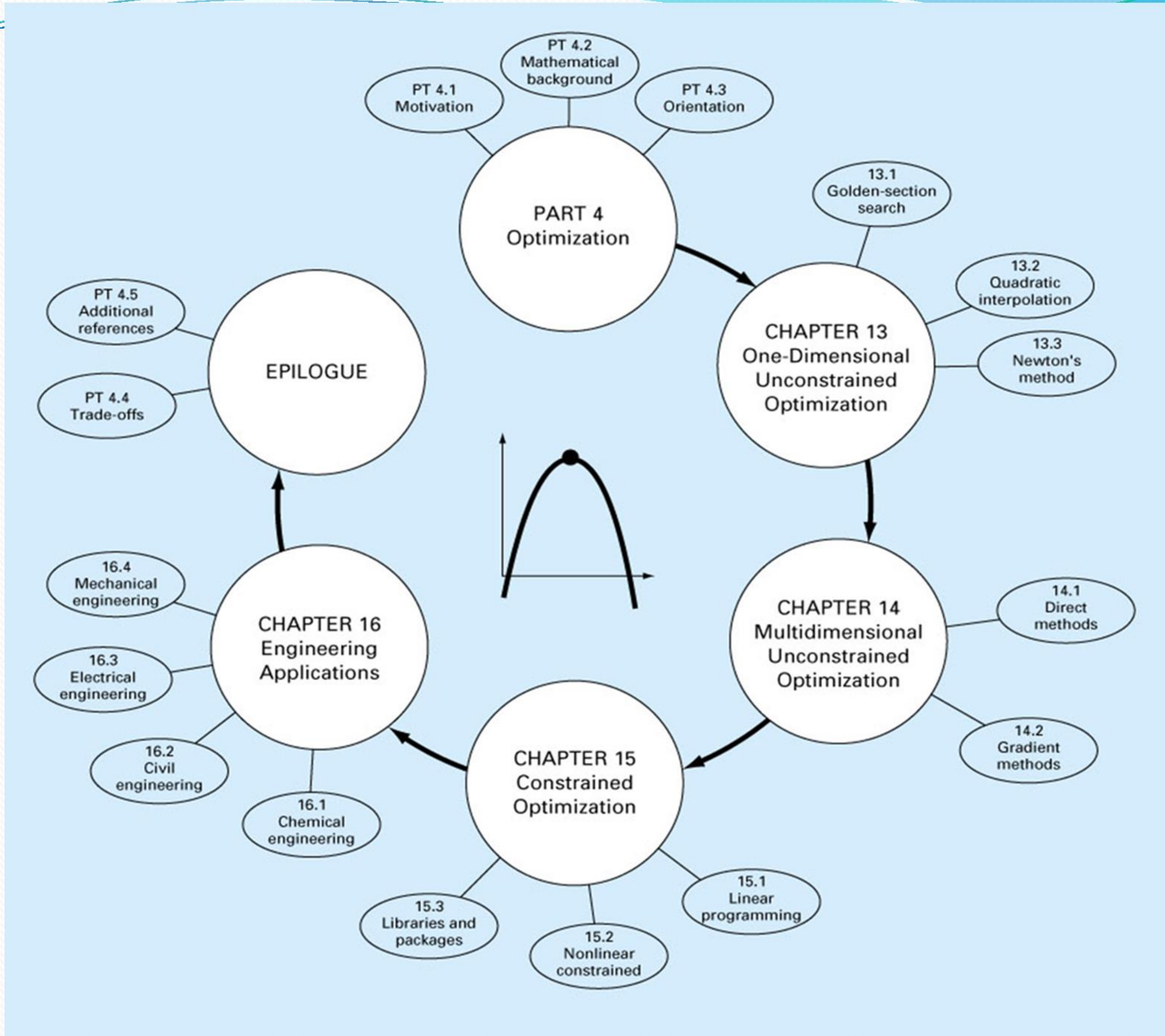
Optimization problems can be classified on the basis of the form of f(x):

If f(x) and the constraints are linear, we have linear programming.

If f(x) is quadratic and the constraints are linear, we have quadratic programming.

If f(x) is not linear or quadratic and/or the constraints are nonlinear, we have nonlinear programming.

When equations(*) are included, we have a constrained optimization problem; otherwise, it is unconstrained optimization problem.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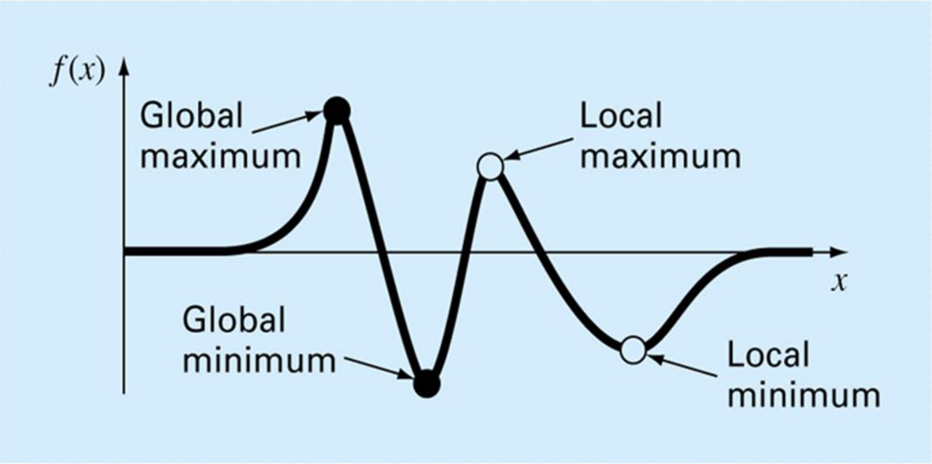
One dimensional optimization

In multimodal functions, both local and global optima can occur. In almost all cases, we are interested in finding the absolute highest or lowest value of a function.



By graphing to gain insight into the behavior of the

function.

Using randomly generated starting guesses and

picking the largest of the optima as global.

Perturbing the starting point to see if the routine

returns a better point or the same local minimum.

Golden section search

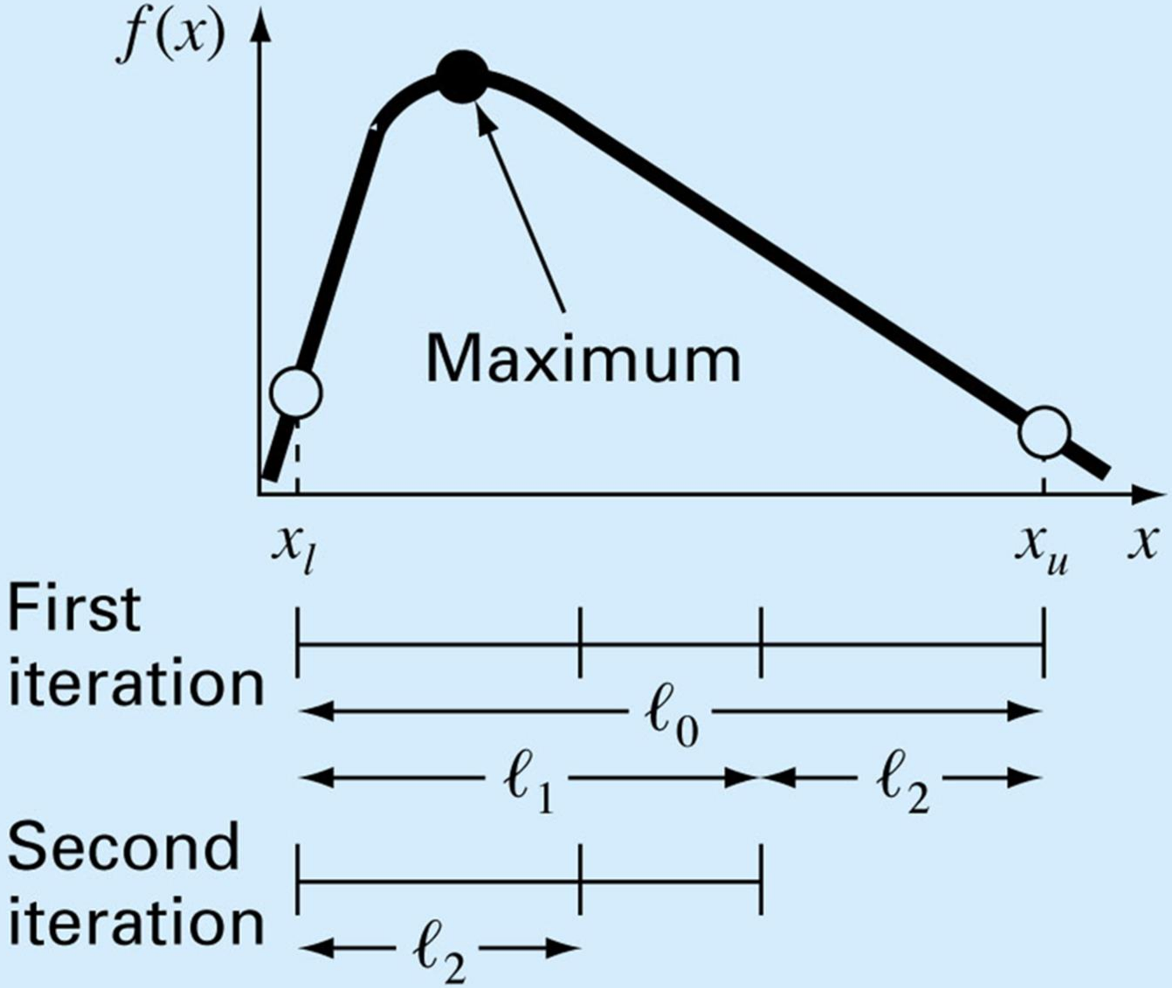
A unimodal function has a single maximum or a minimum in the a given interval. For a unimodal function:

First pick two points that will bracket your extremum [xl, xu].

Pick an additional third point within this interval to determine whether a maximum occurred.

Then pick a fourth point to determine whether the maximum has occurred within the first three or last three points

The key is making this approach efficient by choosing intermediate points wisely thus minimizing the function evaluations by replacing the old values with new value



f(x)
Maximum
x_l
x_u
x
First iteration
ℓ0
ℓ1
ℓ2
Second iteration
ℓ2

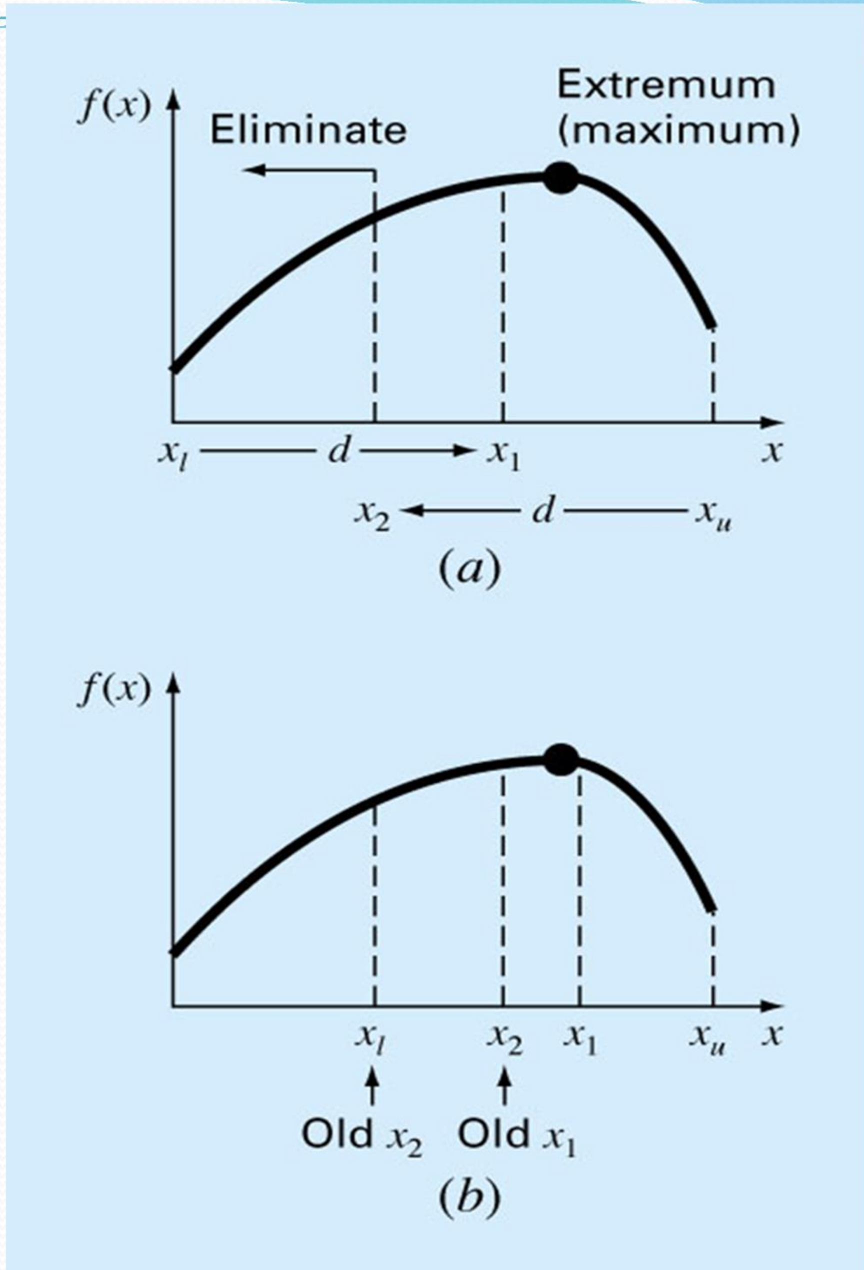
$$l_0 = l_1 + l_2$$

$$\frac{l_1}{l_0} = \frac{l_2}{l_1}$$

$$\frac{l_1}{l_1 + l_2} = \frac{l_2}{l_1} \qquad R = \frac{l_2}{l_1}$$

$$1 + R = \frac{1}{R} \qquad R^2 + R - 1 = 0$$

$$R = \frac{-1 + \sqrt{1 - 4(-1)}}{2} = \frac{\sqrt{5} - 1}{2} = 0.61803$$



f(x)
Eliminate
Extremum
(maximum)
x_l
d
x_1
x
x_2
d
x_u
(a)
f(x)
x_l
x_2
x_1
x_u
x
Old x_2
Old x_1
(b)

$$d = \frac{\sqrt{5}-1}{2}(x_u - x_l)$$

$$x_1 = x_l + d$$

$$x_2 = x_u - d$$

Two results can occur:

If f(x1)>f(x2) then the domain of x to the left of x2 from xl to x2, can be eliminated because it does not contain the maximum. Then, x2 becomes the new xl for the next round.

If f(x2)>f(x1), then the domain of x to the right of x1 from xl to x2, would have been eliminated. In this case, x1 becomes the new xu for the next round.

New x1 is determined as before

$$x_1 = x_l + \frac{\sqrt{5}-1}{2}(x_u - x_l)$$

Newton method

A similar approach to Newton- Raphson method

can be used to find an optimum of f(x) by defining

a

new function g(x)=f'(x). Thus because the same

optimal value x* satisfies both

f'(x*)=g(x*)=0

$$x_{i+1} = x_i \frac{f'(x_i)}{f''(x_i)}$$

We can use the following as a technique to the extremum of f(x).